Scholars Journal of Engineering and Technology
Abbreviated Key Title: Sch J Eng Tech
ISSN 2347-9523 (Print) | ISSN 2321-435X (Online)
Journal homepage: https://saspublishers.com/journal/sjet/home

Buckling Analysis of Piles: A Review

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DOI: 10.36347/sjet.2019.v07i11.001 | **Received:** 17.10.2019 | **Accepted:** 25.10.2019 | **Published:** 11.11.2019

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Abstract **Review Article**

Buckling is a phenomenon that occurs in columns and piles when the applied load exceeds a certain value (critical value), this leading to curvature of straightness. The study of buckling is very important for pile foundation to avoid structural failure and maintain the stability of the structure. The main objective of this paper is to review the behavior of piles subjected to buckling loads. Many researchers have studied several factors that influence the behavior of piles under buckling loads like the ratio between the length of the pile to the diameter, the characteristics of the soil and pile and the interaction between the pile and the soil, etc. Through this paper, it has been easier for researchers to review previous studies of piles behavior and to develop research in this field and taking into account the buckling loads in the design.

Keywords: Buckling load, pile behavior, critical buckling, Literature Review.

INTRODUCTION

Piles are used to transfer the structural loads from structure to the soil and subjected to horizontal, vertical, moment and buckling loads; mostly the piles were made from concrete, steel, wood or combined of these materials and driven in the soil. In the past, experimental and theoretical investigations were undertaken by many researchers to calculate the lateral load and vertical load for single pile or pile groups driven in the several types of soil. Structures like Bridges, Buildings, off-shore structure, towers, and Piers harbor are mostly built on the pile foundations. Many foundations were needed to resist horizontal loads and subjected to large buckling. This case of piles is partially free-standing frequently arise nowadays, the piles are exposed to buckling due to the length of the pile that unsupported, this also because of the eccentricity of the applied load. When the installation of the pile is vertically this may be difficult in practice, Model studies conducted on piles are relatively consistent parameter values are confirmed to those values presented by the author [2].

The current paper show all experimental and theoretical studies of piles subjected to buckling loads and the factors effected on the response of pile such as the end conditions, embedded length of pile and soil properties.

The Problem of Buckling

The response of long and slender piles subjected to axial compressive force is dissimilar from the response of normal piles. This response is defining as a buckling. Buckling has occurred when the axial compressive force on the pile head reaches to a certain value. The pile was bending out sideways at this value. Then, the force at which buckling happens was a design criterion for compression members. Generally, this force is called critical force (or buckling force). The straight configuration was the transition to the bent configuration because the straight configuration was stable at critical force [2]. A theoretical study of the critical force for long members was made by Leonhard Euler in [3]. The studies were depended on the differential equation of the elastic curve:

$$M = EI\left(\frac{d^2y}{dx^2}\right) \qquad ------(1)$$

The critical force for members with other end conditions can be represented in terms of the critical force for a hinged member, which is considered as a basic case. The critical force for a hinged-ends member is, therefore:

$$P = \frac{EI\,\pi^2}{L^2} \qquad ----(2)$$

The general formula of Euler's equation for other boundary condition at the bottom and top of the member is presented as:

$$P = N \frac{EI\,\pi^2}{L^2} \qquad ----(3)$$

Where,

P = Buckling force
EI = Stiffness of column
L = Length of column
N = Number of times the strength of hinged columns.
The limitation of factor (N) differs with end conditions as shown in Table-1.

Table-1: Limitation of Euler's Formula

End condition	N	Le
Fixed ends	4	$\frac{L}{2}$
One end fixed, the other hinged	2	0.7 L
Both ends hinged	1	L
One end fixed, the other free	$\frac{1}{4}$	2 L

If the piles are driven through soft soil to the firm soil at a small length below the ground surface the piles are considered as a short member. While if the piles are driven with large length through very soft water or soil, then the piles are considered as a long member. Then, the possibility of pile buckling must be taken into account as one of the failure modes [4].

