Scholars Journal of Arts, Humanities and Social Sciences
OPEN ACCESS
Abbreviated Key Title: Sch J Arts Humanit Soc Sci
ISSN 2347-9493 (Print) | ISSN 2347-5374 (Online)
Journal homepage: https://saspublishers.com/journal/sjahss/home

Stability Analysis of the Time-Delay Worker Retraining Model with Validity Period

WEI Yu-fen[1*], ZHU Huan[2]

[1]WEI Yu-fen, Sciences College, Heilongjiang Bayi Agriculture University, Daqing-163319, China
[2]Huan Zhu, Sciences College, Heilongjiang Bayi Agriculture University, Daqing-163319, China

*Corresponding author
WEI Yu-fen

Article History
Received: 25.12.2018
Accepted: 05.01.2019
Published: 16.01.2019

DOI:
10.36347/sjahss.2018.v07i01.001

Abstract: This study investigated the worker retraining model with a validity period of private enterprises. The threshold that determines worker retraining and the sufficient conditions that determine the global stability of both the non-negative equilibrium point and unique positive equilibrium point are given. Finally, numerical simulation is presented using Matlab.
Keywords: simple balance, non-trivial equilibrium, threshold value, global stability
Mathematics Subject Classification: 37C20 37C75 O175.13

Introduction

Retraining of employees is an important human resource investment. Through employee retraining, employees can clarify job responsibilities, tasks and goals, improve knowledge and skills, and have their own quality and business ability that are compatible with the realization of enterprise goals. It is now a necessity to study the law of technology update and improvement in communication of the same workers and put forward a control strategy for retraining workers so as to maximize the benefits of enterprises and workers.

Time-delay differential equations have been widely used in simulation of population change, transmission of infectious diseases, biological science, physics, control theory, and other fields [1-8]. Fred Brauer [9] studied a class of time-delay infectious disease models with nonlinear birth and vertical transmission, proving that Hopf branches can be generated at the endemic equilibrium point under certain conditions, indicating the time delay has a great impact on the spreading of the diseases. JinHua Ye[10] studied a stage structure and Holling Ⅲ type of predator-prey system function. The system is uniformly persistent provided sufficient conditions are given; HaiFeng Huo[11] studied the stability of the smoking cessation model under the influence of education in a class of public health and provided the local stability of the smokeless equilibrium point and smoking equilibrium point. Zhao Tao[12] studied a type of time-delay SEIR computer virus propagation model, taking the latency of computer virus as bifurcation parameter, and discussed the local asymptotic stability of the model. The basic worker training model proposed by Brauer and Carlos [13] is as follows:

$$\begin{cases} P' = qK - \dfrac{\beta}{K}PM + \delta R - \mu P \\ R' = (1-q)K - (\delta+\mu)R - aR \\ M' = \dfrac{\beta}{K}PM - (r+\mu)M \\ U' = -\mu U + \alpha R \\ I' = rM - \mu I \end{cases} \qquad (1.1)$$

Literature [14] has analyzed the stability of the model, and concluded that under certain conditions, the non-negative equilibrium point and the positive equilibrium point of the model have local stability and global stability.

Where P(t), R(t), M(t), U(t), I(t) denote the number of technical workers, regular workers, returning workers, foreman, non-expendable workers (injured or pregnant) and sabotage workers at time t, respectively. Because the workers who cannot be eliminated after a certain period of time, owing to their own needs, want to be retrained, they need to return to the workers. Therefore, this study considered adding this transformation time into the model, namely time delay. Through the characteristic equation analysis method and Hurwitz's decision theorem, it obtained the local stability and global stability of the non-negative equilibrium point, the global stability of the positive equilibrium point and the sufficient conditions for the existence of local asymptotic stability, and used Matlab to verify the relevant conclusions.

