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# Modeling of Extreme Temperature Using min-Generalized Extreme Value Distribution

H. A. Alaswed[1*]

[1]Assistant Professor at Statistics Department, Faculty of Science, Sebha University, Libya

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***Corresponding author:** H. A. Alaswed
Assistant Professor at Statistics Department, Faculty of Science, Sebha University, Libya

**Abstract** **Original Research Article**

Extreme value theory (EVT) is one of major importance in many fields of applications where extreme values may appear and have detrimental effects and finally Generalized Extreme Value (GEV) distribution model is found as the best fitted distribution model. Extreme minimum temperature using 40 years of data is studied. Minimum of four different time periods (monthly, quarterly, half yearly and yearly) are fitted to the minimum Generalized Extreme Value (MGEV) distribution. The first objective of this study is to describe and model the behaviour of extreme minimum temperature in Sebha by using the MGEV distribution. The second object, we determine the effect of using different size blocks, and illustrate the best four different time (Monthly, Quarterly, Half yearly and yearly) selection periods that are suitable for modelling with the MGEV distribution. The method of probability-weighted moments (PWM) has been used to estimate the unknown parameters; and its corresponding Deviance Test (DT) approach to test the goodness of fit. The results show that minimum Weibull distribution as special case of MGEV distribution is the most an appropriate choice of all selection periods (quarterly half yearly and yearly) are significant except only monthly of all three periods are significant to be fitted by minimum Frechet model. We will use the R programming with packages of fExtreme, evir and ismev to calculate, parameter estimation, testing and diagnostic plots.

**Keywords**: Application on real data; Deviance test (DT); Minimum Generalized Extreme Value Distribution (MGEVD); Model checking; Probability-weighted moments (PWM).



## 1. INTRODUCTION

Extreme value theory (EVT) is unique as a statistical discipline in that it develops techniques and models for describing the unusual rather than the usual. By definition, extreme values (EV) are scarce, meaning that estimates are often required for levels of a process that are much greater than have already been observed. This implies an extrapolation from observed levels to unobserved levels, and EVT provides a class of models to enable such extrapolation. EV techniques are also becoming widely used in many other disciplines. For example: insurance and finance to meteorology and hydrology. Early references include the work of [1], who identified one possible limit distribution for maxima. In [2] showed that extreme limit laws can be only one of three types are called the Gumbel, Frechet, and Weibull distributions respectively. In, [2] presented some simple and useful sufficient conditions for the weak convergence of the largest order statistic to each of the three types of limit distributions. In [3] established a rigorous foundation of the EVT when he provided necessary and sufficient conditions for the weak convergence of the sample extremes. His work was refined by [4]. There are many areas where EVT plays an important role; see, for example, [5]. In [5] describes the common approaches of EV analysis including the block minima (or maxima) method and the threshold excess models [6] focus particularly on applications of extreme value analysis (EVA) in the engineering areas. In [7] we use EV methods to model hydrological and droughts, also design and analyse experiments to compare treatments with extreme responses, using corrosion experiments to illustrate their approach. by, [8] also summarizes the minima and maxima domain of attraction of these three types of parametric limiting distributions. Methods for modelling extremes of natural phenomena, such as winds, temperatures, waves and floods, are based on the generalized extreme value distribution (GEVD). In, [8-10], the GEVD is useful when the data consist of a set of minima or maxima. EVT is unique as a statistical discipline in that it develops techniques and models for describing the unusual rather than the usual. EVT, differs from other typical statistical techniques and it is based on the analysis of the minima (or maxima) value in a selected time period.

The remainder of this paper is organized as follows: Section 2 briefly describes the theoretical of minimum generalized extreme value (MGEV) distribution. Section 3, introduces the Fitting of the GEV Distribution by using probability weighted moments. While, the model Checking is presented in section 4. Section 5, explores the case study. Section 6, introduced the results and discussion. While, Section 7, provides some summary conclusions and suggests some areas for future research.

## 2. Minimum GEVD (min-GEVD)

The three limiting distributions (Gumbel, Frechet and Weibul) are embedded in the minimum generalized extreme value (MGEV) distribution with a probability density function (pdf) is given by:

$$g_{\min}(x)=\begin{cases}\frac{1}{\sigma}[1+\gamma(\frac{x-\mu}{\sigma})]^{\frac{1}{\gamma}-1}\exp\{-[1+\gamma(\frac{x-\mu}{\sigma})]^{\frac{1}{\gamma}}\} & \gamma\neq 0\\ \frac{1}{\sigma}\exp(\frac{x-\mu}{\sigma})\exp\{-\exp[\frac{x-\mu}{\sigma}]\} & \gamma=0\end{cases} \qquad 1$$

Where the support is $x\geq\mu-\frac{\sigma}{\gamma}$ if $\gamma>0$ or $x\leq\mu-\frac{\sigma}{\gamma}$ if $\gamma<0$.