Gramholm [5] is the earliest procedure to studies the problem of buckling. He observed that the driven pile has normal dimensions through soft soil, buckling most not occupied except in the very soft soil. However, with increased using the long piles and slender piles that extend for a large distance above the ground line, the possibility of pile buckling should have been reconsidered, a large quantity of research has been performed to estimate the buckling of the pile with more accurate. Subgrade reactions theory was employed as analytical methods, although the elastic-continuum theory was used afterward [4].

The subgrade-reaction solution was available analysis and described the elastic buckling force of partially and fully embedded piles are offered.

Fully Embedded Piles

For the slender and long piles embedded in extremely soft soils, the possibility of pile buckling must be taken into consideration, because the lateral support from the soft soil is incompetent to prevent pile buckling, so the investigator presented arithmetical solutions to estimate the critical pile buckling relies on the differential equation below:

$$E_p I_p \left(\frac{d^4 p}{dz^4}\right) + \frac{P\, d^2 p}{dz^2} + k_h d_p = 0 \quad -------- (4)$$

Where,

$E_p I_p$ = Flexural stiffness of pile.
k_h = Modulus of horizontal subgrade reaction.
d = Pile diameter or width.
ρ = Horizontal pile displacement.
z = Depth of pile from the ground surface.

Partially Embedded Piles

When the piles projected above the ground level for large distance, like similar structures and marine, piles buckling force should be taken into account. The competition is simplified by taken in account that the piles have the properties of a long compression member with a depth equal to the un-supported depth plus a depth from the ground line to a visible point of fixity (depth of fixity) [4].

LITERATURE REVIEW

This section presents a historical background regarding the studies dealt with the buckling analysis of the fully and partially embedded piles.

Granholm [6] study the effect of the driven pile in soft soil and showed that the buckling occurs in soft clay. The study suggested equation below for interaction between the pile and elastic soil.

$$\frac{1}{Pult} = \frac{1}{Ps} + \frac{1}{Pcr} \qquad (5)$$

Where,

Pult = Pile collapse loading,
Pcr = Elastic buckling loading
(*Ps*) = Short column loading capacity of pile section.

Timoshenko, P. S [7] discussed the buckling for pin end pile and give solution when no load- transfer occurs, equation (6) shows the buckling load.

$$\frac{Pcr}{PE} = \mathrm{m}^2 + \frac{\beta^2}{m^2} \qquad (6)$$

Where

PE = Euler load of a strut in air $= \frac{\pi^2 E_p I_p}{L^2}$
$\beta^2 = \frac{k_h\, d\, L^4}{\pi^4\, E_p I_p}$
L = pile length.
EPIP = Flexural stiffness of pile.
k_h = Modulus of horizontal subgrade reaction.
d = Pile size.
m = number of buckled half-waves.

The (m)-value is obtained at the condition that (*Pcr*) is a minimum. Thus, when (*β* = 0), (*Pcr*) is a minimum for (m =1) and (*Pcr* = *PE*). As (*β*) increases, that is (*kh*) increases, the number of buckled half-waves, and hence (*Pcr*), increases.

Brandtzaeg and Harboe [8] implemented many experimental to study the buckling for many piles embedded in various soil and pulled the pile after the test. The test showed that the buckling occurs in the top zone of soil, the axial load is transmitted to the soil by adhesion for all type of soil

Bjerrum [9] suggested that buckling occur if

$$I_p/_{A^2} = \frac{\sigma_{max}^2}{4\ k_h E_p\ d} \quad (7)$$

Where,

σmax = yield stress of the pile
A = cross-sectional area of pile
the $I_p/_{A^2}$ was calculated for several steel pile sections embedded in soft clay.

Davisson [10] discussed the buckling loads for pinned and fixed pile head condition and uniform subgrade modulus by using Klohn and Hughes [11] study. The obtained results were compared to experimental results from a full-scale pile loading. The fixity pile head gives significantly increase in the buckling resistance other than pinned head condition. Also, studied the effect of buckling for a group of timber piles (2 x 2), and shows the failure of buckling occurred suddenly and without obvious warning but the failure occur progressively for single pile under structural load.

