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Temperature Distribution along a Cone-Cylinder Cathode of an MPD Arcjet

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Review Article

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Abstract: The purpose of the present paper is to obtain an analytical solution of one-dimensional steady heat conduction problem of a cone-cylinder shape cathode of a magnetoplasmadynamic thruster. The exact solution of one-dimensional steady heat conduction is developed to couple conical-cylindrical section of the cathode applying perfect thermal contact conditions. One-end of the cathode is heated whereas the other-end is water cooled. Along the surface of the cathode, heat is transferred from the surface of the cathode to the surrounding air. This analysis of heat transfer is considered a forced convection to the ambient. The analytical solution may easily employ to calculate the temperature distribution and heat flux from the analytical solutions. The exact solution of one-dimensional steady heat conduction can be used to obtain preliminary thermal analysis of the cathode.
Keywords: Biot number, cathode, heat conduction, MPD, thin rod.

INTRODUCTION

The cathode of a magnetoplasmadynamic (MPD) must be maintained at high temperature for emission of electrode with moderate electric field with minimum material erosion. A cathode is to be exposed to severe thermal environments resulting to sharp temperature rise at the cathode root. Knowledge of the temperature distribution along the conical-cylinder shape thin rod is needed for the thermal analysis and design of the cathode.

Shih *et al.* [1] have not considered conical shape of the cathode in the solution of heat conduction problem. In practice, a 2% thoriated tungsten cathode with a semi-cone α of $15 - 30^0$ is commonly selected in the MPD devices [2]. Numerical solution of one-dimensional steady heat conduction heat conduction is carried out using fourth-order Runge-Kutta scheme by Mehta [3]. A transient heat transfer analysis [4] is employed to solve one-dimensional heat conduction finite difference method. Thermal erosion of cathode [5] is calculated using finite element method. Intermittent heating and cooling of an electrode is analyzed in Ref. [6]. Recently Asadi and Khoshkho [7] have presented an exact solution for temperature distribution arising convection-radiation along a constant cross sectional area fin. Heat conduction analysis of thin rod is presented in many text book [8–10].

It is the purpose of this paper to develop an analytical solution of the heat conduction equation that will be useful for preliminary studying the effect of geometrical parameters on the temperature distribution along the conical-cylindrical cathode.

Analysis

We consider a thin rod cathode consisting of a conical and a cylinder attached to form thin rod cathode of a magnetoplasmadynamic as depicted in Fig. 1. With the assumptions of constant thermo-physical properties, one-dimensional steady heat conduction equation can be written for a conical region as

$$x_{10}^2(T_{10})_{xx} + 2x_{10}(T_{10})_x - \frac{2h_c}{k\delta_1} w x_{10}(T_{10} - T_g) = 0, \qquad x_a < x_{10} \geq w \tag{1}$$

And for a cylinder region as

$$(T_{20})_{xx} - \frac{2h_c}{k\delta_1}(T_{20} - T_g) = 0, \qquad w \leq x_{20} > b_n \tag{2}$$

Subject to following boundary conditions including with the perfect thermal contact at the cone and the cylinder sections of the thin rod

$$T_{10} = T_a \quad \text{at} \quad x_{10} = x_a$$
$$T_{10} = T_{20} \quad \text{at} \quad x_{10} = x_{20} \tag{3}$$
$$(T_{10})_x = (T_{20})_x \quad \text{at} \quad x_{10} = x_{20}$$
$$T_{20} = T_w \quad \text{at} \quad x_{20} = b_n$$

The above one-dimensional heat conduction equations are non-dimensionalized using the following variables:

$$T_1 = \left[\frac{T_{10} - T_g}{T_0 - T_g}\right],$$

$$T_2 = \left[\frac{T_{20} - T_g}{T_0 - T_g}\right],$$

$$x_1 = \frac{x_{10}}{w},$$

and

$$x_2 = \frac{x_{20}}{w}$$

The non-dimensional one-dimensional steady heat conduction equation for the conical region is

$$x_1^2 (T_1)_{xx} + x_1 (T_1)_x - 2Bi^2 (x_1 T_1) \tan\alpha = 0\,, \qquad \xi < x_1 \geq 1 \tag{4}$$

and for the cylinder region

$$(T_{20})_{xx} - Bi^2 T_2 = 0 \qquad 1 \leq x_2 > b \tag{5}$$

Subjected to following boundary conditions

$$T_1 = T_a \quad \text{at} \quad x_{10} = \xi$$
$$T_1 = T_2 \quad \text{at} \quad x_1 = x_2 \tag{6}$$
$$(T_{10})_x = (T_{20})_x \quad \text{at} \quad x_1 = x_2$$
$$T_2 = T_w \quad \text{at} \quad x_2 = b_n$$

