

## Compute the Orthometric Height in Baghdad City

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**Abstract:** The principal purpose of this research is to employ the GPS observations to predict the orthometric height. Specialist software such as MS-Excel has been used to develop the required regression models. Vertical control points within the network that include ellipsoidal, orthometric and geoid height data were investigated. Although the theoretical relationship between these height types is simple in nature, discrepancies among the combined height data and its practical implementation have proven to be quite challenging due to numerous factors. Two primary challenges have been chosen as research objectives; (i) developing a mathematical model to compute the geoid undulation from GPS and precise leveling, (ii) correcting the observed geoid undulation by least square method. To address these objectives, a general empirical-based procedure has been adopted with statistical analysis for assessing the performance of the selected parametric models. Additional numerical studies included the obtaining of geoid models (local geoid), scaling the GPS-derived ellipsoidal height matrix, and evaluating the orthometric heights obtained from national/regional adjustments of leveling data. Finally, the orthometric height results obtained from the developed mathematical and adjustment models can be adequately and reliably used for the city of Baghdad.

**Keywords:** Orthometric height, Ellipsoidal height, Geoid Undulation, Geodetic coordinates.

### INTRODUCTION

The geoidal undulation can be defined as the separation of the ellipsoid reference with the geoid surface measured along the normal ellipsoid as shown in Figure 1. The combined use of GPS, leveling, and geoid height information has been used as a key procedure in various geodetic applications. These three types of height information are considerably different in several aspects such as physical meaning, surface reference definition, observational methods, and accuracy; nevertheless, they all should fulfill the following simple geometrical relationship:

$$h = H + N \quad (1)$$

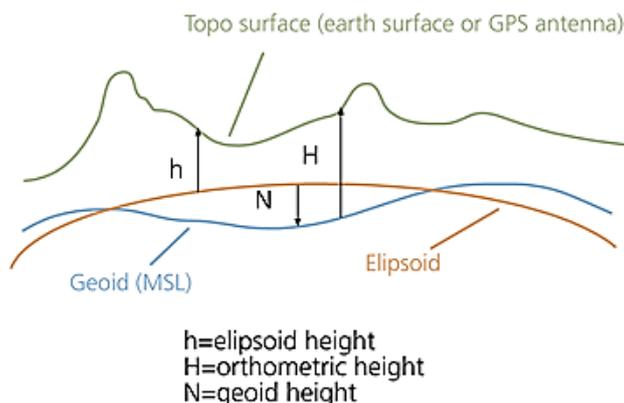
Where:

h is the ellipsoidal height,

H is the orthometric height or mean sea level, and

N is the geoid undulation.

The GPS technique can be associated with simultaneous 3-D positioning in geodetic aims; however, the GPS derived ellipsoidal heights must be transformed to orthometric heights in order to have physical meaning in surveying or engineering applications.



**Fig-1: Schematic of the geoid undulation (N) between the ellipsoid and geoid (5)**

**The Linear Regression Equation**

Linear regression is an effective approach to predict the relationship between dependent and independent variables. The slope of regression line is important to determine this relationship. The typical form of a simple regression equation can be written in the form

$$Y=a+bX \tag{2}$$

where Y is the dependent variable (that’s the variable that goes on the Y axis), X is the independent variable (i.e. it is plotted on the X axis), b is the slope of the line and a is the y-intercept. The a and b parameters can be calculated from the following equations

$$a = \frac{(\sum y)(\sum x^2)-(\sum x)(\sum xy)}{n(\sum x^2)-(\sum x)^2} \tag{3}$$

$$b = \frac{n(\sum xy)-(\sum x)(\sum y)}{n(\sum x^2)-(\sum x)^2} \tag{4}$$

These parameters explain the relationship between the two variables.