Davisson and Robinson [12] calculated the buckling load and fixity depth for pile under axial load by using equivalent scale and partially embedded pile, by solving the equation for the free-standing pile length and used Toakley [13] method for selecting the axially loading. The study used a series of trigonometric functions to represent the buckling shape, used three functions to represent the difference in an axial load along the pile length and taken two types of the modulus of subgrade reaction linearly varying and constant. Its showed that the end fixity is effected on the buckling load, for small values of (L/l').

Lee [14] carried a series of buckling loads test for pile diameter (0.25 to 0.5 in) embedded in dry sand to study the validity of an equation released by Davisson and Robinson [12] for calculating the critical buckling for the partially embedded pile. The results were acceptable when compared with the appropriate predicted values by Klohn and Hughes report [11].

Valsangkar & Reddy [15] study the influence of the axial load distribution for many boundary conditions and used the Rayleigh-Ritz energy method for beam vibration function. The results showed that buckling load increased because of the skin friction of pile for fully embedded piles. For parabolic axial load distribution, the buckling load was increased less than the linear distribution. The skin friction is very small and can be negligible for the partially embedded pile.

Sovinc [16] was investigate the buckling of piles contain initial curvature due to heavy axial load and the have increased in modulus of subgrade reaction with depth. The investigation computed the critical buckling load for steel pile embedded in elastic-plastic soil. The calculation was checked with a full-scale test (40 m pile) and have agreement results. For pile have initial curvature the buckling is increased rapidly, and the bending moment not affected by soil properties.

Prakash and Sharma [17] used energy methods to obtained solutions for fully embedded vertical piles under buckling loads and fixed top-fixed tip and pinned top-pinned tip boundary conditions. Many effects of factors were studied such as boundary conditions, soil stiffness, and pile length. The result was calculated the critical load for the smallest eigenvalue of the leading principal submatrix based on energy method. Generally, the buckling load was in increased with the increase in the value of (k_0) for constant (n_h) values and with the increase in the value of (n_h) for variable modulus.

Bowles [18] developed a formula suggested by Wang [19] to calculate the buckling load of columns with variable cross-sections. A computer program was simulated to calculate the buckling load for partially and fully embedded. Skin friction effected was considered in the analysis.

Fenu and Serra [20] presented a solution of buckling in pile embedded in much elastic soil layer modeled as Winkler soils. Fourier series was used to represent the distribution of subgrade reaction modulus in each layer. Also, the pile was considered as a compressed column surrounded by springs. The results approved that the buckling load affected by the distribution of soil layers and boundary conditions. when a soft layer over the stiff layer makes that the stiffness was less important.

Al-Obaydi [5] performed an experimental investigation for calculating the critical buckling loads for partially and fully embedded pile in sand soil. pile embedded length, pile upper-end conditions, pile unsupported length, soil density, and type of loading were considered in the study. The results were presented in a dimensionless form and compared with many theoretical methods.

Gabr *et al.*, [21] estimated the buckling load of pile considering skin friction by the developed theoretical model. Rayleigh-Ritz method and minimum potential energy method was taken in selecting the suitable deflection. A six pile with different boundary conditions were used with linearly increasing in subgrade reaction modulus. The result shows that the boundary conditions are the effect on the critical buckling loads when pile length exceeded critical values, especially in the pile head for partially embedded and fully embedded piles. The equivalent buckling length was increased and the axial load to cause buckling load was decreased when decreased the embedment ratio. The influence of embedment length on the buckling loads of partially embedded piles was large than fully embedded piles.

Jwad, Z. M [22] Discussed the behavior of slender piles embedded in weak soil under buckling for fully and partially boundary condition. Winkler elastic model was used to represent the soil and elastic spring was used to model the soil reaction. at first, the linear small-angle bending theory was used to computed the buckling and the result gave agreement value compare with other studies. After that, two indications for the curvature were used at the large-angle to model the response of the free-standing part of a partially embedded pile. The relation between the curvature, the horizontal (lateral) deflection and the vertical axis was written. Also, the relation between lateral deflection along pile length was explained.