The three shape parameters are $\gamma=0$, $\gamma>0$ and $\gamma<0$ that contain distribution of Gumbel (symmetric-tailed), Frechet (heavy-tailed) and Weibull (thin-tailed) respectively, and the cumulative density function (cdf) is given by:

$$G_{\min}(x)=\begin{cases}1-\exp\{-[1+\gamma(\frac{x-\mu}{\sigma})]^{\frac{1}{\gamma}}\} & \gamma\neq 0\\ 1-\exp\{\exp-[\frac{x-\mu}{\sigma}]\} & \gamma=0\end{cases} \qquad 2$$

and the corresponding p-quantile is:

$$x_p=\begin{cases}\mu+\sigma[1-(-\log(1-p))^{\gamma}]/\gamma & \gamma\neq 0\\ \mu+\log(-\log(1-p)) & \gamma=0\end{cases} \qquad 3$$

In general, MGED is generalised representation of the following three specific form of distributions are named:

### 2.1Gumbel, Frechet and Weibull as MGEVDs

The limiting distribution of the minima belongs to one of the three forms known as the Gumbel, Frechet, and Weibull families (and there are corresponding distributions for maximum that we will not explicitly consider here). Here is a list of the three different sub-models by writing down as special cases of **min-GEVD**:

#### 2.1.1 Minimum Gumbel or reversed Gumbel

The minimum Gumbel (pdf) and (cdf) are given by:

$$g_{\min}(x)=\frac{1}{\sigma}\exp\{\frac{x-\mu}{\sigma}\exp-[\frac{x-\mu}{\sigma}]\},\gamma=0 \quad G_{\min}(x)=1-\exp\{\exp-[\frac{x-\mu}{\sigma}]\},\gamma=0 \qquad 4$$

#### 2.1.2 Minimum Frechet or reversed Frechet

The minimum Fréchet (pdf) and (cdf) are given by:

$$f_{\min}(x)=\begin{cases}\frac{\gamma\sigma}{(\mu-x)^2}\frac{\sigma}{\mu-x}\exp[-(\frac{\sigma}{\mu-x})^{\gamma}], & x<\mu,\gamma>0\\ 1 & x>\mu\end{cases}$$

$$F_{\min}(x)=\begin{cases}1-\exp[-(\frac{\sigma}{\mu-x})^{\gamma}], & x\leq\mu,\gamma>0\\ 1 & x>\mu\end{cases} \qquad 5$$

#### 2.1.3 Minimum Weibull distribution:

The minimum Weibull (pdf) and (cdf) are given by:

$$w_{\min}(x)=\frac{\lambda}{\sigma}(\frac{x-\mu}{\sigma})^{\gamma-1}\exp[-(\frac{x-\mu}{\sigma})^{\gamma}],\ x>\mu,\gamma<0 \quad W_{\min}(x)=1-\exp[-(\frac{x-\mu}{\sigma})^{\gamma}],\ x\geq\mu,\gamma<0 \qquad 6$$

The three types may be combined into a single GEV) distribution as in eq-1. in relationship between three distribution, for a frechet distribution can be transformed to Gumbel distributions by the simple transformations, $g=\log(x-\mu)$ also for Weibull distribution can be transformed to Gumbel distributions by the simple transformations, $g=-\log(\mu-x)$.

### 2.2 Block minimum

The choice of the length of blocks implies a trade-off between bias and variance. When the length of the blocks is small, then the approximation of the distributions by the limit is quite poor and this is leading to bias in estimation and extrapolation. Whereas the long blocks on the other hand generate only few data leading to large estimation variance. Consider the collection of $n$ returns, $\{r_1, r_2, ..., r_n\}$. The minimum return of the collection is $r_1$, that is, the smallest order statistic, whereas the maximum return is $r_n$, the maximum order statistic. Specifically, $r_1 = \min_{1\le j\le n}(r_j)$ and $r_n = \max_{1\le j\le n}(r_j)$. Following, we focus on properties of the minimum return $r_1$. One of the ideas used in the literature is to divide the sample into subsamples and apply the EVT to the subsamples. Assume that there are $T$ returns $\{r_j\}_{j=1}^{T}$ available. We divide the sample into $g$ non-overlapping subsamples each with $n$ observations, assuming for simplicity that $T=ng$. In other words, we divide the data as:

$$\{r_1,...,r_n \left| r_{n+1},...,r_{2n} \right| r_{2n+1},...,r_{3n} \left| ... \right\| r_{(g-1)n+1},...,r_{ng} \Big|\} \quad 7$$