The specific model is established as follows:

$$\begin{cases} \dfrac{dP(t)}{dt} = qk - \beta P(t)\dfrac{M(t)}{k} + \delta R(t) - \mu P(t) \\ \dfrac{dR(t)}{dt} = (1-q)k - (\delta+\mu)R(t) - aR(t) + aR(t-\tau)e^{-\mu\tau} \\ \dfrac{dM(t)}{dt} = \beta P(t)\dfrac{M(t)}{k} - (r+\mu)M(t) \\ \dfrac{dU}{dt} = -\mu U - aR(t-\tau)e^{-\mu\tau} + aR(t) \\ \dfrac{dI}{dt} = rM(t) - \mu I \end{cases} \tag{1.2}$$

The parameters $\alpha, \beta, \delta, \mu, K, q, r$ and τ are positive constants, in which $\dfrac{\beta}{K}$ is the contact rate between foreman and regular worker; q is the transformation rate of the training workers into regular workers ; μ is the removal rate of all types of skilled workers ; δ is the transformation rate of the returning workers into regular workers; α is the transformation rate of the returning workers into non-expendable workers; r is the transformation rates of the foremen into sabotage workers; K is a constant. τ is a constant delay for workers to grow from non-expendable workers (injured or pregnant) to regular workers. According to the system (2.1), the total number of workers can be obtained to satisfy the equation:

$$N' = K - \mu N \qquad (1.3)$$

where $N(t) = P(t) + R(t) + M(t) + U(t) + I(t)$,

$$N(t) = \frac{K}{\mu} - (\frac{K}{\mu} - N(0))e^{-\mu t}$$

The stability of the equilibrium points

Existence of equilibrium point

Because the first three equations of model (2.1) do not contain variables U, I , only the first three equations in the model need to be discussed later, and the following model can be obtained:

$$\begin{cases} \dfrac{dP(t)}{dt} = qk - \beta P(t)\dfrac{M(t)}{k} + \delta R(t) - \mu P(t) \\ \dfrac{dR(t)}{dt} = (1-q)k - (\delta+\mu)R(t) - aR(t) + aR(t-\tau)e^{-\mu\tau} \\ \dfrac{dM(t)}{dt} = \beta P(t)\dfrac{M(t)}{k} - (r+\mu)M(t) \end{cases} \tag{2.1}$$

The initial conditions for system (2.4) are

$P(t)=\phi_1(t)\in C\{[-\tau,0],R_+\}$, $R(t)=\phi_2(t)\in C\{[-\tau,0],R_+\}$
$M(t)=\phi_3(t)\in C\{[-\tau,0],R_+\}$, $\phi_1(0)\geq 0$, $\phi_2(0)\geq 0$, $\phi_3(0)\geq 0$ (2.2)

It is easy to verify that, if $t\geq 0$ for system (4), the solution under initial condition (2.2) is unique.

The non-negative equilibrium $E_0=(P_0,R_0,M_0)$ and positive equilibrium $E^*=(P^*,R^*,M^*)$ of system (2.1) satisfy the following combined equations.

$$\begin{cases} qk-\beta P(t)\dfrac{M(t)}{k}+\delta R(t)-\mu P(t)=0 \\ (1-q)k-(\delta+\mu)R(t)-aR(t)+aR(t-\tau)e^{-\mu\tau}=0 \\ \beta P(t)\dfrac{M(t)}{k}-(r+\mu)M(t)=0 \end{cases} \quad (2.3)$$

From (2.3) we obtain

$$E_0=(P_0,R_0,M_0)=\left(\frac{1}{\mu}[qk+\frac{(1-q)\delta k}{\mu+\delta+a(1-e^{-\mu\tau})}],\frac{(1-q)k}{\mu+\delta+a(1-e^{-\mu\tau})},0\right)$$

That $\bar{R}=\dfrac{\beta(\varepsilon q+(1-q)\delta)}{\varepsilon\mu(r+\mu)}$, when $\bar{R}>1$, the unique positive equilibrium point E^* of system (2.4) can be obtained.

$$E^*=(P^*,R^*,M^*)=\left(\frac{k(r+\mu)}{\beta},\frac{(1-q)k}{\mu+\delta+a(1-e^{-\mu\tau})},\frac{\mu k(\bar{R}-1)}{\beta}\right)。$$

Theorem 2.1. If $\bar{R}>1$, there exist a non-negative equilibrium $E_0=(P_0,R_0,M_0)$ and a unique positive equilibrium $E^*=(P^*,R^*,M^*)$ for system (2.1).

