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A New Member in the Lindley Class of Distributions with Flexible Applications

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Abstract **Review Article**

In this paper, a new lifetime distribution in the class of Lindley is developed and it is called "Doje Distribution". It is flexible in modeling lifetime data. The mathematical properties which include moments, the shape of the distribution, Quantile function, hazard function, survival function, stochastic ordering, mean deviation, Bonferroni and Lorenz curve, order statistic, and Renyi entropy have been studied. The maximum likelihood estimation has also been discussed. Two-lifetime data sets were utilized to demonstrate its usefulness. The results show that the proposed Doje distribution is better than Lindley, Ishita, Pareto, Chris-Jerry, Sushila, and Rama's distribution in the instances of the data used.

Keywords: Doje distribution, Exponential distribution, Estimation, Gamma distribution, Lindly distribution

1. INTRODUCTION

In the last two decades, research has taken a new turn in the direction of probability modeling due to the desire to model evolving life events that defy the use of standard probability distributions. Usually, one parameter distribution provides simplicity and mathematical tractability, and yet in many scenarios best fits real-life data except in very minor cases where many parameter distributions are used to elucidate features of a given event.

The journey to mixing proportion for developing new one parameter distributions started with the earlier work of [9] followed by numerous distributions in that class namely [7, 20, 23-30]. Extensions and modifications exist for both the Lindley class and other distributions. Important to this article among others are [3-6, 8, 10-19, 21, 22, 31, 32].

Two arguments against distributions in the class of Lindley distribution are that of simplicity of the method and that any so-called extension of Lindley is simply a change of mixing proportion and the shape parameter of the Gamma distribution which is a component of the mixture. However, considering various distributions in this class reveals that Lindley distribution is not as suitable for much real-life data as these other distributions. Hence, a need for the modification of the mixing proportion, the number of component mixtures, and the shape parameter of the Gamma distribution. One commonality among these class of distributions is that almost all of them possesses an increasing failure rate shape with their peculiarities contained in the volume of data and data structure. Consider for instance that the Lindley distribution does not fit the popular data on the survival times of Guinea pigs injected with studied by [2] while Chris-Jerry distribution fits it well.

In this paper, we propose a one-parameter lifetime distribution of the class of Lindley distribution and call it the Doje distribution. The pdf is a mixture of two distributions namely exponential distribution with scale parameter θ and gamma distribution having a shape and scale parameters 7 and θ respectively with a mixing proportion $p=\frac{\theta^6}{\theta^6+720}$. The mixture is of the form $f_{doje}(x,\theta) = pExp(x,\theta)+(1-p)\,Gamma(x,7,\theta)$. Hence, the pdf is given as follows;

$$f_{doje}(x) = \frac{\theta^7}{\theta^6 + 720}\left(1 + x^6\right)e^{-\theta x} \tag{1}$$

The cdf is given as

$$F_{Doje}(x) = 1 - \left\{1 + \frac{1}{\theta^6 + 720}\left(\theta^6 x^6 + 6\theta^5 x^5 + 30\theta^4 x^4 + 120\theta^3 x^3 + 360\theta^2 x^2 + 720\theta x\right)\right\}e^{-\theta x} \tag{2}$$