and write the observed returns as $r_{in+j}$, where $1 \le j \le n$ and $i=0,1,...g-1$. Note that, each subsample corresponds to a sub-period of the data span. When $n$ is sufficiently large, we hope that the EVT applies to each subsample. Let $r_{1,i}$ be the minimum of the $i^{th}$ subsample (i.e., $r_{1,i}$ is the smallest return of the $i^{th}$ subsample), where the subscript $n$ is used to denote the size of the subsample. When $n$ is sufficiently large, $x_{1,i}=(r_{1,i}-\beta)/\alpha$ should follow an EVD, and the collection of subsample minima $\{ r_{1,i} : i=1,2,...,g \}$ can then be regarded as a sample of g observations from that EVD. Specifically, we define:

$$r_{1,i} = \min_{1\le j\le n}\{r_{(i-1)n+j}\},\ i=1,2,...,g \qquad 8$$

The collection of subsample minima $\{ r_{1,i} \}$ is the data we use to estimate the unknown parameters of the EVD. Clearly, the estimates obtained may depend on the choice of sub-period length $n$. When $T$ is not a multiple of the subsample size $n$, several methods have been used to deal with this issue. First, one can allow the last subsample to have a smaller size. Second, one can ignore the first few observations so that each subsample has size $n$, for more details, see [11, 12].

### 3. Fitting the min-GEV Distribution

The min-GEV distribution can be fitted using various methods. We focus on probability-weighted moments (PWM), it has described how a continues distribution can be fitted with PWM in [10]. The PWM method is a variation of the method of moments. For a continuous random variable $x$ with a pdf $f(x;\theta)$ and a cumulative distribution function $F(x;\theta)$, the PWM estimators are obtained by setting the first $k$ weighted-moments of the random variable equal to the corresponding weighted-sample moments, then solving the resultant system of equations. More precisely, let:

$$M(r,s,t)=E\{X^r[F(x;\theta)]^s[1-F(x;\theta)]^t\}, \quad 9$$

Where r, s, and t are real numbers, be the probability-weighted moments of order $r, s$ and $t$ of the random variable $x$ (see, [13]). PWM is most useful when the inverse distribution function $X_p = F^{-1}(p;\theta)$ can be written in closed form, for then we may write:

$$M(r,s,t)=\int_{-\infty}^{\infty} X^r[F(x;\theta)]^s[1-F(x;\theta)]^t\} \qquad 10$$

Or

$$M(r,s,t)=\int_0^1 X^r p^s[1-p]^t \qquad 11$$

Where we have made the change of variable $p=F(x_p;\theta)$. The corresponding weighted-sample moments are:

$$m(r,s,t)=\frac{1}{n}\sum_{i=1}^{n} x_{i:p}^r p_{i:n}^s[1-p_{i:n}]^t \qquad 12$$

Where $x_{i:p}^r$ is the $i^{th}$ sample order statistic and $p_{i:n}=\frac{i}{n+1}, \quad i=1,2,...,n$ is a corresponding plotting position. For appropriate choices of $\alpha,\beta \ge 0$. Other alternative can be found in [10].

The PWM estimators are then found by solving the system of equations:

$$M(r,s,t)=m(r,s,t) \ldots\ldots\ldots\ldots\ldots\ldots 13$$

and the resultant estimators are denoted by $\hat{\theta}_{PWM}$,

In practice, such computations are nearly always carried out using some package, evir or fExterem allows for fitting GEV distribution, for more details see [14-16].

## 4. Model Checking

The reason for fitting a statistical model to data is to make conclusions about some aspect of the population from which the data were drawn.

### 4.1 Selecting Models by Graphical Methods

In this section, three important graphical plots to select a suitable to make conclusions about some aspect of the population from which the data were drawn. Now, we will focus on some graphical methods for checking whether a fitted model is in agreement with the data such that, probability -paper (P-P) plot, Quantile Quantile (Q-Q) plot and Return level (R-L) plot by [9], which are often more informative for our purposes and that deals with the problem of selection models by diagnostic plots. Moreover, many popular estimation methods from EVT turn out to be directly based on these graphical tools. Firstly, we describe the P-P Plot.

#### 4.1.1 P-P Plot in min-GEVD

A probability plot is a comparison of the empirical and fitted distribution functions. The empirical distribution function evaluated in the $i^{th}$ ordered block minimum, $P_{i:n}$ and the fitted distribution function in the same point is:

$$\hat{F}(x_i) = \exp\{-[1+\hat{\gamma}\frac{x_i-\hat{\mu}}{\hat{\sigma}}]\frac{-1}{\hat{\gamma}}\} \qquad 14$$

In practise the plot of points are:

$$(\hat{F}(x_i), \frac{i}{n+1}, i = 1,2,...n) \qquad 15$$

When a P–P plot is employed in the EV model, if there is a strong deviation of the P–P plot from the main diagonal in the unit square indicates that the given model is incorrect (or the estimates of the location and scale parameters are inaccurate) or the EV model is untenable.