The stability of the non-negative equilibrium

Theorem 2.1. IF $0<\bar{R}\leq 1$, $\tau\neq 0$, the non-negative equilibrium E_0 is a local stability; if $\bar{R}>1$, It is not a stability.

Proof: The characteristic equation corresponding to the linearized system of model (2.1) at A_{E_0} is

$$f(\lambda)=(\lambda+\mu)(\lambda+\mu+\delta+a(1-e^{-\mu\tau}))(\lambda-\frac{\beta}{K}P_0+r+\mu) \quad (2.4)$$

There are always two negative roots for the eigenvalues of A_{E_0}, $\lambda_1=-\mu<0$。 $\lambda_2=-[\mu+\delta+a(1-e^{-\mu\tau})]<0$. And the other roots are determined by the following equation

$$\lambda-\frac{\beta}{K}P_0+r+\mu=0 \quad (2.5)$$

That is

$$\lambda_3=\frac{\beta}{K}P_0-(r+\mu)=\frac{\beta}{K}\frac{1}{\mu}[qK+\frac{(1-q)K}{\mu+\delta+a(1-e^{-\mu\tau})}]-(r+\mu)$$
$$=\frac{\beta}{\mu[\mu+\delta+a(1-e^{-\mu\tau})]}[q(\mu+\delta+a(1-e^{-\mu\tau})+(1-q)\delta]-(r+\mu)$$

$$= \frac{\beta}{\mu[\mu+\delta+a(1-e^{-\mu\tau})]}[q(\mu+\delta+a(1-e^{-\mu\tau}))+(1-q)\delta-(r+\mu)\mu(\mu+\delta+a(1-e^{-\mu\tau}))]$$

$$= \frac{\beta[q(\mu+\delta+a(1-e^{-\mu\tau}))+(1-q)\delta]}{\mu[\mu+\delta+a(1-e^{-\mu\tau})]\overline{R}}(\overline{R}-1) \le 0$$

When $\overline{R} \le 1$, all the characteristic roots of the characteristic equation A_{E_0} are negative, so E_0 is locally asymptotically stable; When $\overline{R} > 1$, $\lambda_1 > 0$, E_0 is unstable in region D. The proof is complete.

Theorem 2.2. IF $\overline{R} < 1$, and $\min\left\{\left(1-\frac{\delta}{2\mu}\right),(a(1-e^{-\mu\tau})+\delta+\mu-\frac{\delta}{2\mu})\right\} \ge 0$ then the non-negative equilibrium point E_0 is a local stability; if $\overline{R} > 1$, It is not a stability.

Proof: Let $x = P - P_0$, $y = R - R_0$, $z = M - M_0$, then model (1) can be deformed into

$$\begin{cases} x'(t) = -\mu x(t) + \delta y(t) - \frac{\beta P_0}{k} z(t) \\ y'(t) = -(\mu+a+\delta)y(t) + ay(t-\tau)e^{-\mu\tau} \\ z'(t) = -(r+u)z(t) + \frac{\beta P_0}{k} z(t) \end{cases} \quad (2.6)$$

Construct the Lyapunov function $V_1(t) = \frac{x^2}{2\mu} + \frac{y^2}{2} + \frac{z^2}{2(r+u)}$, and derive the derivative of the orbit along the model (2.6)

$$\frac{dV_1}{dt} = -x^2(t) + \frac{\delta}{\mu}x(t)y(t) - \left(1-\frac{\beta p_0}{k(r+u)}\right)z^2(t) + ay(t)y(t-\tau)e^{-\mu\tau}$$
$$-(\delta+a+\mu)y^2(t) - \frac{\beta p_0}{\mu k}x(t)z(t)$$

$$\frac{dV_1}{dt} \le -x^2(t) + \frac{\delta}{\mu}x(t)y(t) - \left(1-\frac{\beta p_0}{(r+\mu)k}\right)z^2(t) + ay(t)y(t-\tau)e^{-\mu\tau} - (\delta+a+\mu)y^2(t)$$

Let $V(t) = V_1(t) + \frac{ae^{-\mu\tau}}{2}\int_{t-\tau}^{t} y^2(u)du$. Due to the $\frac{dy}{dx}\int_{t-\tau}^{t} y^2(u)du = y^2(t) - y^2(t-\tau)$, $y \le \frac{A}{b}$, we can obtain