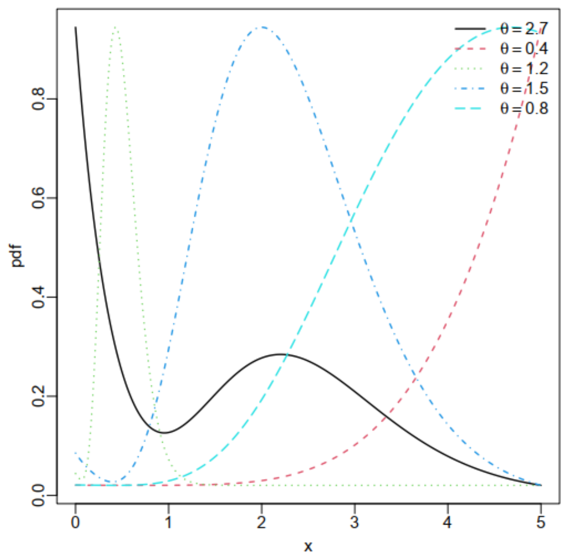

Figure 1: pdf of Doje distribution

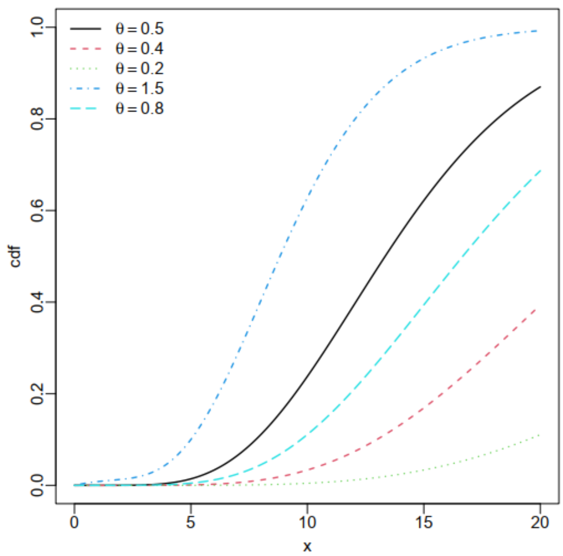

Figure 2: cdf of Doje distribution

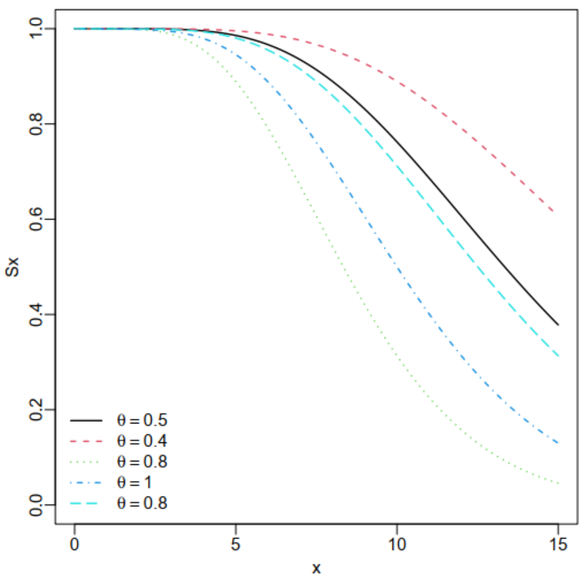

Figure 3: survival function of Doje distribution

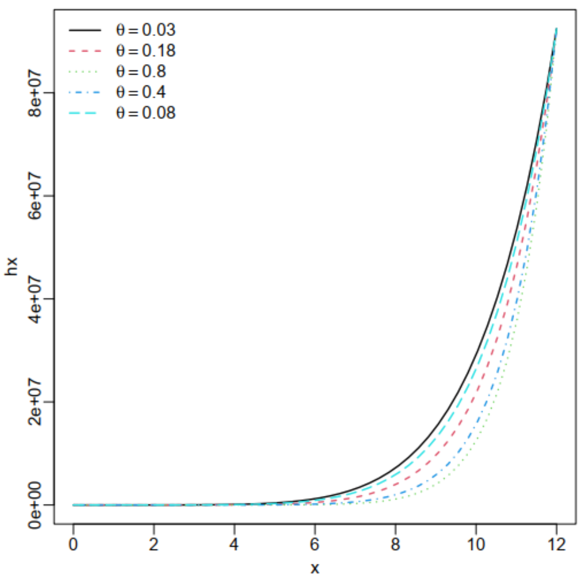

Figure 4: hazard function of Doje distribution

The motivation for the Doje distribution is to suggest another one-parameter lifetime distribution that will better model some scenarios than the existing one-parameter distributions. Doje distribution is also mathematically tractable except for the quantile function which is a very common issue in the literature. The simplicity of the distribution is credited to the scale parameter θ which is the only parameter. The distribution can perfectly fit an increasing hazard rate situation as can be seen in Figure 4.

The rest of this article is organized as follows; in section 2, the characteristics of the proposed distribution are derived and studied. In section 3, the estimation of the parameter using the method of maximum likelihood is carried out. We illustrate the importance of the proposed distribution using two-lifetime data sets and conclude the article in section 5.

2. Characteristics of Doje distribution

We discuss some characteristics of the proposed distribution.

2.1 Moment

Let $X \sim$ Doje (θ) distribution with pdf in eq 1, then the r^{th} crude moment is given by

$$\mu_k = E(x^k) = \int_0^\infty x^k f(x)dx = \frac{\theta^6 k! + (k+6)!}{\theta^k(\theta + 720)} \tag{3}$$

The first, second, third, and fourth crude moments are obtained by replacing k with 1,2,3 and 4 respectively in eq

$$\mu = \frac{\theta^6 + 5040}{\theta(\theta^6+720)}; \quad \mu_2' = \frac{2(\theta^6+20160)}{\theta^2(\theta^6+720)}; \quad \mu_3' = \frac{6(\theta^6+60480)}{\theta^3(\theta^6+720)}; \quad \mu_4' = \frac{24(\theta^6+151200)}{\theta^4(\theta^6+720)} \tag{4}$$

The variance is given as follows

$$\sigma^2 = \frac{\theta^{12} + 11520\theta^6 - 10886400}{\theta^2(\theta^6+720)^2} \tag{5}$$

2.2 Reliability Analysis

For $X \sim$ Doje (θ), survival function is given by

$$R_{doje} = \left\{1 + \frac{\theta}{\theta^6+720}(\theta^5x^6 + 6\theta^4x^5 + 30\theta^3x^4 + 120\theta^2x^3 + 36\theta x^2 + 720x)\right\}e^{-\theta x} \tag{6}$$

The Hazard or failure rate of the function is;

$$h(x) = \frac{\theta^7(1+x^6)}{\theta^6 + \theta^6x^6 + 6\theta^5x^5 + 30\theta^4x^4 + 120\theta^3x^3 + 360\theta^2x^2 + 720\theta x + 720} \tag{7}$$

The cumulative hazard function is given as follows;

$$CH_{Doje} = -ln(1-F_{Doje}) = -ln(1 + \frac{\theta}{\theta^6+720}(\theta^5x^6 + 6\theta^4x^5 + 30\theta^3x^4 + 120\theta^2x^3 + 36\theta x^2 + 720x) - \theta x \tag{8}$$

The reversed hazard rate function and odds function of Doje distribution are given as follows;

$$RH_x = \frac{f_{Doje}}{F_{Doje}} = \frac{\theta^7(1+x^6)}{\theta^6x^6 + 6\theta^5x^5 + 30\theta^4x^4 + 120\theta^3x^3 + 36\theta^2x^2 + 720\theta x} \tag{9}$$

$$Odd_{Doje} = \frac{F_{Doje}}{1-F_{Doje}} = \frac{\theta^6x^6 + 6\theta^5x^5 + 30\theta^4x^4 + 120\theta^3x^3 + 36\theta^2x^2 + 720\theta x}{\theta^6 + \theta^6x^6 + 6\theta^5x^5 + 30\theta^4x^4 + 120\theta^3x^3 + 36\theta^2x^2 + 720\theta x + 720} \tag{10}$$

2.3 The shape of Doje Distribution

The shape of the Doje distribution is obtained by first taking the derivative of the pdf and equating it to zero.

$$\frac{d}{dx}f(x) = \frac{\theta^7}{\theta^6+720}\frac{d}{dx}(1+x^6)e^{-\theta x} = \frac{\theta^7}{\theta^6+720}\left[-\theta e^{-\theta x} + 6x^5e^{-\theta x} - x^6\theta e^{-\theta x}\right] = 0; \quad -\theta + 6x^5 - \theta x^6 = 0; \quad x = \theta^{\frac{1}{5}} \quad or \quad x = \frac{6-\theta}{\theta} \tag{11}$$

2.4 Quantile function

The quantile function of Doje distribution is obtained using $F(x_q) = P(X \le x_q) = q$ for $0 < q < 1$.

$$\begin{aligned} q &= 1 - \left\{1 + \frac{1}{\theta^6+720}\left(\theta^6x^6 + 6\theta^5x^5 + 30\theta^4x^4 + 120\theta^3x^3 + 360\theta^2x^2 + 720\theta x\right)\right\}e^{-\theta x} \\ (\theta^6+720)(1-q) &= \left[\theta^6 + 720 + \theta^6x^6 + 6\theta^5x^5 + 30\theta^4x^4 + 120\theta^3x^3 + 360\theta^2x^2 + 720\theta x\right]e^{-\theta x} \end{aligned} \tag{12}$$

which has no closed-form solution.