**Prediction of Linear Regression Equation**

Regression equation expresses the linear relationship between the Y and X variables. In regression analysis, the independent variable (X) is used to estimate the dependent variable (Y). The relationship between the variables is assumed to be linear and both variables must be at least of interval scale. The least squares criterion is used to determine the unknown parameters of the regression equation by minimizing the sum of the squares of the differences between vertical distance of the actual Z-value (observed) and the predicted Z-value. Standard error is a statistical parameter used to estimate or measure the scatter or dispersion of the observed values around the line of regression. It can be calculated from the following formula;

$$S_{y,x} = \sqrt{\frac{\sum(Y-Y')^2}{n-2}} \tag{5}$$

**Methodology and Model Prediction**

The multiple regression equation (MRE) is the mathematical technique used to solve some problems in all branches of science. This traditional technique only accommodates coordinate transformations relating with two datum. In many instances, particularly for many classical local datum, there are known datum and can be changed into a realized unknown datum, where the ellipsoidal surface is known datum and the geoid surface is unknown datum. Various methods have been proposed to address this problem. One of the most popular method is the multiple regression formula where polynomial functions represent the variations as a function of position in terms of the difference of latitude, longitude, and height (or X, Y, and Z coordinates) [3]. Depending on the degree of variability in the distributions, an approximation may be carried out using 2<sup>nd</sup>, 3<sup>rd</sup> and higher degree polynomials. In the case of geoid undulation, it is possible to use any degree that can minimize the distortion in the checkpoints. Polynomial approximation functions themselves are subjected to variations, as different approximation characteristics may be achieved by different polynomial functions. The simplest of all polynomials is the general polynomial function [5]. The polynomial technique

can be classified into two models, the first is a real number polynomial model and the second is a complex number polynomial model. The first model is the general model, the formula is:

$$N = A_0 + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + \dots + A_{nn} U^2 V^2 \tag{6}$$

Where  $A_0, \dots, A_{nn}$  are the coefficients,  $N$  is the geoid undulation, and  $U$  and  $V$  are the available data.

This model has been adopted by several researches with mean value where  $U$  and  $V$  are used relative to central evaluation points.

**Experimental Work and Data Collection**

The experimental work of the current study includes collecting geodetic coordinates with orthometric height for selected points within the Baghdad city national geodetic network developed by pole Service Company. Whereas Figure

**2 shows the main steps of the fieldwork**

Table 1 lists the geodetic coordinates for selected horizontal and vertical points. The points were observed in the DGPS method and corrected by using online positioning user service. Also by using local correction by Leica Geo office software 8.3, the h-values computed from GPS observations (Table 2) and H-values of the Pole Service Company can be seen in Table 3 and Figure 3.

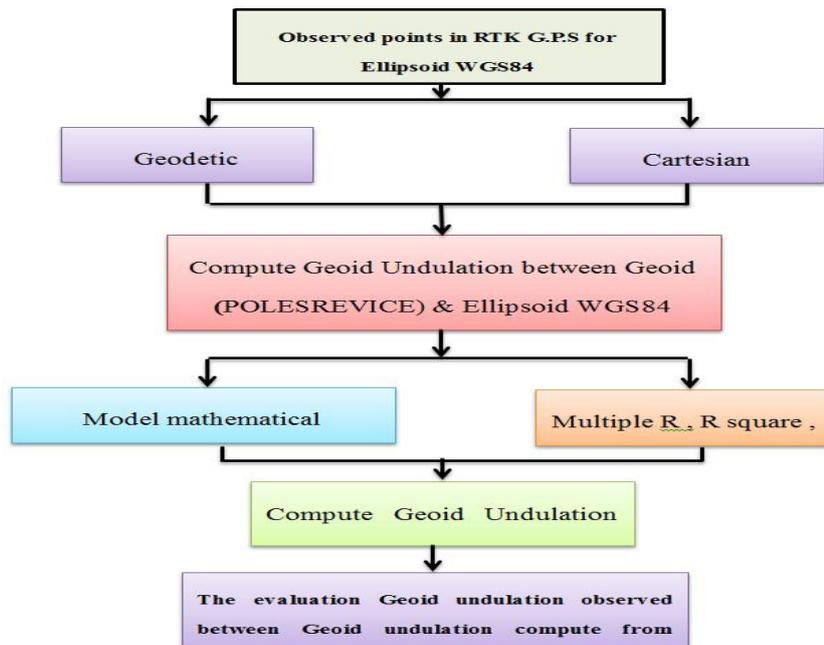


Fig-2: A flowchart for the fieldwork methodology

Table-1: Geodetic coordinates for selected horizontal and vertical points of the national geodetic network