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{ae^{-\mu\tau}}{2}\left[y^2(t) - y^2(t-\tau)\right]$$

$$\le -x^2(t) - (\delta+a+\mu)y^2(t) - \left(1-\frac{\beta p_0}{(\mu+r)k}\right)z^2(t) + \frac{\delta}{\mu}x(t)y(t) + ae^{-\mu\tau}y^2(t)$$

$$\le -\left(1-\frac{\delta}{2\mu}\right)x^2(t) - (a(1-e^{-\mu\tau})+\delta+\mu-\frac{\delta}{2\mu})y^2(t) - \left(1-\frac{\beta p_0}{(\mu+r)k}\right)z^2(t)$$

$$\le -\left(1-\frac{\delta}{2\mu}\right)x^2(t) - (a(1-e^{-\mu\tau})+\delta+\mu-\frac{\delta}{2\mu})y^2(t) - (1-\overline{R})z^2(t)$$

If $\min\left\{\left(1-\frac{\delta}{2\mu}\right),(a(1-e^{-\mu\tau})+\delta+\mu-\frac{\delta}{2\mu})\right\}\geq 0$, then $V'(z)<0$. When $\bar{R}>1$,

According to the Lyapunov stability theorem [16], the only non-negative equilibrium point E_0 of model (1) is globally asymptotically stable. The proof is complete.

The stability of the equilibrium points

Theorem 3 If $\bar{R}\geq 1$, and $\min\left\{\mu\bar{R}-\frac{1}{2},\delta+\mu+a(1-e^{-\mu\tau})\right\}\geq\frac{\delta}{2(r+\mu)})$,then the equilibrium points E^* is globally asymptotically stable.

Proof: Let $x=P-P^*$, $y=R-R^*$, $z=M-M^*$, then model (1) can be deformed into

$$\begin{cases} x'(t)=(-\frac{\beta M^*}{k}-\mu)x(t)+\delta y(t)-\frac{\beta P^*}{k}z(t) \\ y'(t)=-\left(\mu+a+\delta\right)y(t)+ay(t-\tau)e^{-b\tau} \\ z'(t)=\frac{\beta M^*}{k}x(t)+[\frac{\beta P^*}{k}-(r+\mu)]z(t) \end{cases} \tag{2.7}$$

Construct the Lyapunov function $V_1(t)=\frac{x^2}{2(r+\mu)}+\frac{z^2k}{2\beta M^*}+\frac{y^2}{2}$, and derive the derivative of the orbit along the model (3.6).

$$\begin{aligned} \frac{dV_1}{dt}&=\frac{(-\frac{\beta M^*}{k}-\mu)}{(r+\mu)}x^2(t)+\frac{\delta}{(r+\mu)}x(t)y(t)+ae^{-\mu\tau}y(t)y(t-\tau)-(\mu+a+\delta)y^2(t) \\ &\quad+zx+\frac{K}{\beta M^*}(\frac{\beta P^*}{K}-(r+u))z^2 \qquad (2.8) \\ &\leq-\frac{1}{(r+\mu)}(\frac{\beta M^*}{k}+\mu)x^2(t)+\frac{\delta}{(r+\mu)}x(t)y(t)+\frac{ae^{-\mu\tau}}{2}(y^2(t)+y^2(t-\tau))-(\delta+a+\mu)y^2(t) \\ &\quad+zx+\frac{K}{\beta M^*}(\frac{\beta P^*}{K}-(r+u))z^2 \end{aligned}$$