2.5 Distribution of the Order Statistics

Suppose $X_1, X_2, ..., X_n$ is a random sample of $X_(r); (r = 1,2,..n)$ are the $r^(th)$ order statistics obtained by arranging X_r in ascending order of magnitude

$$f_{r:n}(x;\theta) = \frac{n!}{(r-1)!(n-r)!} f_{doje}[F_{doje}]^{r-1}[1-F_{doje}]^{n-r} = \frac{n!}{(r-1)!(n-r)!} \frac{\theta^7}{6+720}(1+x^6)e^{-\theta x}$$
$$\times \left\{1-\left[1+\frac{\theta}{\theta^6+720}\left(\theta^5x^6+6\theta^4x^5+30\theta^3x^4+120\theta^2x^3+360\theta x^2+720x\right)\right]e^{-\theta x}\right\}^{r-1}$$
$$\times \left\{\left[1+\frac{\theta}{\theta^6+720}\left(\theta^5x^6+6\theta^4x^5+30\theta^3x^4+120\theta^2x^3+360\theta x^2+720x\right)\right]e^{-\theta x}\right\}^{n-r} \quad (13)$$

The pdf of the largest order statistics is obtained by setting $r = n$

$$f_{n:n}(x;\theta) = \frac{n\theta^7}{6+720}(1+x^6)e^{-\theta x}\left\{1-\left[1+\frac{\theta}{\theta^6+720}\left(\theta^5x^6+6\theta^4x^5+30\theta^3x^4+120\theta^2x^3+360\theta x^2+720x\right)\right]e^{-\theta x}\right\}^{n-1} \quad (14)$$

The pdf of the smallest order statistics is obtained by setting $r = 1$

$$f_{1:n}(x;\theta) = \frac{n\theta^7}{6+720}(1+x^6)e^{-\theta x}\left\{\left[1+\frac{\theta}{\theta^6+720}\left(\theta^5x^6+6\theta^4x^5+30\theta^3x^4+120\theta^2x^3+360\theta x^2+720x\right)\right]e^{-\theta x}\right\}^{n-r} \quad (15)$$

2.6 Moment Generating Function

The moment-generating function of the Doje distribution is given as

$$M_x(t) = E(e^t) = \int_0^\infty e^{tx} f_x dx = \int_0^\infty e^{tx}\frac{\theta^7}{\theta^6+720}\left(1+x^6\right)e^{-\theta x}dx = \frac{\theta^7}{\theta^6+720}\left[(\theta-t)^{-1}+720(\theta-t)^{-7}\right] \quad (16)$$

The characteristics function of Doje distribution is given by

$$\psi_x(t) = \int_0^\infty e^{itx} f_x dx = \int_0^\infty e^{itx}\frac{\theta^7}{\theta^6+720}\left(1+x^6\right)e^{-\theta x}dx = \frac{\theta^7}{\theta^6+720}\left[(\theta-it)^{-1}+720(\theta-it)^{-7}\right] \quad (17)$$

2.8 Odd function

The odd function is the reliability tool for modeling data sets that show a monotone hazard rate. It is defined to be the ratio of the CDFs to the survival function

$$O(x;\theta) = \frac{F(x,\theta)}{S(x,\theta)} = \frac{-(\theta^6x^6+6\theta^5x^5+30\theta^4x^4+120\theta^3x^3+360\theta^2x^2+720\theta x)}{\theta^6+720+\theta^6x^6+6\theta^5x^5+30\theta^4x^4+120\theta^3x^3+360\theta^2x^2+720\theta x} \quad (18)$$

2.9 Stochastic ordering of Doje distribution

The stochastic ordering of a non-negative continuous random variable is a tool for comparing the behavior of system components. A random variable X is said to be smaller than another random variable Y in the stochastic order $(X \le_{st} Y)$if $F_Y(x) \ge F_Y(x)\ \forall_x$ Hazard order $(X \le_{hr} Y)$ if $h_X(x) \ge h_y(x)\ \forall_x$. Mean residual life order $(X \le_{mrl} Y) if\, m_X(x) \ge m_Y(x)\ \forall_x$ Likelihood ratio order $(X \le_{lr} Y)$ if $\frac{f_X(x)}{f_Y(y)}$ decreases in x.

This implies that $X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{st} Y \Rightarrow X \le_{mrl} Y$

Theorem 1. Let $X \sim Doje(\theta_1)$ and $Y \sim Doje(\theta_2)$. if $\theta_1 \ge \theta_2$ then $X \le_{lr} Y$ hence $X \le_{lr} Y$ hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$

$$\frac{f_x(x)}{f_y(x)} = \frac{\frac{\theta_1^7}{\theta_1^6+720}(1+x^6)e^{-\theta x}}{\frac{\theta_2^7}{\theta_2^6+720}(1+x^6)e^{-\theta x}} = \frac{\theta_1^7(\theta_2^6+720)(1+x^6)e^{-\theta_1 x}}{(\theta_1+720)(1+x^6)e^{-\theta_2 x}}$$

Taking the natural log of the ratio will yield

$$ln\frac{f_x(x)}{f_y(x)} = ln\frac{\theta_1^7(\theta_2+720)}{\theta_2^7(\theta_1+720)} + ln\frac{1+x^6}{1+x^6} + (\theta_2-\theta_1)x$$

Differentiating the natural log of the ratio w.r.t x will yield

$$\frac{d}{dx}ln\frac{f_x(x)}{f_y(x)} = \theta_2-\theta_1$$

If $\theta_2 > \theta_1, \frac{d}{dx}ln\frac{f_x(x)}{f_y(x)} < 0$, and $\frac{f_x(x,\theta_1)}{f_y(x,\theta_2)}$ is decreasing in x. (19)