#### 4.1.2 Q–Q Plots in min- GEVD

In EV model, one must keep in mind that there is an additional parameter, namely the shape parameter, besides the location and scale parameters. We suggest applying a Q–Q plot with the unknown shape parameter having been replaced by an estimate. Q-Q plot is useful for the visual discrimination between distributions. The quantile plot is a representation of the points:

$$(\hat{F}^{-1}(\frac{i}{n+1}), x_i, i = 1,2,...n) \qquad 16$$

In the ideal situations the plot should show a linear function. Departures from linearity in the quantile plot also indicate model failure. If there is a stronger deviation of the Q–Q plot from a straight line, then either the estimate of the shape parameter is inaccurate or the model selection is untenable. If the parametric model fits the data well, this graph must have a linear form. Thus, the graph makes it possible to compare various estimated models and choose the best. The more linear the P-P plot and Q-Q plots, the more appropriate the model in terms of goodness of fit. Also, if the original distribution of the data is more or less known, the P-P plot and Q-Q plots can help to detect outliers.

#### 4.1.3 Return level (RL) in min-GEVD

The return level play an important role in the extreme value analysis. The return level plot represents the points:

$$(\log(-\log(1-p_{i:n})), \log x_{i:n}), 0 < p < 1 \qquad 17$$

Confidence intervals are usually added to this plot to increase its information. Furthermore, to use this plot as a model diagnostic one, the empirical estimates of the return level function are also added. For suitable models the model based curve and empirical estimates should be in agreement.

### 4.2 Selecting Models by Hypothesis Testing

For testing the data come from a d.f. of Gumbel defined in (3), the hypothesis testing can be written as:

$$H_0 : \gamma = 0 \;\; versus\, H_1 : \gamma \neq 0 \;\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots. \; 18$$

So under $H_0$, on the Deviance test (DT) is defined as:

$$DT = 2\{\log(L_1)\log(L_0)\} \approx \chi^2_{1,1-\alpha} \;\; \ldots\ldots\ldots\ldots\ldots\ldots \; 19$$

Where $L_0$ and $L_1$ are the value of the log likelihood under the $H_0$ and $H_1$ hypothesis respectively. Under $H_0$ the $DT$ is with a $\chi^2_1$ distribution, The $H_0$ will be rejected at significance $\alpha$ -level if $DT > \chi^2_{1,1-\alpha}$ or P-value less than 0.05 ( $P-value < 0.05$). For more details see [8]. Here, we are particularly interested in the case $\gamma < 0$, motivated by the conclusion drawn in the graphical preliminary analysis. Some recent references for tests selection of extreme value models, see [6, 8].

## 5. Case Study

Studies on extreme temperatures are beneficial to human understanding of extreme events will benefit from knowledge about the behaviour of minimum observation of -temperatures, as appropriate policies and plans can be drawn to prepare the general public for changes due to extreme temperatures. The data employed in this study consist of minimum temperature from the Sebha Meteorological Department during the period from 1981-2020 from consist of 40 year and we partition the sample size in four period (g=4) into non-overlapping sub-periods (monthly, quarterly, half yearly and yearly) to obtain estimates of the scale, location and shape parameters for the subperiod of minima { $r_{1,i}$ }. The first period of the data from 1981-200 consist of 20 year minimum blocks and the second period from 2001-2020 consist of 20 year blocks. The objective here is to implement various EVT to model extreme min-temperature. Data analysis is carried out using R package of fExtreme, evir and ismev for descriptive statistics, parameter estimation and diagnostic plots, see [14-16].

## 6. RESULTS AND DISCUSSION

The first step for data analysis is to see the graphical behaviour and present descriptive statistical summary for min-temperature data. The block minima of three periods are displayed in Fig 1.