Wa'el M. AL-Bawwab [23] carried out a test to study the behavior of single pile embedded in soft clay under critical buckling load. The experimental consist of twenty pile under bulking load in free air, twelve pile used to balance the experimental equipment, eighteen pile under the lateral loading to calculated (n_h), and twenty-four pile under buckling long the piles, all piles were fully embedded into soft clay. the analytical solution revealed that for piles having (99 ≤ L/r ≤ 165) cannot ignore the danger of buckling because of the buckling point less than its yield stress. But the buckling can be ignored for (L/r > 165) if (P_{cr} / P_E) is more than (F_y/α_E) because of the pile yield before reaching to the critical buckling.

Gabr *et al.*, [24] carried out many tests for the slender pile to evaluate the critical buckling, the soil was support horizontally based on the subgrade reaction. The coefficient of the subgrade reaction (k_h) was distribution along the pile length. Linear (p-y) behavior was assumed to represent the lateral load-deflection. For fully embedded pile and free at the top and the pile length greater than (10 m), the buckling load (Pcr) was increased about (59%) when change the subgrade reaction from constant to linearly increasing. The test shows that the boundary conditions at the bottom of pile have the minimum influence of (Pcr) when the pile depth exceeded (3.3) for free pile top, (7.6) for the pile pinned-top and (5.6) for the pile fixed-sway top condition.

Howard A. P. and Magnum P [25] Used Lpile software to investigate buckling resistance and the lateral strength of helix footing in different soil conditions. Using fixed rotation to modeled the performing analysis, also the buckling resistance was estimated as a free translation end conditions for pinned end pile conditions. The buckling resistance was found to impart limits on the permissible vertical load that can be put to standard pipe, high strength structural tube and solid square shaft in very loose to loose sands and very soft to soft clays. For competent soil conditions, the buckling was not important. Buckling was found to occur over a (7 ft to 12 ft) long section of shaft regardless of the remaining length of the shaft within the same weak soil stratum. The results were compared with other using underpinning bracket reaction and have a good agreement computation.

Heelis *et al*., [26] presented a non-dimensional solution for the beam-column pile embedded in the Winkler foundation. The beam-column was supported by end-bearing and skin friction that increase along the pile length. The analytical results were compared with experimental studies of the problem and discussed the variation between them. It was provided that more general and overcomes many of the restrictive methods inherent in previous work.

Hughes *et al.*, [27] discussed the influence of extreme scour and summarized the studies of the influence of the modulus of subgrade reactions of soil on bridge pile pushover capacity and bent buckling. The analysis showed that pushover loads and bent buckling were not sensitive to the modulus of subgrade reactions (k_0) unless when ($k_0 \leq 1.36\, N/cm^3$). Also, the result showed that the pile fixity can be assumed about (1.5 m) under the soil surface unless (k_0) is very small. At last, it concluded that the result of the critical buckling calculated by (FB-Pier Program) and Granholm equation have a good agreement when compare with the predicted equation below.

$$\left(P_{cr} = \frac{2\pi^2 E_p I_p}{L^2}\right) \qquad (8)$$

Zou *et al*., [28] discussed the stability of bridge piles consist of high-rise cap under buckling, that used in harbor engineering. also, discussed the limitations and advantages of traditional analytical approaches. Then, presented a new method to consider the nonlinear interaction between pile and soil based on the nonlinear finite element method and stability problem. ANSYS software used to discuss the effect of different factors on the critical buckling campsites (Pcr). It's observed that, the critical buckling decrease about 32.5 % when increase the free length of the top pile from (2.5 m) to (5.0 m), similarly the increasing location and degree of starting bending and necking along the pile length have same change. Moreover, when the inclination degree is increased would result in an obvious linear cut down of (Pcr). Finally, the method was used to balance the data that get from a site loading, a good agreement results between the measured failure load and the calculated buckling load.