2.10 Information and asymptotic behavior of Doje Distribution

Entropy is the quantity of uncertainty or randomness in a system. It is an information measure for non-negative $\omega \neq 1$. The Reny Entropy for Doje distributed random variable X is

$$R_\omega(x) = \frac{1}{1-\omega}\left\{log\int_0^\infty f(x)^\omega dx\right\} = \frac{1}{1-\omega}log\left\{\int_0^\infty \frac{\theta^7}{\theta^6+720}(1+x^6)e^{-\theta x}\right\}^\omega dx = \frac{1}{1-\omega}log\left\{\int_0^\infty \frac{\theta^{7\omega}}{(\theta^6+720)^\omega}(1+x^6)^\omega e^{-\theta\omega x}dx\right\} \quad (20)$$

From binomial theorem, $(1+x)^j = \sum_{j=0}^\infty \binom{j}{k}x^k$, therefore $(1+x^6) = \sum_{k=0}^\infty \binom{\omega}{k}(x^6)^k$ and

$$R_\omega(x) = \frac{1}{1-\omega}log\left\{\frac{\theta^7}{\theta^6+720}\int_0^\infty \sum_{k=0}^\infty \binom{\omega}{k}(x^6)^k e^{-\theta\omega x}dx\right\} = \frac{1}{1-\omega}log\left\{\frac{\theta^{7\omega}}{(\theta^6+720)^\omega}\sum_{k=0}^\infty \binom{\omega}{k}\frac{\Gamma_{6k+1}}{(\theta\omega)^{6k+1}}\right\} \quad (21)$$

2.11 Mean Residual Life Function

For a continuous distribution with pdf $f(x)$ and cdf $F(x)$, the mean residual life function is defined as

$$\begin{aligned} m(x) = E(X-x|X>x) &= \frac{1}{1-F(x)}\int_x^\infty [1-F(x)]dt \\ &= \frac{\theta^6 e{-}\theta x + 720e{-}\theta x + \gamma_{(7,x)} + 6\gamma_{(6,x)} + 30\gamma_{(5,x)} + 120\gamma_{(4,x)} + 360\gamma_{(3,x)} + 720\gamma_{(2,x)}}{\theta(\theta^6+720+\theta^6x^6+6\theta^5x^5++30\theta^4x^4+120\theta^3x^3+360\theta^2x^2+720\theta x)e{-}\theta x} \end{aligned} \quad (22)$$

2.12 Bonferroni and Lorenz Curve

Bonferroni indices have applications not only in Economics to study income and poverty distribution, but also in other fields like reliability, demography, insurance, and medicine. The Bonferroni and Lorenz curves are defined as

$$\begin{aligned} B(b) &= \frac{1}{p\mu_1'}\int_0^q xf(x)dx = \frac{\theta^8}{p(\theta^6+5040)}\left\{\frac{\gamma(2,q)}{\theta^2}+\frac{\gamma(8,q)}{\theta^8}\right\}; \quad \text{and} \\ L(b) &= \frac{1}{\mu_1'}\int_0^q xf(x)dx = \frac{\theta^8}{(\theta^6+5040)}\left\{\frac{\gamma(2,q)}{\theta^2}+\frac{\gamma(8,q)}{\theta^8}\right\} \end{aligned} \quad (23)$$

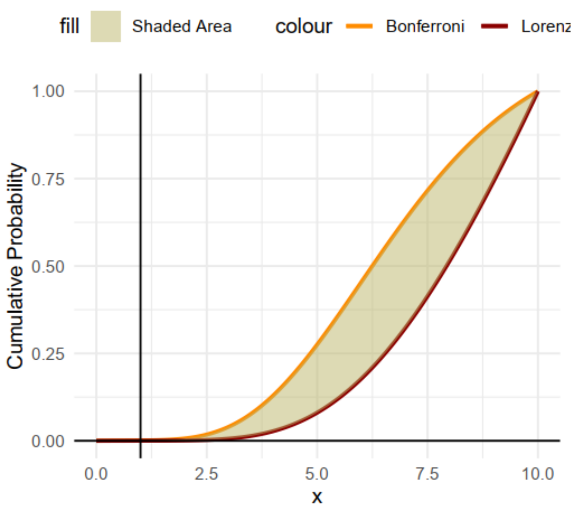

Figure 5: Bonferroni and Lorenz curve

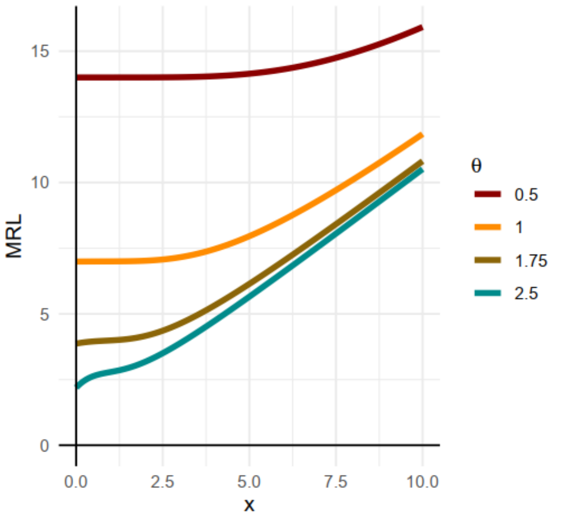

Figure 6: Mean Residual Function

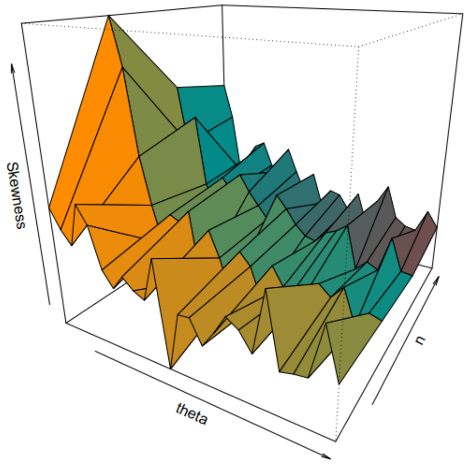

Figure 7: Skewness of Doje distribution

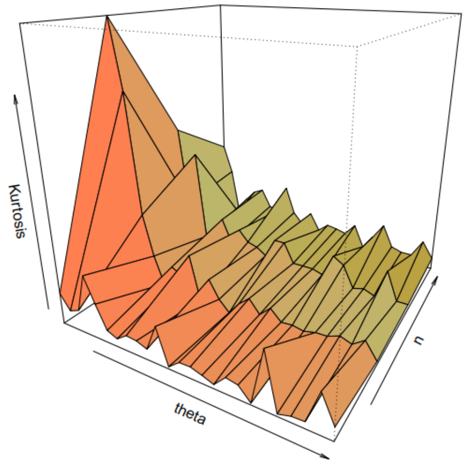

Figure 8: Kurtosis of Doje distribution

3 Maximum likelihood Estimation

Let $(x_1, X_2, ..., x_n)$ be random samples drawn from the Doje distribution, then the likelihood function is given as

$$\ell(f_{doje}(x,\theta)) = \prod_{i=1}^{n} \frac{\theta^7}{\theta^6+720}(1+x^6)e^{-\theta x} = \frac{\theta^{7n}}{(\theta^6+720)^n} e^{-\theta \sum x_i} \prod_{i=1}^{n}(1+x^6) \tag{24}$$

