Scholars Journal of Engineering and Technology (SJET) **ISSN 2347-9523 (Print)**
Abbreviated Key Title: Sch. J. Eng. Tech. **ISSN 2321-435X (Online)**

A Unit of Scholars Academic and Scientific Society, India
www.saspublishers.com

The Solutions to a Fractional Nonlinear Burgers Equation Modeled Liquid Phenomenon

Xin Wang
402 Team, the Fourth Oil Production Branch, No. 3 Oil Production Company, Daqing Oilfield Company Limited. Daqing 163113, China

Original Research Article

***Corresponding author**
Xin Wang

Article History
Received: 20.12.2017
Accepted: 28.12.2017
Published: 30.12.2017

DOI:
10.36347/sjet.2017.v05i12.008

Abstract: By the integration method, some exact solutions to conformal fractional Burgers equation modeled liquid phenomenon is obtained. These solutions include rational solutions, exponential function solutions and trigonometric function solutions.
Keywords: Conformal fractional derivative; integration method; exact solutions; nonlinear Burgers equation.

INTRODUCTION

Job-shop scheduling problem (JSSP), as a typical NP problem, is one of the most intractable problems among all combinational optimization problems till now. Over the years, people have put forward various methods to solve the problem including enumeration, construction and heuristic methods based on priority rules, relaxation method, moving bottleneck method, neural network, and ant colony system (ACS) method, simulated annealing (SA), genetic algorithm (GA), and tabu search method. For example, Giffler and Thompson put forward priority dispatching rule for production scheduling in 1960 and Gere W.S. proposed a heuristic algorithm based on priority dispatching rule for JSSPs in 1966[1,2]. In addition, Balas first solved scheduling problem of machines by using enumeration method based on disjunctive graphs in 1969[3]. In general, the technologies and methods for solving JSSPs are mainly divided into two types: approximate solution and optimization methods. Although solutions to the problem can be quickly obtained by using approximate solution method, they cannot be guaranteed to be optimal. By using optimization method, globally optimal solution can be acquired.

INTRODUCTION

Fractional derivatives have been applied extensively to many problems in various of fields such as physics, biology, engineering and so on. Recently, the conformal fractional derivative is proposed to provide a new concept and a tool for studies of some physical problems, and hence attracts many scholars' attentions [1-5].

In the paper, we consider the conformal fractional Burgers equation. By the integration method, we obtain its exact solutions. These solutions include rational solutions, exponential function solutions and trigonometric function solutions. Burgers equation is a famous equation which appears in many field such as liquid, biology and complex net and so on. The conformal fractional Burgers equation is a generalization of usual Burgers equation.

CONFORMAL FRACTIONAL DERIVATIVE

For a function f = f (t), $\mathbf{t} \in (\mathbf{0}, +\infty)$, and for a given $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$, the conformal fractional derivative is defined by[5]

$$\mathbf{D}_t^\alpha \mathbf{f}(\mathbf{t}) = \mathbf{lim}_{\mathbf{h}\to\mathbf{0}} \frac{\mathbf{f}(\mathbf{t}+\mathbf{h}\mathbf{t}^{\mathbf{1}-\boldsymbol{\alpha}})-\mathbf{f}(\mathbf{t})}{\mathbf{h}}. \tag{1}$$

If the limitation exists, we call the function $\mathbf{f}(\mathbf{t})$ to be $\boldsymbol{\alpha}$ −derivatible at *t*. The basic properties of the conformal fractional derivative can be given as follows [4,5]:

(1) $\mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{t}) \pm \mathbf{g}(\mathbf{t})) = \mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{t})) \pm \mathbf{D}_t^\alpha(\mathbf{g}(\mathbf{t}))$

(2) $\mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{t})\mathbf{g}(\mathbf{t})) = \mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{t}))\mathbf{g}(\mathbf{t}) + \mathbf{D}_t^\alpha(\mathbf{g}(\mathbf{t}))\mathbf{f}(\mathbf{t})$,