No.	Name of point	Latitude( $\phi$ )	Longitude( $\lambda$ )	Orthometric ELEV.
1	20108	33° 14' 8.9152"	44° 29' 34.2367"	57.20
2	20081	33° 22' 23.7172"	44° 31' 33.1741"	60.70
3	20073	33° 24' 14.7614"	44° 18' 3.5463"	40.20
4	20080	33° 20' 14.0295"	44° 23' 42.3256"	85.90
5	527003	None	none	33.314
6	525504	None	none	35.358
7	517301	None	none	40.077
8	523002	None	none	33.966
9	524001	None	none	32.180
10	511701	None	none	36.554
11	32/2	None	none	36.797

**Table-2: Observed geodetic coordinates for selected horizontal and vertical points of the national geodetic network (Ellipsoid WGS84 /ITRF2008)**

No.	Name of point	Latitude( $\phi$ )	Longitude( $\lambda$ )	Ellipsoid Height(h)
1	20108	33° 14' 8.9474"	44° 29' 23.0654"	55.233
2	20081	33° 22' 23.7309"	44° 31' 21.9874"	58.933
3	20073	33° 24' 14.7503"	44° 17' 52.3269"	39.247
4	20080	33° 20' 14.0360"	44° 23' 31.1269"	84.838
5	527003	33° 21' 53.9122"	44° 15' 0.7972"	32.295
6	525504	33° 18' 19.7270"	44° 16' 35.0450"	34.141
7	517301	33° 25' 40.9742"	44° 25' 1.83023"	38.451
8	523002	33° 17' 35.1032"	44° 27' 15.9473"	32.333
9	524001	33° 13' 36.02694"	44° 22' 27.86923"	32.180
10	511701	33° 21' 37.9443"	44° 21' 18.3075"	35.637
11	32/2	33° 22' 39.4728"	44° 19' 05.8638"	35.916

**Table-3: The geoid undulation between orthometric height (POLESREVICE) and ellipsoid height (WGS84/ITRF2008)**

No.	Name of point	Ellipsoid Height	Orthometric Height	Geoid Undulation
1	20108	55.233	57.20	-1.967
2	20081	58.933	60.70	-1.767
3	20073	39.241	40.20	-0.959
4	20080	84.838	85.90	-1.062
5	527003	32.295	33.314	-1.019
6	525504	34.141	35.358	-1.217
7	517301	38.451	40.077	-1.626
8	523002	32.333	33.966	-1.633
9	524001	30.389	32.180	-1.791
10	511701	35.637	36.554	-0.917
11	32/2	35.916	36.797	-0.881