Taking the log of ℓ and differentiating with respect to θ yields the result below

$$\begin{aligned} \psi = \ell(x,\theta) = 7nln\theta - nln(\theta^6+720) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} ln(1+x^6) \\ \frac{d\psi}{d\theta} = \frac{7n}{\theta} - \frac{n}{(\theta^6+720} - \sum_{i=1}^{n} x_i = 0 \end{aligned} \tag{25}$$

4 Applications

In this section, two real-life data are used to illustrate the performance of the proposed model.

4.1 Application to Time-to-Failure of Turbocharger Data

The first data set refers to the time-to-failure (103h) of the turbocharger of one type of engine given in Xu et al. [32].

Table 1: The data consist of 40 observations. For these data, we compare the fits of the Doje distribution and those of other competitive

1.6	2.0	2.6	3.0	3.5	3.9	4.5	4.6	4.8	5.0	5.1	5.3	5.4	5.6	5.8	6.0	6.0	6.1
6.3	6.5	6.5	6.7	7.0	7.1	7.3	7.3	7.3	7.7	7.7	7.8	7.9	8.0	8.1	8.3	8.4	8.4
8.6	8.7	8.8	9.0														

The measures of model performance for the distributions are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), Hannan–Quinn information criterion (HQIC), Cramer von Mises (W^*), Anderson Darling (A^*), while the Kolmogorov-Smirnov (K-S) statistic and the p-value determine the fitness of the distribution to the data.

From table 2, the proposed model fits the data on time-to-failure of turbocharger more than the competing distributions with a $p-value$ =0.4998

Table 2: The Estimate of the parameter, analytical measures of fitness and model performance based on Time-to-Failure of Turbocharger data

Dist.	$\hat{\theta}$	-LL	AIC	CAIC	BIC	HQIC	W*	A*	K-S	P-value	Rank
Doje	1.1165	87.58	177.1565	177.2618	178.8454	177.7672	0.1900	1.2770	0.1309	0.4998	1
L	0.2844	104.3	210.6068	210.712	212.2956	211.2174	0.1861	1.2500	0.2948	0.0019	8
I	0.4605	95.70	193.4042	193.5095	195.0931	194.0149	0.1843	1.2394	0.2177	0.0452	5
A	0.4502	96.88	195.7624	195.8676	197.4512	196.3730	0.1709	1.1599	0.2298	0.0292	6
P	0.6251	91.81	185.6204	185.7257	187.3093	186.2311	0.1811	1.2214	0.1833	0.1360	2
CJ	0.4375	99.21	200.4134	200.5186	202.1023	201.0240	0.1546	1.0614	0.2628	0.0797	4
S	0.3016	102.0	205.9105	206.0158	207.5994	206.5212	0.1949	1.3013	0.2724	0.0053	7
RD	0.6211	92.18	186.3594	186.4646	188.0482	186.9700	0.1720	1.1676	0.1845	0.1313	3

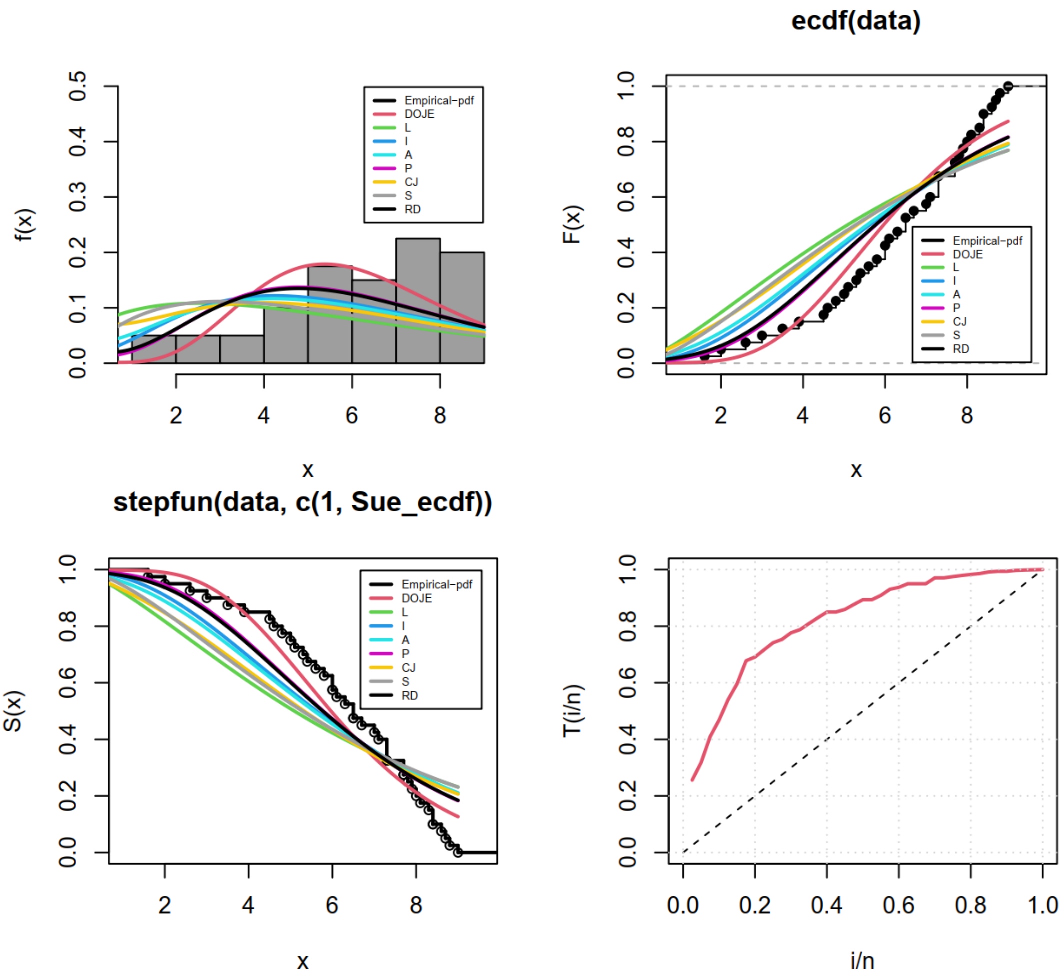

Figure 9: The density, empirical CDF, empirical survival function, and TTT-plots for the time-to-failure of Turbocharger data