The exact solutions of the one-dimensional steady heat conduction equation in the conical section is

$$T_1 = x_1^{0.5}\left[c_1 I_1\left(2Bix_1^{0.5}\, tan\alpha\right)\right] + c_2 K_1\left(2Bix_1^{0.5}\, tan\alpha\right) \tag{7}$$

and for the cylinder section is

$$T_2 = Ae^{(Bix_2)} + Be^{-(Bix_2)} \tag{8}$$

The constants of the above equations are

$c_1 = (T_a - Qc_2)/P$

$$c_2 = \frac{\left(\frac{VT_a}{P} + \frac{X}{Z}\right)\left(U - \frac{TM}{Z}\right) - \left(\frac{RT_a}{P} + \frac{T}{Z}\right)\left(Y - \frac{MX}{Z}\right)}{\left(S - \frac{QR}{P}\right)\left(Y - \frac{MX}{Z}\right) - \left(W - \frac{QV}{P}\right)\left(U - \frac{MT}{Z}\right)}$$

$A = (1 - BM)/Z$

$$B=\frac{\left(\frac{VT_a}{P}+\frac{X}{Z}\right)\left(S-\frac{QR}{P}\right)-\left(\frac{VT_a}{P}+\frac{T_s}{Z}\right)\left(W-\frac{QV}{P}\right)}{\left(U-\frac{MT_S}{Z}\right)\left(W-\frac{QR}{P}\right)-\left(S-\frac{QR}{P}\right)\left(Y-\frac{MX}{Z}\right)}$$

P = $\xi^{-1/2}$ I_1 (2Bi tan α $\xi^{1/2}$),

Q = $\xi^{-1/2}$ K_1 (2Bi tan α $\xi^{1/2}$),

R = I_1 (2Bi tan α),

S = K_1 (2Bi tan α),

T_S = $-e^{Bi}$,

U = $-e^{-Bi}$,

V= Bi I_2(2Bi tan α),

W = $-K_2$ (2Bi tan α),

X = -Bi e^{Bi},

Y = Bi e^{Bib},

Z = e^{Bib},

M = e^{-Bib},

$\xi = x_0/w$,

$b = b_n/w$,

Following relations for the heat flux at the tip of the cone, at the end of the cathode and heat transfer to the surrounding are derived

$$q_a = Bi\,tan\alpha\left[c_1 I_2\left(2Bi\,tan\alpha\,\xi^{1/2}\right)-c_2 K_2\left(2Bi\,tan\alpha\,\xi^{1/2}\right)\right] \tag{9}$$

$$q_w = \left[Ae^{(Bib)} - Be^{(-Bib)}\right] \tag{10}$$

$$\begin{aligned} q_g = &\left[c_1\left\{I_2\left(2Bi\,tan\alpha\,\xi^{1/2}\right)\right\}-\xi I_2\left(2Bi\,tan\alpha\,\xi^{1/2}\right)+c_2\left\{K_2\left(2Bi\,tan\alpha\,\xi^{1/2}\right)\right\}-K_2\left(2Bi\,tan\alpha\,\xi^{1/2}\right)\right] \\ &+A\left\{e^{(Bib)}-e^{(Bi)}\right\}+B\left\{e^{-(Bi)}-e^{(Bib)}\right\} \end{aligned} \tag{11}$$

In the above equations expression for the heat fluxes are non-dimensionlized. It is interesting to mention here that Equations (9) to (11) are function of Biot number.

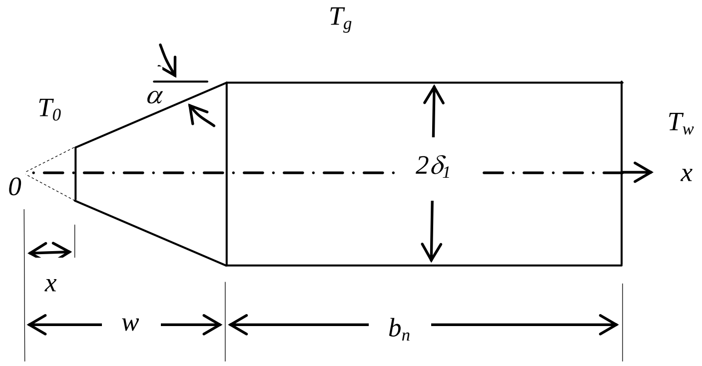

Fig-1: Geometry of tungsten electrode

RESULTS AND DISCUSSION

Analytical results of the exact solutions of one-dimensional heat conduction equation are used to obtain temperature profile over the surface of the thin rod. Temperature distributions along the cone-cylinder thin rod for semi-cone angle of 15^0 and 30^0 are depicted in Fig.2 for Biot number Bi = 1.06. It is observed that temperature decreases rapidly along the nose-cone portion of the cathode, while in the remaining section of the cathode, the temperature varies almost linearly. The decrease in semi-cone angle of cone increases the heat transfer due to the decrease in the cross-sectional area in the semi-cone portion of the cone.