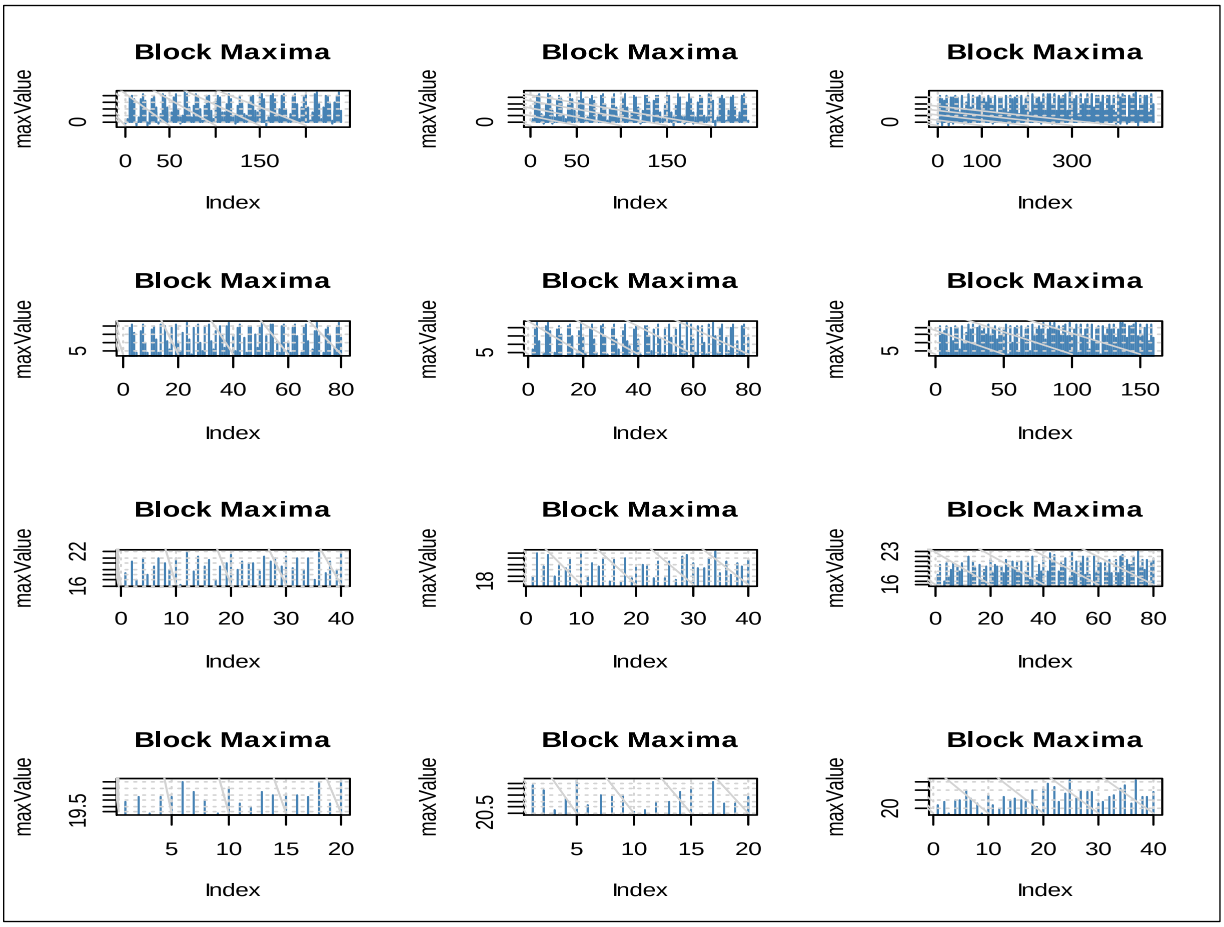


**Fig 1: Left panel: plots the yearly block minima from 1981-2000. Mid panel from 2001-2020 and right panel from 1981-2020**

### 6.1 Summary Statistics

Firstly, it is useful to have an overview of the statistical behaviour of the given data, it is useful to have an idea about the tail index of the underlying distribution. we present some results based on descriptive statistics of min-temperature application data and the results of four period (Monthly, Quarterly, Half yearly and Yearly) of each choices of part are reported respectively in Table-1 to Table-3.

**Table 1: Descriptive statistics of first period**

| Period | First period (1981-2020) | | | |
|---|---|---|---|---|
| Statistic | **M** | **Q** | **H** | **Y** |
| Min | 11.99 | 13.40 | 16.23 | 19.39 |
| Max | 22.2 | 22.2 | 22.2 | 22.2 |
| Mean | 10.59 | 14.13 | 19.75 | 20.86 |
| Median | 10.21 | 15.85 | 19.95 | 20.87 |
| **Skewnes** | **0.02** | **0.45-** | **0.42-** | **0.07-** |

M= **M**onthly, Q= **Q**uarterly, H= **H**alf yearly, Y=**Y**early

**Table 2: Descriptive statistics of second period**

| Period | second period (2001-2020) | | | |
|---|---|---|---|---|
| Statistics | **M** | **Q** | **H** | **Y** |
| Min | 12.88 | 14.93 | 17.62 | 20.47 |
| Max | 23.33 | 23.33 | 23.33 | 23.33 |
| Mean | 11.62 | 14.84 | 20.66 | 21.75 |
| Median | 11.14 | 17.87 | 20.77 | 21.85 |
| **Skewne** | **0.04** | **0.50-** | **0.28-** | **0.21-** |

**Table 3: Descriptive statistics of total period**

| Period | Total period (1981-2020) | | | |
|---|---|---|---|---|
| Statistics | **M** | **Q** | **H** | **Y** |
| Min | 11.99 | 13.40 | 16.23 | 19.39 |
| Max | 23.23 | 23.23 | 23.23 | 23.23 |
| Mean | 10.87 | 14.49 | 20.20 | .2124 |
| Median | 10.86 | 16.62 | 20.48 | .2130 |
| **Skewnes** | **0.02** | **0.47-** | **0.28-** | **0.06-** |