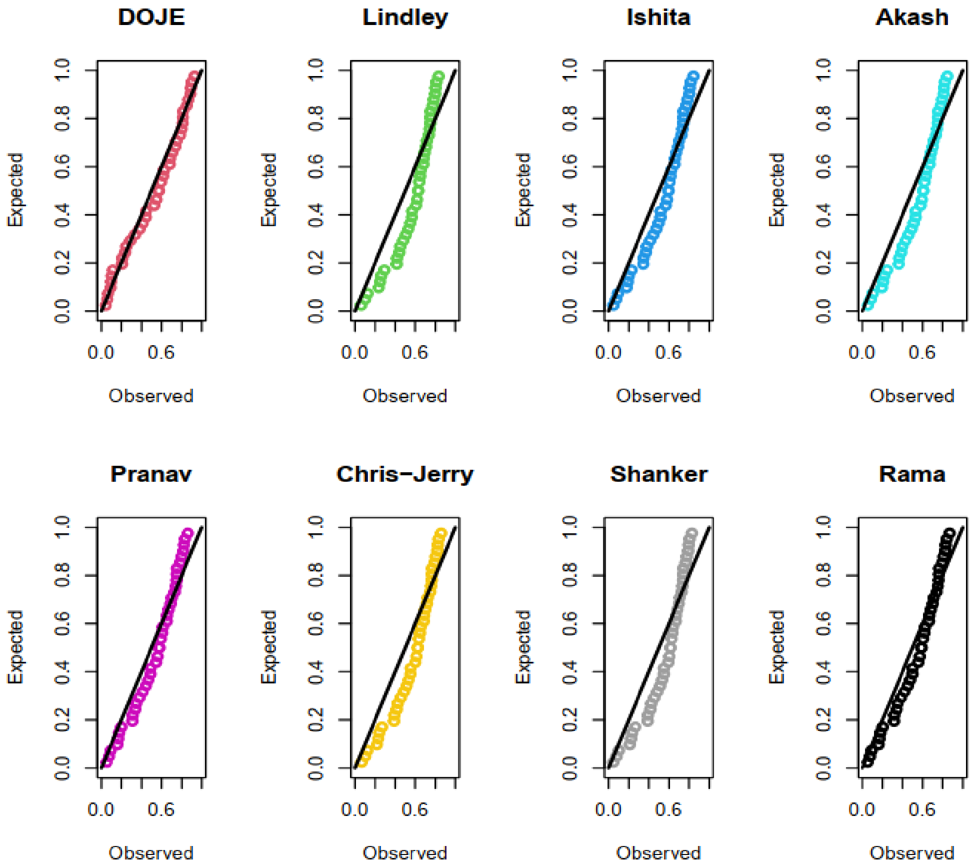

Figure 10: The PP-plot for time-to-Failure of Turbocharger data

4.2 Application to Ordered Lifetimes of Blood Cancer Patients Data

The second data represents 40 patients suffering from blood cancer (leukemia) from one Ministry of Health hospital in Saudi Arabia (see Abouammoh, Ahmad, and Khalique [1]). The ordered lifetimes (in years) are given

Table 3: Ordered Lifetimes of Blood Cancer Patients Data

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036	2.162	2.211	2.37
2.532	2.693	2.805	2.91	2.912	3.192	3.263	3.348	3.348	3.427	3.499	3.534
3.767	3.751	3.858	3.986	4.049	4.244	4.323	4.381	4.392	4.397	4.647	4.753
4.929	4.973	5.074	5.381								

Table 4: The Estimate of the parameter, analytical measures of fitness and model performance based on Ordered Lifetimes of Blood Cancer Patients data

Dist.	$\hat{\theta}$	-LL	AIC	CAIC	BIC	HQIC	W*	A*	K-S	P-value	Rank
Doje	2.0500	70.09	142.1735	142.2788	143.8624	142.7842	0.0827	0.5747	0.1038	0.7823	1
L	0.5270	80.25	162.5012	162.6064	164.1900	163.1118	0.2084	1.2974	0.2406	0.0195	8
I	0.8054	75.78	153.5668	153.6721	155.2557	154.1774	0.1527	0.9730	0.1697	0.1994	4
A	0.8006	76.34	154.6894	154.7947	156.3783	155.3000	0.1499	0.9562	0.1879	0.1185	5
P	1.0800	73.49	148.9727	149.0780	150.6616	149.5833	0.1072	0.7005	0.1421	0.3948	3
CJ	0.8024	77.07	156.1463	156.2515	157.8352	156.7569	0.1484	0.9474	0.2141	0.0510	6
S	0.5487	78.92	159.8358	159.9411	161.5247	160.4465	0.2107	1.3105	0.2147	0.0501	7
RD	1.1001	73.30	148.6007	148.7059	150.2896	149.2113	0.1090	0.7118	0.1393	0.4191	2