Let $V(t)=V_1(t)+\frac{ae^{-\mu\tau}}{2}\int_{t-\tau}^{t}y^2(u)du$, we can obtain

$$\begin{aligned} \frac{dV}{dt}&\leq-\frac{1}{(r+\mu)}(\frac{\beta M^*}{k}+\mu)x^2(t)+\frac{\delta}{(r+\mu)}x(t)y(t)+zx+\frac{K}{\beta M^*}(\frac{\beta P^*}{K}-(r+u))z^2 \\ &\quad+\frac{ae^{-\mu\tau}}{2}(y^2(t)+y^2(t-\tau))-(\delta+a+\mu)y^2(t)+\frac{ae^{-\mu\tau}}{2}(y^2(t)-y^2(t-\tau)) \qquad (2.9) \\ &\leq-\frac{1}{(r+\mu)}(\frac{\beta M^*}{k}+\mu)x^2(t)+\frac{\delta}{(r+\mu)}x(t)y(t)-\left[(\delta+a+\mu)-ae^{-\mu\tau}\right]y^2(t) \\ &\quad+zx+\frac{K}{\beta M^*}(\frac{\beta P^*}{K}-(r+u))z^2 \\ &\leq-\frac{1}{(r+\mu)}(\frac{\beta M^*}{K}+\mu-\frac{\delta+r+\mu}{2(r+\mu)})x^2(t)-\left[(\delta+a+\mu-ae^{-\mu\tau}-\frac{\delta}{2(r+\mu)}\right]y^2(t) \end{aligned}$$

$$+zx+\frac{K}{\beta M^*}(\frac{\beta P^*}{K}-(r+u))z^2$$

$$\leq -\frac{1}{(r+\mu)}(\mu\bar{R}-\frac{1}{2}-\frac{\delta}{2(r+\mu)})x^2(t)-\left[(\delta+a+\mu-ae^{-\mu\tau}-\frac{\delta}{2(r+\mu)}\right]y^2(t)-\frac{1}{2\mu(\bar{R}\text{-}1)}z^2(t)$$

From (2.9), we can see that if $\min\left\{\mu\bar{R}-\frac{1}{2},\delta+\mu+a(1-e^{-\mu\tau})\right\}\geq\frac{\delta}{2(r+\mu)})$, then $\frac{dV}{dt}\leq 0$. When $\bar{R}>1$,

According to the LaSalle invariance principle [17], the non-negative equilibrium point E^* is globally asymptotically stable. The proof is complete.

By substituting the equilibrium point E_0 and E^* of model (4) and the fourth and fifth equations of model (1) into model (1), the non-negative equilibrium point of model (1) can be obtained as follows:

$$E_0=\left(\frac{1}{\mu}[qk+\frac{(1-q)\delta k}{\mu+\delta+a(1-e^{-\mu\tau})}],\frac{(1-q)\delta k}{\mu+\delta+a(1-e^{-\mu\tau})},0,\frac{(1-q)ak}{\mu[\mu+\delta+a(1-e^{-\mu\tau})]},0\right).$$

Similarly, the only positive equilibrium point of model (1) can be obtained as follows:

$$E^{*'}=\left\{\frac{k(r+\mu)}{\beta},\frac{(1-q)k}{\mu+\delta+a(1-e^{-\mu\tau})},\frac{(\bar{R}-1)\mu k}{\beta},\frac{(1-q)ak}{\mu[\mu+\delta+a(1-e^{-\mu\tau})]},\frac{rk(\bar{R}-1)}{\beta}\right\}.$$

According to the limit equation theory, if $\bar{R}<1$, $\min\left\{\left(1-\frac{\delta}{2\mu}\right),(a(1-e^{-\mu\tau})+\delta+\mu-\frac{\delta}{2\mu})\right\}\geq 0$. The non-negative equilibrium point is the global asymptotic stability point of model (1).

If $\bar{R}\geq 1$ and $\min\left\{\mu\bar{R}-\frac{1}{2},\delta+\mu+a(1-e^{-\mu\tau})\right\}\geq\frac{\delta}{2(r+\mu)})$, $E^{*'}$ Is the global asymptotic stability point of model (1).

NUMERICAL SIMULATION

(1) $\tau=0$ and $K=50$

We choose a set of parameters: $q=0.031$, $\delta=0.5$, $\beta=0.158$, $\mu=0.2$, $\alpha=0.3$, $r=0.3$, then $\bar{R}=0.8145$, The model has only one non-negative equilibrium point $E_0=(2.5775, 0.9690, 0, 1.4535, 0)$. Let $P(0)=4$, $R(0)=3$, $M(0)=0$, $U(0)=8$, $I(0)=0$, We can see E_0 is globally asymptotically stable from Fig. 1.