From Table 1 to Table 3, it can be seen that the results of all descriptive statistics at min-temperature of each period of four choices (Monthly, Quarterly, Half yearly and yearly) and the results is shown in Table-1 & Table-3. The all descriptive statistics (min, max, mean, median, mode, and skewness) of yearly in all Table are large than, the other Table. The sample mean value of the first period (10.59) in Table-1 is smaller than Quarterly (13.13) Half yearly (19.75) and yearly (20.85). But the sample means of second period (21.75) is large than the first (20.85) and total (21.30) period. After partitioning the data into different selection periods, it is observed that as the selection period increases, the difference between the minimum and maximum gets smaller, and the coefficient of variation decreases. This indicates that the min- temperature data is less dispersed from the mean as the selection period increases. The estimated skewness of all period in each three part (monthly, quarterly and half yearly) show that the distribution is negative skewed, it can be noted that the distribution have a left tail (short tailed). The skewness are negative (here, $sk < 0$) for three period, that mean the distribution has left tail ($\gamma < 0$), so there is a good reasons to think that the distribution of these data an appropriate by using **min-Weibull** modelling fitting, except other three periods of monthly are positive skewed ($sk > 0$) and indicate that the distribution of data has right tail and **min-Frechet** distribution is an appropriate. We apply PWM to estimate the three parameters of distribution, scale, location and shape, and summarizes the results in Table 4 to Table 6, for the application of the block minima data after partitioning the data into three different selection periods, for the various sample sizes that can be viewed as effected on PWM.

From Table-4 to Table-6, summarizes some estimation results of three parameter via the PWM, we get the estimates of parameters. This analysis is based on the series of minimum temperature over the period 1981-2020, of four choices (Monthly, Quarterly, Half yearly and yearly) are reported in the Table-4 - 6, and the results are stable. The results listed in Table 4-6, show the point estimates of shape for three periods of data are negative and indicate that the distribution of data has left tail and **min-Weibull** distribution is an appropriate for these data of **min-temperature except** other two selection periods first and second of monthly is positive ($\gamma > 0$) and indicate that the distribution of data has right tail and **min-Frechet** distribution is an appropriate. Also the P-value of the **DT** of all each period is smaller than the significance levels (P-value<0.01) and it also shows that all three selection periods of each time, the min-Weibull is good for all model fitting, the shape parameter $\gamma < 0$ for these data, except other three periods of monthly (P-value > 0.01). This is confirmed by the standard diagnostic graphical checks.

### 6.2 Diagnostic Plots

In order to get an idea about the tail behaviour of the distribution. We present in Fig 3 & 4, the various diagnostic plots for assessing the accuracy of the model fitted for an application data of min-temperature are shown in four different plots; P-P-plot, Q-Q-plot, Return level plot and density plot.

**Table 4: PWM of the EVD of first period**

| Period | First period (1981-2020) | | | |
|---|---|---|---|---|
| Parameter | **M** | **Q** | **H** | **Y** |
| Scale $\sigma$ | 0.78 | 1.57 | 7.52 | 8.94 |
| Location $\mu$ | 20.61 | 19.39 | 13.83 | 9.46 |
| Shape $\gamma < 0$ | 0.39 | -0.59 | -0.89 | -0.69 |
| Domain of Distribution | $\gamma > 0$ | $\gamma < 0$ | $\gamma < 0$ | $\gamma < 0$ |
| DT (P-value) | 22.78 (0.06) | 69.87 (0.00) | 246.5 (0.00) | 812.8 (0.00) |

**Table 5: PWM of the EVD of Second period**

| Period | second period (2001-2020) | | | |
|---|---|---|---|---|
| Parameter | **M** | **Q** | **H** | **Y** |
| Scale $\sigma$ | 0.74) | 1.60 | 7.80 | 9.16 |
| Location $\mu$ | 21.45) | 20.28 | 14.51 | 9.91 |
| Shape $\gamma < 0$ | 0.22 | -0.48 | -0.89 | -0.67 |
| Domain of Distribution | $\gamma > 0$ | $\gamma < 0$ | $\gamma < 0$ | $\gamma < 0$ |
| DT (P-value) | 23.22 (0.33) | 71.19 (0.00) | 249.56 (0.00) | 820.49 (0.00) |

**Table 6: PWM of the EVD of total period**

| Period | Total period (1981-2020) | | | |
|---|---|---|---|---|
| Parameters | **M** | **Q** | **H** | **Y** |
| Scale $\sigma$ | 0.89 | 1.62 | 7.46 | 8.86 |
| Location $\mu$ | 20.95 | 19.76 | 13.83 | 9.50 |
| Shape $\gamma < 0$ | 0.27 | -0.41 | -0.79 | -0.63 |
| Domain of Distribution | $\gamma > 0$ | $\gamma < 0$ | $\gamma < 0$ | $\gamma < 0$ |
| DT (P-value) | 52.22 (0.02) | 146.16 (0.00) | 501.56 (0.00) | 1636.42 (0.00) |