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POLSERVICE-GEOKART

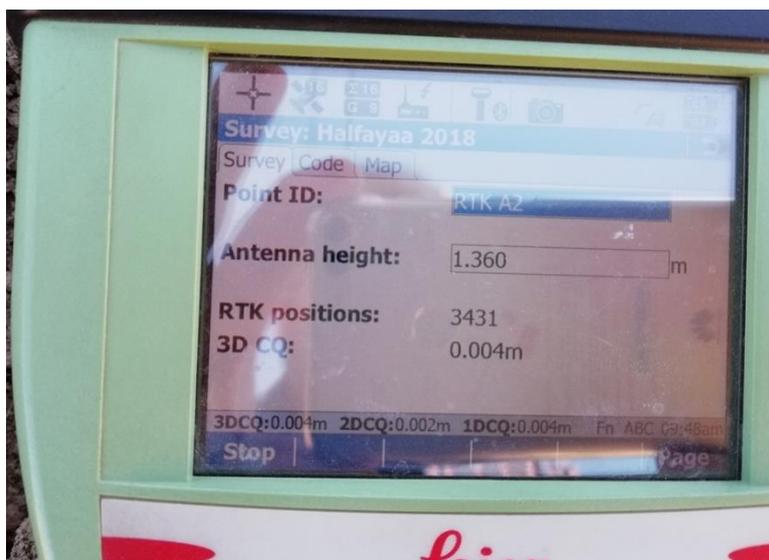
**POINT TRAVERSE HEIGHTS**

REFERENCE LEVEL: FAO PORT - 1978  
 MEAN SQUARE ERROR AFTER ADJUSTMENT: MO= 1.9 MM/KM

ITEM	POINT NO	HEIGHT IN METERS
1	0 230 80	34.051
2	3 117 02	34.685
3	3 117 04	34.351
4	3 118 01	33.697
5	3 118 03	33.279
6	3 118 04	33.368
7	3 118 06	32.884
8	3 124 03	34.383
9	3 124 05	33.899
10	3 124 06	34.052
11	3 128 01	34.060
12	3 128 02	35.097
13	3 128 03	34.832
14	3 133 01	34.337
15	3 133 03	35.315

**Fig-3: Vertical point from (POLESERVICE) report [6]**

The GPS observation process continues for a period until reaching high accuracy as shown in Figure 4 and 5.



**Fig-4: Accuracy of 3D CQ in observation**

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FILE: BASE0760.18o OP1522134658456

NGS OPUS SOLUTION REPORT
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All computed coordinate accuracies are listed as peak-to-peak values.
For additional information: https://www.ngs.noaa.gov/OPUS/about.jsp#accuracy

USER: engineer200529@gmail.com DATE: March 27, 2018
RINEX FILE: base076l.18o TIME: 07:14:47 UTC

SOFTWARE: page5 1603.24 master54.pl 160321 START: 2018/03/17 11:07:00
EPHEMERIS: igr19926.eph [rapid] STOP: 2018/03/17 14:00:00
NAV FILE: brdc0760.18n OBS USED: 6194 / 6620 : 94%
ANT NAME: LEIGS15 NONE # FIXED AMB: 30 / 39 : 77%
ARP HEIGHT: 1.26 OVERALL RMS: 0.015(m)

REF FRAME: IGS08 (EPOCH:2018.2069)

X: 3814582.387(m) 0.026(m)
Y: 3724874.272(m) 0.062(m)
Z: 3489028.731(m) 0.014(m)

LAT: 33 22 39.48488 0.018(m)
E LON: 44 19 5.87701 0.035(m)
W LON: 315 40 54.12299 0.035(m)
EL HGT: 35.952(m) 0.052(m)

UTM COORDINATES
UTM (Zone 38)
Northing (Y) [meters] 3693360.368
Easting (X) [meters] 436590.422
Convergence [degrees] -0.37505351
Point Scale 0.99964957
Combined Factor 0.99964393

BASE STATIONS USED
PID DESIGNATION LATITUDE LONGITUDE DISTANCE(m)
TEHN 693053.6
NICO 1023129.3
BHR4 999077.9

This position and the above vector components were computed without any
knowledge by the National Geodetic Survey regarding the equipment or
field operating procedures used.
    
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Fig-5: NGS OPUS solution report

Figure 4 also demonstrates the accuracy of the observation, the number of satellites available and the vertical and horizontal accuracy of the point. After completing the observations for all points, the geoid undulation values for the selected points were calculated based on the orthometric H-values (obtained by pole service company) and the ellipsoid h-values (obtained from equation 1) (See Table 3). Equation 7 represents the use of regression equation for geoid undulation and the extract of parameters in terms of the difference of coordinates.