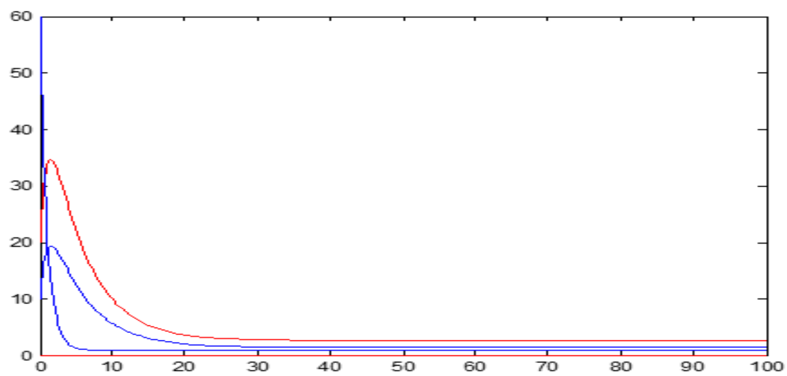

图2 非负平衡点的全局稳定性

Fig-2: The global stability of working-present equilibrium point

We choose a set of parameters: $\beta = 0.9$, $q = 0.05$, $\mu = 0.1$, $\delta = 0.01$, $\alpha = 0.01$, $r = 0.01$, then $\bar{R} = 10.5682 > 1$, Model has a stable unique positive equilibrium point in addition to a non-negative equilibrium point $E^* = (6.11, 395.83, 17.04, 39.58, 5.31)$ .Let $P(0)=2$, $R(0)=30$, $M(0)=700$, $U(0)=20$, $I(0)=10$ Let $P(0)=3$, $R(0)=30$, $M(0)=2$, We can see E^* is globally asymptotically stable from Fig. 2.

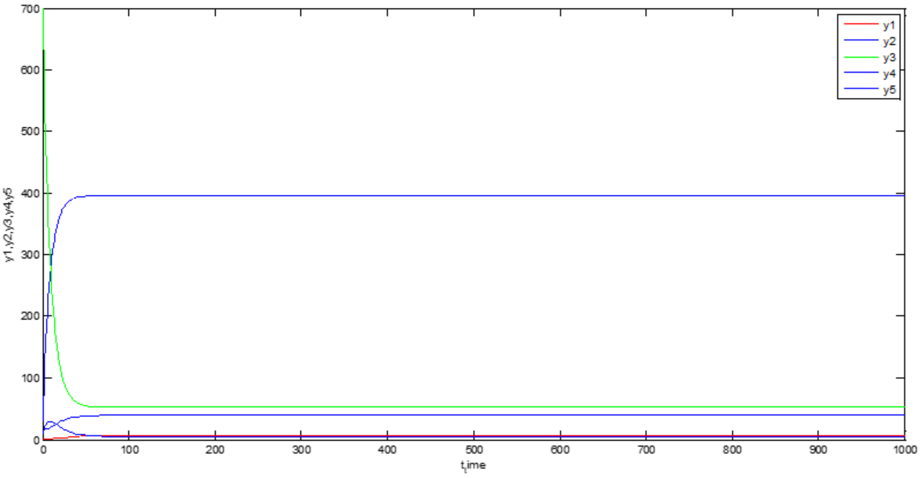

图3 唯一正平衡点的全局稳定性

Fig-3: The global stability of training again equilibrium point

(2) $\tau \neq 0$, $K = 50$

We choose a set of parameters: $q = 0.5$, $k = 50$, $\delta = 0.32$, $\beta = 0.25$, $\mu = 0.80$, $\alpha = 0.26$, $r = 0.22$, $\tau = 9$, then the model is

$$\begin{cases} \dfrac{dP}{dt} = 0.5K + 0.0050\mathrm{PM} - 0.2P + 0.32R \\ \dfrac{dR}{dt} = 0.5K - 1.1200R - 0.26R + 0.26R(t-1)e^{-0.8\tau} \\ \dfrac{dU}{dt} = -0.8U + 0.26R - 0.26R(t-1)e^{-0.8\tau} \\ \dfrac{dI}{dt} = 0.22M - 0.8I \end{cases} \quad (14)$$

By calculation $\bar{R} = 0.1966 < 1$, the model has only one non-negative equilibrium point $E_0 = (38.1625, 22.1201, 0, 0.2819, 0)$.Let $P(0)=6$, $R(0)=3$, $M(0)=2$, $U(0)=5$, $I(0)=3$, we can see E_0 is globally asymptotically stable from Fig. 1.