Feng *et al*., [29] performed a three-dimensional series analysis by used finite element to study the behavior of partially embedded piles with the diameter (0.8 m) in elastic soils. The numerical results were compared with other analytical methods. The result shows that the pile embedded length does not influence on eigenvalue critical buckling of partial condition. When the buckling was exceeding the critical evaluate, which was relies on subgrade modulus of interaction and the flexural rigidity of piles. the critical length for (η = 3.0) was recommended by equation (9)

$$.L_c = \eta\sqrt{\frac{E_p I_p}{k_s}} \qquad (9)$$

Equation (9) was estimated by Davisson and Robinson [12] to calculate the eigenvalue buckling capacities of a partially pile condition when buckle happened at the pile. When the critical length is more than the embedded part then the eigenvalue buckling equation was more complicated.

Zhou *et al.*, [30] Studied the influence of different modulus of subgrade reaction on eigenvalue buckling capacities with different ratios of pile embedment, ABAQUS program used to model the problem, the beam and spring model used to represent the pile and soil system respectively. Analytical methods were used to estimate the buckling capacities of the partially embedded pile, the Winkler foundation was used with linearly increasing and constant subgrade reaction. Non-dimensional parameters were used to dealt with the numerical results and presented in the (J_R-S_R) curves. The analytical results showed that the existence of turn points in the curves which separated the curves into flat parts and steep parts.

The critical value (J_{Rcr}) of turn points was influenced slightly by distributions of modulus of subgrade reaction but differ with the ratios of embedment piles. The parameters (J_R) were related to The buckling modes of piles. When the value of (J_{Rcr}) is greater than the value of (J_R) for a given embedment ratio. The deflection has happened along the un-embedded part of the pile and the most embedded part, this meant that the soil around the pile cannot support the pile laterally. When the value of (J_{Rcr}) less than (J_R), the deflection was limited by the un-embedded part of the pile. Also, the study concluded that the Tip resistance and skin friction have a small influence on the eigenvalue buckling capacities of piles.

Wei Lu and Dong Zhao [31]. Used energy method to studied the buckling stability of steel piles embedded with several soil layer and described the properties of soil around the pile. The constraints were simplified at the bottom of the pile by fixing the upper free. The interaction between soil and pile was considered in the account, the calculated pile length was analyzed, and the pile length equation was used in layered and uniform foundation, also explained the equation with examples. The analytical equations are very simple, defines all parameters, easy in used and can be considered ad a reference to calculated pile length on foundation work.

Walid El. and Hany El [32] developed the three-dimensional analytical model by used finite difference method to investigate the response of end-bearing piles under buckling. At the begging, the buckling was calculated and verified by the used theoretical solution during an idealized Euler's method. After that, a parametric study was carried out to investigate the influence of the flexural pile stiffness and the soil stiffness on the response of buckled end bearing pile at unloading and loading conditions. The results appeared that the flexural pile stiffness and the soil stiffness have a large effect on the buckling load and buckling length. The stiffness of pile flexural should be relatively close with soil stiffness to avoid large values of horizontal displacement on the pile length.

Magdy I. and Ali M [33] developed the elastic stability model for partially embedded steel piles under to axial load and taking into account all the parameters that influencing of the buckling capacities. These parameters are the modulus of elasticity of soil, the ratio between embedded length and pile length, the stiffness of flexural and overall length of the pile. The minimum potential energy method and the finite element method are carried out to calculate the buckling capacities and effective length of the pile. Elastic media was used to represent the soil. The result was being presented in charts to show the influence of the parameters on the effective pile length. The results of the finite element method and the energy technique method was compared with the experimental test to obtain the coefficient of effective pile length.

CONCLUSION

From previous studies above, the studying of the response of piles under buckling loads (theoretically and experimentally) begging by Gramholm at 1929 and it continues to this moment. it was suggested more mathematical equations for calculating the buckling capacities of piles. All research in this area has taken many factors that influence into the behavior of piles under buckling loads, such as the characteristics of the soil and pile, diameter/length ratio of the pile and the interaction between the pile and the soil, etc. There is still more scope to study the behavior of piles under buckling loads.

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