$$N = 7.312239 + 3.531372 \phi - 2.84779 \lambda \tag{7}$$

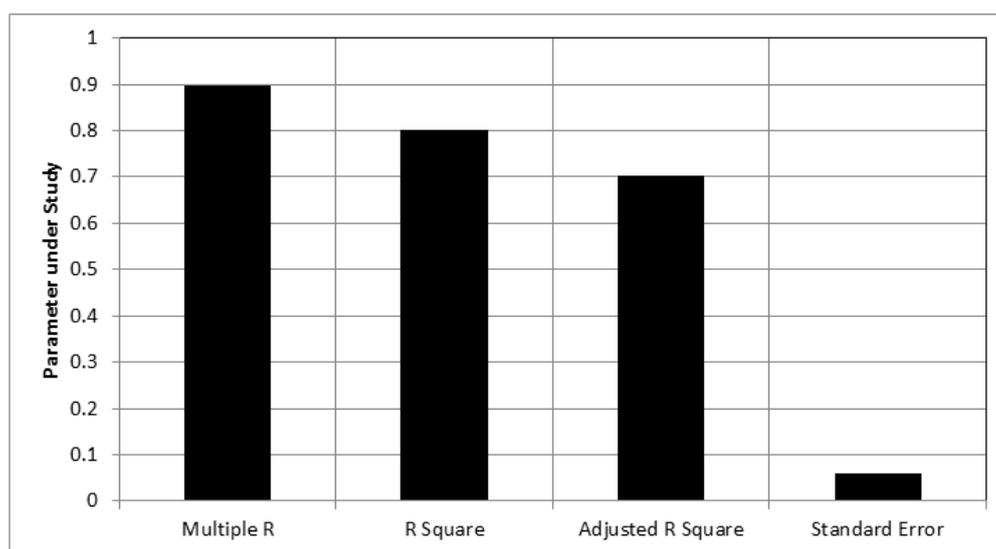
	A	B	C	D	E	F
1		<i>df</i>				
2	Regression		2	SUMMARY OUTPUT		
3	Residual		4			
4	Total		6	Regression Statistics		
5				Multiple R		0.912728054
6		<i>Coefficients</i>		R Square		0.833072501
7	Intercept	7.312239231		Adjusted R Square		0.791340626
8	Lat. Variable	3.531372582		Standard Error		0.062751321
9	Long. Variable	-2.847797353		Observations		11
10						
11						

Fig-6: Regression equation from Excel sheet

Based on geoid undulation equation and the H-values obtained from the pole service company, a comparison between the observed points and the points calculated by equation (7) can be made as shown in Table 4. It can be noticed from Table 4 and Figure 7 the results ranged from (2-5) cm only. This accuracy is quite good for field surveys.

**Table-4: The discrepancies between the observed geoid undulation values and those computed from regression equation in Baghdad**

No.	Name of point	Ellipsoid Height	Orthometric Height (m)	Geoid Undulation Observed (m)	Geoid Undulation Computed (m)	Standard Error
1	49/7	33.309	44.361	-1.341	-1.392	0.051
2	47/1	33.346	44.358	-1.299	-1.253	0.046
3	29/1	33.31	44.499	-1.774	-1.782	0.008
4	521505	33.329	44.528	-1.846	-1.797	0.049
5	20/2	33.235	44.371	-1.673	-1.682	0.009
6	525001	33.278	44.284	-1.308	-1.282	0.026
7	24/5	33.225	44.401	-1.782	-1.803	0.02
8	528701	33.279	44.536	-2.011	-1.956	0.055



**Fig-7: Selected regression statistics**

**CONCLUSION**

The research employed empirical data and statistical approach to compute and evaluate the accuracy of geoid undulation in Baghdad city. Based on available data obtained from Pole Service Company, eleven points have been selected; their coordinates and orthometric heights have been determined using precise levelling and GPS. The work reveals that adopting in plane geodetic coordinates can yield reliable geoid undulation values over the city of Baghdad. The overall accuracy ranges from (2 to 5) cm only. Hence, calculating orthometric heights using the simple equation  $H = h - N$  is quite beneficial. The results are quite useful in Baghdad city.

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