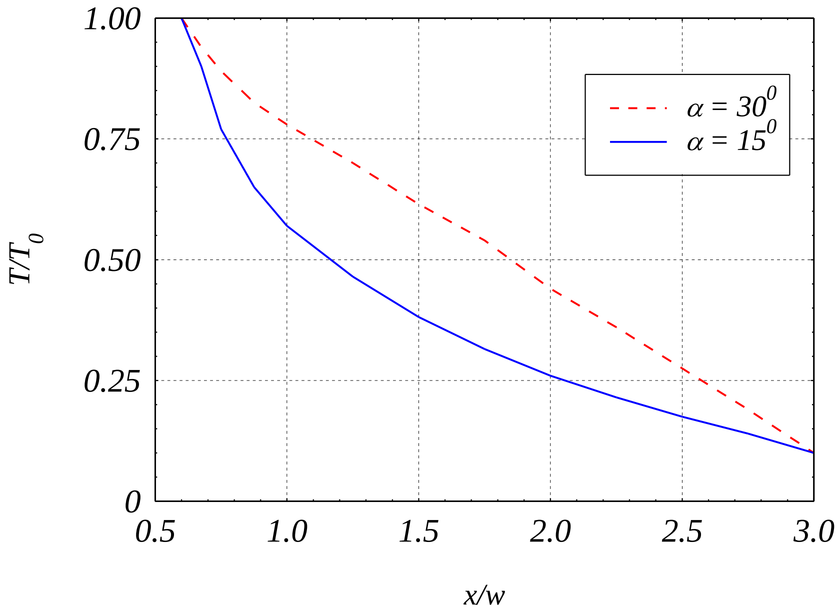

Fig-2: Temperature distribution along the cone-cylinder thin rod

The non-dimensional heat flux at the tip of the cone q_g, at the end of cylinder q_w and heat flux to the ambient q_a are calculated using Eqs. (9) – (11) and tabulated in Table 1. The exact solution is useful in finding the safe limit for the cathode in case of a coolant failure.

Table-1: Values of calculated non-dimensional heat flux

α	q_g	q_a	q_w
15^0	-0.368×10^2	0.702×10^2	-0.084×10^2
30^0	-0.771×10^2	0.339×10^2	-0.201×10^2

It is important to mention here that the temperature at the tip of the cone is about 3000 K which is below the melting point of tungsten [1]. At the cathode, the electron component has to satisfy the appropriate emission law. It would be inappropriate to discuss here electron emission theories. However, it is worth to say that a theory of thermionic emission under the influence of high temperature has been derived by Richardson [11].

CONCLUSION

An exact solution of one-dimensional steady heat conduction equation is obtained for cone-cylinder thin rod configuration. Temperature distribution along the cathode is presented for different semi-cone angle. The values of non-dimensional heat flux are computed by differentiation of exact solutions of one-dimensional steady state heat conduction expressions at the tip, the end and on the surface of the cathode. One advantage of seeking analytical solution is that the solutions do represent a clear functional relation among the geometrical parameters.

Nomenclature

A_x = Area of the cathode spot, m^2
b_n = length of the cylindrical rod, m
h_c = convective heat transfer coefficient, W/m^2K
I_1, I_2 = modified Bessel function of first kind of first and second order, respectively
K_1, K_2 = modified Bessel function of second kind of first and second order, respectively
k = thermal conductivity, W/mK
Bi = Biot number, $(2h_c w^2/k)^{1/2}$
N_1, N_2 = non-dimensional constant
q_a = heat flux from the arc, W/m^2
q_g = heat dissipation to the ambient, W/m^2
q_w = heat flux from the cathode to water, W/m^2
T = Temperature, K
w = length of the conical section of the cathode, m
x = axial coordinate, m
α = semi-cone angle, deg
δ_1 = radius of the cathode, m

Subscripts
w = Temperature of the cooling water
10 = conical
20 = cylinder
a = maximum operating temperature of the cathode root
g = ambient temperature

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