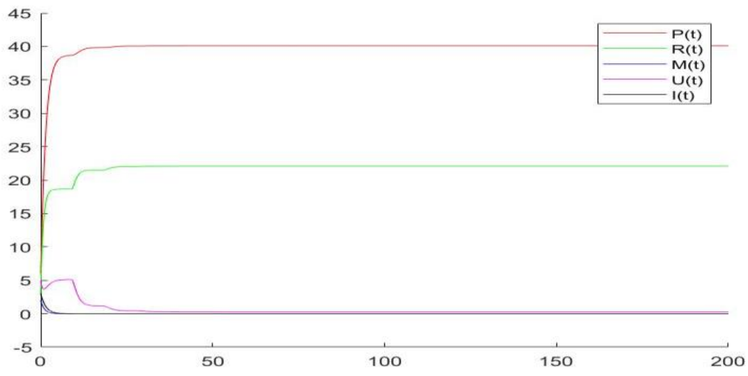

Fig-2: The global stability of working-present equilibrium point

We choose a set of parameters: $\beta = 0.89$, $q = 0.43$, $\mu = 0.05$, $\delta = 0.27$, $\alpha = 0.46$, $r = 0.97$, $k = 50$, $\tau = 19$, then the model is

$$\begin{cases} \frac{dP}{dt} = 0.43K - 0.0178PM + 0.27R - 0.05P \\ \frac{dR}{dt} = 0.5700K - 0.7800R + 0.46R(t-35)e^{-0.0050} \\ \frac{dM}{dt} = 0.0180PM - 1.0200M \\ \frac{dU}{dt} = -0.05U + 0.46R - 0.46R(t-35)e^{-0.0050} \\ \frac{dI}{dt} = 0.97M - 0.05I \end{cases} \quad (15)$$

By calculation $\overline{R} = 15.8370 > 1$, Model (15) has a stable unique positive equilibrium point in addition to a non-negative equilibrium point $E^* = (57.3034, 88.4285, 41.6770)$. Let $P(0)=3$, $R(0)=30$, $M(0)=2$, We can see E^* is globally asymptotically stable from Fig. 2.

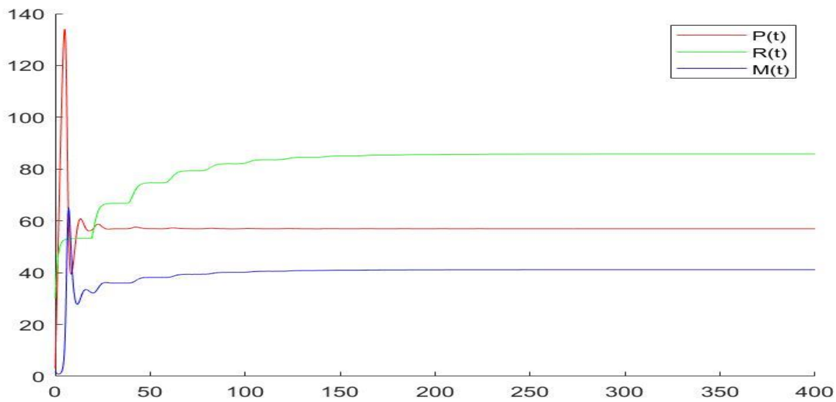

Fig-3: The global stability of training again equilibrium point

CONCLUSION

This study analyzed the stability of the retraining model of workers in the same department in private enterprises by means of the characteristic equation analysis method and Hurwitz decision theorem. The following conclusions are drawn from the research. When the number of returning workers is very large or the rate of transfer from returning workers to non-expendable workers is large, there is a non-negative equilibrium point in the model, and the retraining needs to be carried out continuously. This requires that the number of returning workers and the rate of transfer of returning workers to non-obsolete workers must be strengthened in real life. From the threshold conditions for worker retraining, one can see that the greater the transfer rate from returning workers to non-expendable workers, fewer the workers removed. The shorter the duration of retraining of workers who are non-expendable, the less likely it will be to retrain. Therefore, in order to control the reduction of the number of retraining workers who cannot be eliminated, it is necessary to a pay close attention to the important impact of workers' technical level, increase the proportion of retraining for returned workers, and reduce the transfer rate from returned workers to non-eliminated workers.