(3) $\mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{t})/\mathbf{g}(\mathbf{t})) = \frac{\mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{t}))\mathbf{g}(\mathbf{t}) \pm (\mathbf{g}(\mathbf{t}))\mathbf{f}(\mathbf{t})}{\mathbf{g}^2(\mathbf{t})}$,

(4) $\mathbf{D}_t^\alpha(\mathbf{f}(\mathbf{g}(\mathbf{t})) = \mathbf{t}^{\mathbf{1}-\boldsymbol{\alpha}}\mathbf{g}^{\mathbf{1}-\boldsymbol{\alpha}}\mathbf{D}_g^\alpha(\mathbf{f}(\mathbf{g}(\mathbf{t}))$,

(5) $\mathbf{D}_t^\alpha(\mathbf{f})(\mathbf{t}) = \mathbf{t}^{\mathbf{1}-\boldsymbol{\alpha}}\mathbf{f}'(\mathbf{t})$,

(6) $\mathbf{D_t^{\alpha}(t^p) = pt^{p-\alpha}}$.

These properties can be easily proven. The more detailed properties and proofs can be found in[4,5].

The solutions to fractional nonlinear Burgers equation

Consider the following fractional nonlinear Burgers equation

$$D_t^{\alpha}u+auu_x + bu_{xx} = 0, \tag{2}$$

where *a* and *b* are two constants.

Take the wave transformation

$$\boldsymbol{\theta = kx - \frac{\omega}{\alpha}t^{\alpha}}, \tag{3}$$

where k, ω are real numbers. Substituting it into the Burgers equation gives an ordinary differential equation

$$-\omega u' + akuu' + bk^3u'' = 0. \tag{4}$$

where the prime is the derivative with respect to θ. Further, integrating it yields

$$u' = a_2(u^2 + a_1u + a_0). \tag{5}$$

where

$$a_2 = -\frac{a}{2bk^2}. \tag{6}$$

$$a_1 = -\frac{2\omega}{ak}. \tag{7}$$

and a_0 is an arbitrary constant. Further, we have

$$\int \frac{1}{u^2+a_1u+a_0}du=a_2(\theta - \theta_0), \tag{8}$$

where θ_0 is an integral constant. Denote

$$\Delta= a_1^2 - 4a_0. \tag{9}$$

We have the following three cases to give the solutions to fractional Burgers equation:

Case 1: $\Delta= 0$. Then we know that

$$u^2 + a_1u + a_0 = (u - \alpha)^2. \tag{10}$$

so we get the following solution

$$u_1 = \alpha - \frac{1}{a_2(\theta-\theta_0)}. \tag{11}$$

From the above solution, we can see that the solution has a singular point.

Case 2: $\Delta> 0$. Then we know that

$$u^2 + a_1u + a_0 = (u - \alpha)(u - \beta). \tag{12}$$

so we get the following solutions

$$u_2 = \frac{\alpha+\beta e^{a_2(\alpha-\beta)(\theta-\theta_0)}}{1+e^{a_2(\alpha-\beta)(\theta-\theta_0)}}. \tag{13}$$

$$u_3 = \frac{\alpha-\beta e^{a_2(\alpha-\beta)(\theta-\theta_0)}}{1-e^{a_2(\alpha-\beta)(\theta-\theta_0)}}. \tag{14}$$

These two solutions are exponential function solutions.

Case 3: $\Delta< 0$. Then we know that

$$u^2 + a_1u + a_0 = (u - \alpha)^2 + \beta^2. \tag{15}$$

so we get the following solution

$$u_4 = \beta\tan^2(a_2\beta(\theta - \theta_0)) + \alpha. \tag{16}$$

This is a trigonometric function solution. The solution is not continuous and has singular points.

CONCLUSIONS

We use traveling wave transformation to reduce the conformal fractional Burgers equation to an ordinary differential equation, and hence obtain the exact traveling wave solutions in varied forms. Our results also show that the evolution patterns of the models described by conformal fractional Burgers equations are rich.

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