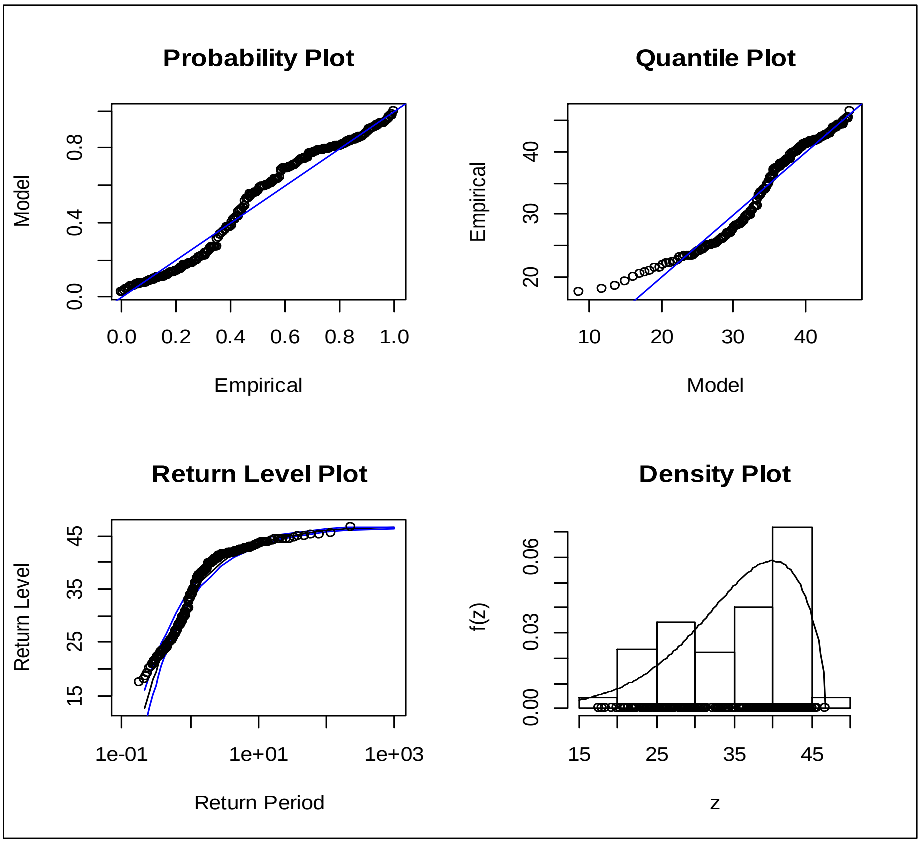


**Fig 2: Four different plots; P-P, Q-Q, R-L and density plot of first period (1981-200)**

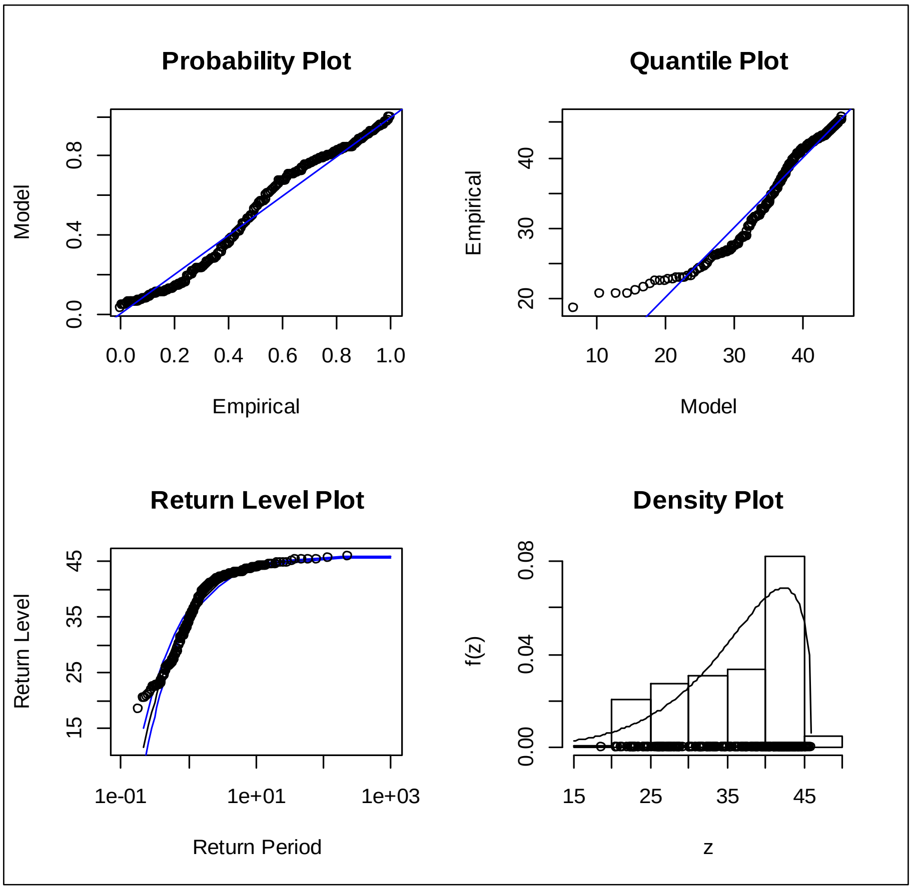


**Fig 3: Four different plots; P-P, Q-Q, R-L and density plot data of second period (2001-2020)**

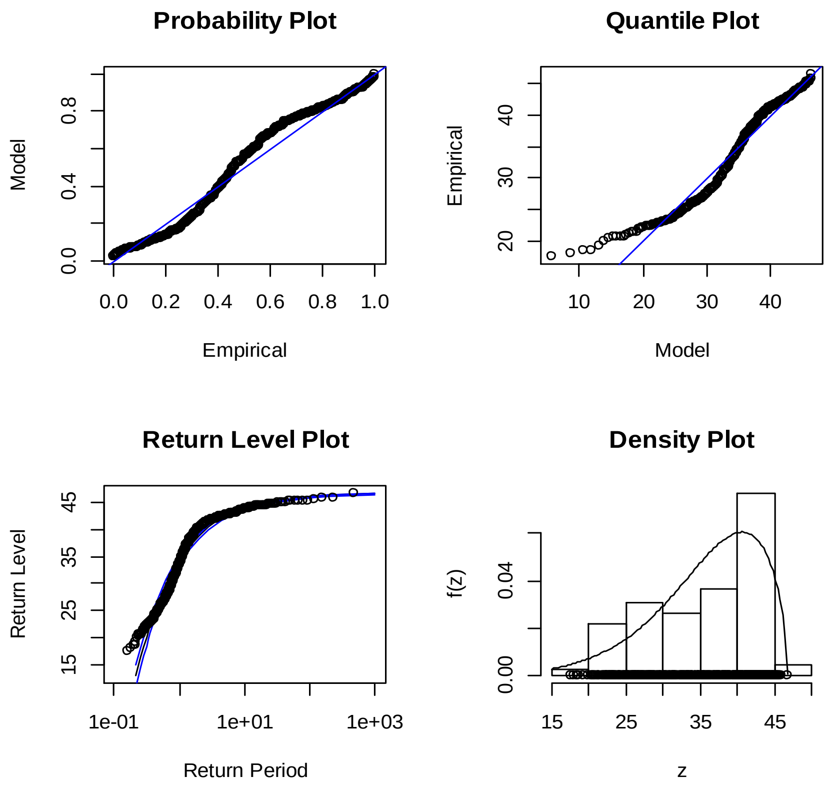


**Fig 4: Four different plots; P-P, Q-Q, R-L and density plot data of total period (1981-2020)**

Both the P-P plot and the Q-Q plot, are roughly a straight line of unit slope passing through the origin. So the **min-Weibull** model can be considered to be a reasonably good fit of the data. The return level curve asymptotes to a finite level as a consequence of the negative estimate of shape. Finally, the correspond density estimate seems consistent with the histogram of the data. Consequently, all four diagnostics plots support the fitted **min-Weibull** model.

There's no big difference between the three fitted models, the diagnostics show that **min-Weibull** model fit is reasonable here.

## 7. CONCLUSION

In this paper we focused on the extreme min-temperature data and illustrated how EVT can be used to model extreme temperature. This paper investigates some theoretical and practical aspects of the use **min-GEV** distribution. This distribution is one of the three probability distributions used to model extreme events. The choice among three distributions (Gumbel, Frechet, and Weibull) depends on the domain of attraction of the relevant tail of the parent distribution. Here we explore the use of min-EVD to model min-temperature. All selection periods, the parameters are estimated by using PWM. The skewness are negative (left tail) for all selection periods except other two selection of first and second periods of yearly are positive (right tail). Model diagnostics which include P-P plot, Q-Q plot, RL plot and density plot show a goodness of fit tests show that min-Weibull distribution using the different selection periods, except monthly of three period. Consequently, diagnostic plots lend support to the fitted min-Ferechet model. Finally, the corresponding density estimate seems consistent with the histogram of the data. Consequently, all four diagnostic plots lend support to **min-Weibull** model are the best fit. The plots also indicate that the shape parameter appears to be negative extremes, indicating that the min-temperature may have a thin tail. Overall, the result indicates that the distribution of each period belongs to **min-Weibull.** Using deviance Test (DT), the value is highly significant compared with significance level, and therefore provides strong evidence in favour of **min- Weibull** mode except Montly of first and second period is min-Ferechet model and the result are almost similar as model diagnostics plot.

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