Acknowledgements

The authors would like to thank the anonymous referees for their careful reading of the original manuscript, and for their valuable comments and suggestions for improving the results as well as the exposition of this article. This work was supported by the National Natural Science Foundation of China (No.31702289)

REFERENCES

1. Xu Fei, LI Shu-min. The Stability Analysis of the Enterprise Competition Model withTwo Time Delays [J]. Journal of Jiangxi Normal University (Natural Science). 2018, 42(05):518-526.
2. Zhu Limei, LI Yong-xiang. Existence of Periodic Solutions for Second order Differential Equations with Multiple Delays [J].Journal of Jilin University(Science Edition). 2017,55(05):1077-1083.
3. Yan Rong-jun, Wei Yu-ming, FENG Chun-hua. Existence of Three Positive Solutions for Fractional Differential Equation of Boundary Value Problem with p-Laplacian Operator and Delay[J].Journal of Guangxi Normal University(Natural Science Edition). 2017,35(03):75-82.
4. Liu Xinag, Qiu Zhipeng. Global stability analysis of delayed vector-host epidemic model [J]. Journal of Nanjing University of Science and Technology. 2016,40(05):589-593.
5. INABA H. Threshold and Stability Results for an Age-structured Epidemic Model [J]. Math. Biol. 1990(28):411-413.
6. Kretzschmar M, Jager JC, etc. The Basic Reproduction Ratio R_0 for a Sexually Transmitted Disease in a Pair Formation Model with Two Types of Pairs [J]. Math. Bio sci, 1994(124):181-205.
7. Xue Ying, Xing Zuo-liang. Stability of an SIR Epidemic Model with Vaccinal Immunity and a Varying Total Population Size [J]. Acta Analysis Function Alias Application, 2007(9):169-175.
8. Agarwalm, Vermav. Stability and Hopf Bifurcation Analysis of a SIRS Epidemic Model with Time Delay [J]**.** J. of Appl. Math and Mech, 2012, 8(9),1-16.
9. Fred Brauer. Castillo-Chavez. Biomathematics-mathematical models in population biology and epidemiology [M]. Tsinghua university press, 2013.
10. Ye Jin-hua Zhu huan. Uniform Persistence for a Prey-predator System with Stage-structure and Holling Ⅲ-type Functional Response [J]. Journal of Heilongjiang Bayi Agricultural University. 2017,29(1)：144-146.
11. HUO Hai-feng, SHE Yu-xing, M ENG Xin-you. Stability of a class of smoking curtailing model accomodated to public health educational campaigns [J]. Journal of Lanzhou University of Technology, 2010(3):135-138.
12. Zhang zhi-shuang. Dynamic Properties of a Class of computer virus Transmission models with double delays [D]. HarBin Institute of Technology, 2013.
13. Fred Brauer Carlos Castillo-Chavez. Mathematical Models in Population Biology and Epidemiology [M]. Beijing: Tsinghua University Press, 2013, 107-136.
14. Wei Yu-fen, Zhu Huan. The construction and stability analysis of the differential dynamics model of the same skilled workers in private sector [J], Acta Mathematica Applicata Sinica, 2018,41(05):711-720.
15. Cheng Lan-sun. Mathematical Ecology Model and Research Method [M]. Beijing: Science Press, 1988: 391-393.
16. Zhang Jin-yan, Feng Bei-ye. Geometric Theory and Bifurcation Problem of Ordinary Differential Equation [M]. Beijing: Peking University Press. 2000: 1-177.
17. Ma Zhi-en, Zhou Yi-cang. Qualitative and Stable Methods for Ordinary Differential Equations [M]. Beijing: Science Press. 2001: 24-27.