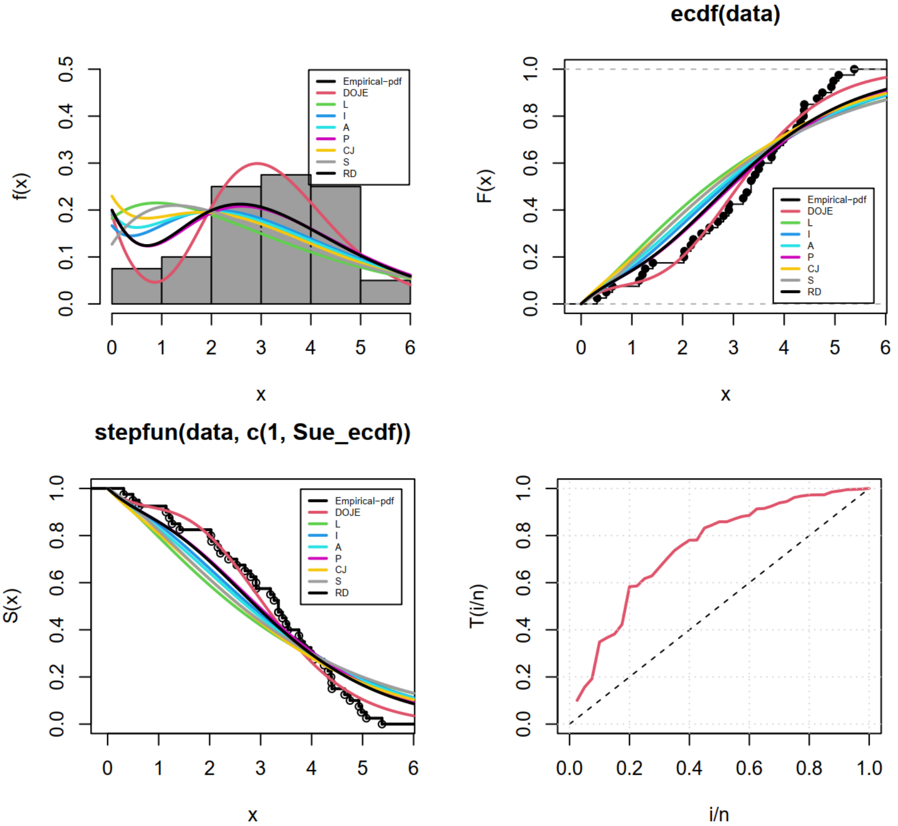

Figure 11: The density, empirical CDF, empirical survival function, and TTT-plots for Ordered Lifetimes of Blood Cancer Patients Data

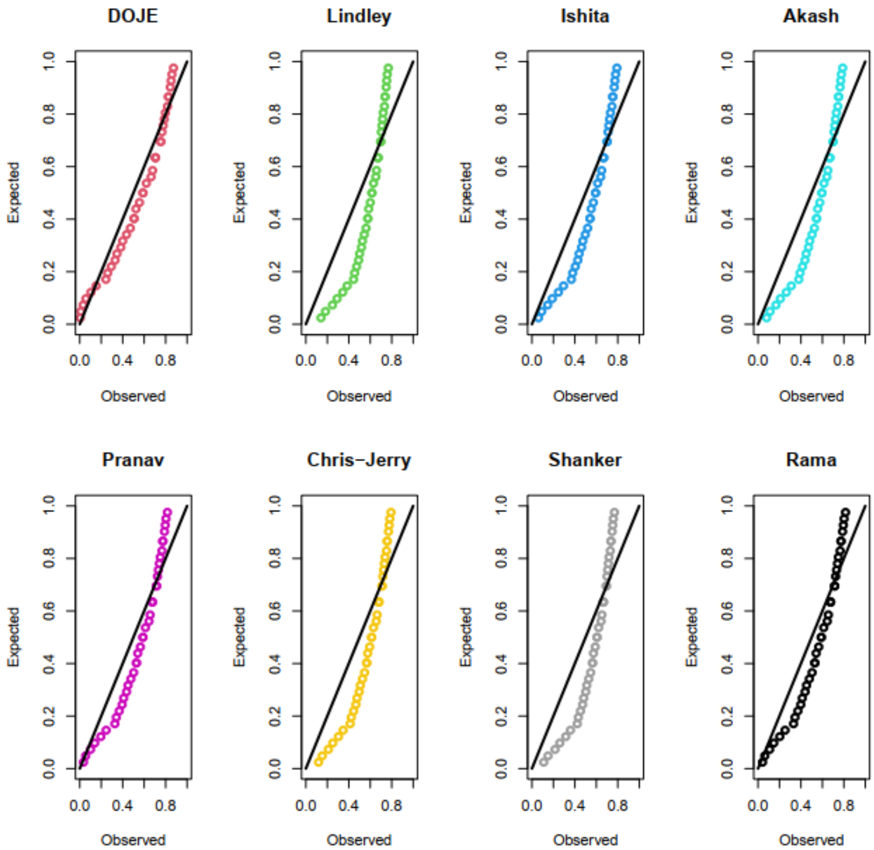

Figure 12: The PP-plot for Ordered Lifetimes of Blood Cancer Patients Data

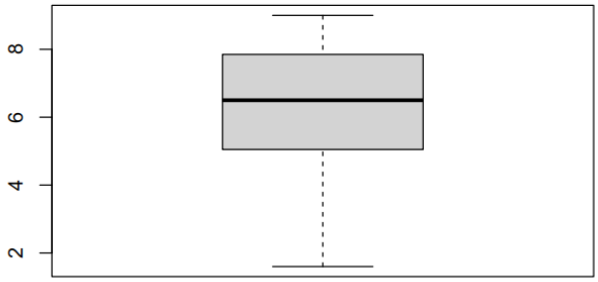

Figure 13: Boxplot for time-to-Failure of Turbocharger data

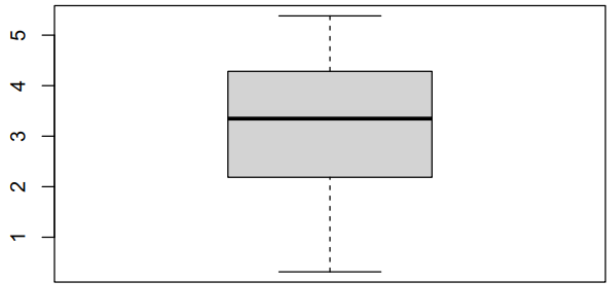

Figure 14: Boxplot for Ordered Lifetimes of Blood Cancer Patients Data

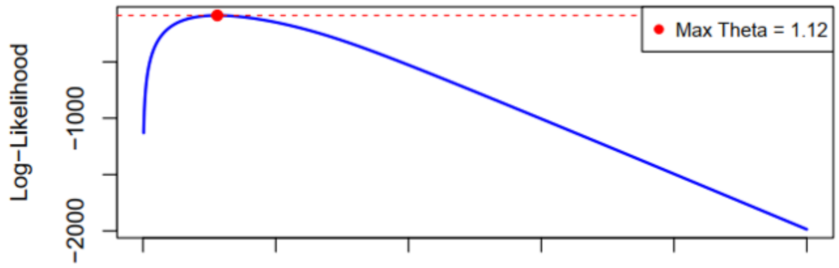

Figure 15: Profile log-likelihood for time-to-Failure of Turbocharger data

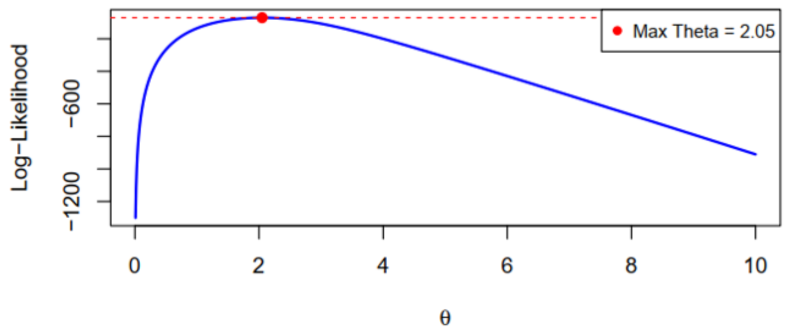

Figure 16: Profile log-likelihood for Ordered Lifetimes of Blood Cancer Patients Data

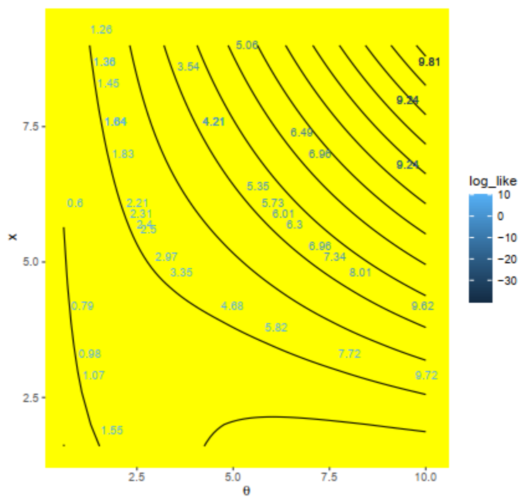

Figure 17: contour for a Doje loglike time-to-failure of Turbocharger data

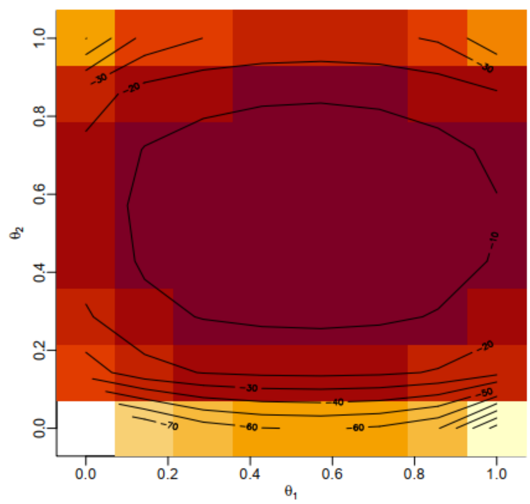

Figure 18: contour for mixture of two Doje loglike time-to-Failure of Turbocharger data

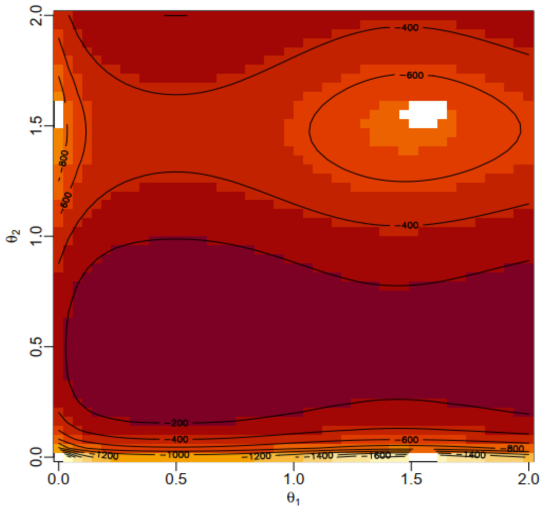

Figure 19: contour for a mixture of two simulated Doje loglike

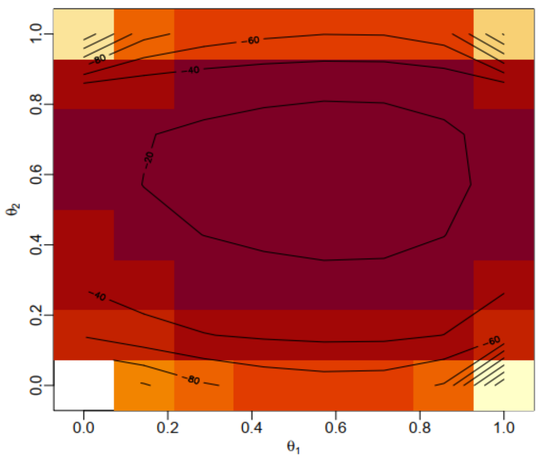

Figure 20: contour for a mixture of two Doje loglike for Ordered Lifetimes of Blood Cancer Patients Data

Figures 14, 15, and 16 shows no correlation between the two Doje-distributed random variables. Figure 14 we closed contours suggesting higher density or concentration of data which is also observed in figure 16 while in figure 13 and 15, there is a spread-out of the data suggesting lower density. The outlier points are visible in the four contour plots.

5. CONCLUSION

A new lifetime distribution was proposed in this article. The statistical properties such as moments, moment generating function, characteristic function, Order statistic, stochastic ordering, odd function, Renyi entropy, Bonferroni and Lorenz curve, survival, and hazard function were studied. The maximum likelihood function was also discussed. The data used to fit the distribution was timeto-failure turbocharger data and Blood cancer patients data. The AIC, BIC, -LL, CAIC, and P-value of Doje distribution and other distributions such as Lindley, Ishita, Chris-jerry, and Rama were compared, and Doje was found to have a better fit. The histogram shows that Doje is well-fitted.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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