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Distributional Properties of Generalized Diversity Index Based on two-key Species Abundance Models

Gebriel M. Shamia[1*], Ala Eddin Al-Shareef[2]

[1]Department of Statistics, Faculty of Science, University of Benghazi, Benghazi, Libya
[2]Institute of Science and Technology, Benghazi, Libya

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***Corresponding author:** Gebriel M. Shamia
Department of Statistics, Faculty of Science, University of Benghazi, Benghazi, Libya

Abstract **Original Research Article**

In the area of ecological research, the study of species diversity of a community or population seems to have been fully developed. However, the problem of how the distributions and expectations of the sample diversity indices are affected by population diversity has received little attention. This paper is concerned with methods of moments of improved generalized diversity index, $N(\alpha,\beta)$ due to Shamia's (2013) proposal which includes special cases. This improved index is a further generalization due to Good as described by Backowski *et al.*, (1997). In this article, the first four central moments of $N(\alpha,\beta)$ are derived for both general species relative abundance models: the Broken-Stick model and Sequential-Breakage model within a range of (α,β) considered. This allows the determination of the skewness and kurtosis of $N(\alpha,\beta)$ and thus gives information about the behaviour of the distribution of the improved index. The results are applied for comparing the diversities of the communities based on the samples n>s, and they yielded certain desirable monotonicity properties for large samples. It can be also shown that such indices are asymptotically normally distributed.

Keywords: Mathematical ecology, diversity indices, community structure, species abundance models, large sample.

1. INTRODUCTION

Improvement of some diversity indices is important to find meaningful measures that describe the systems under study. In addition, any diversity measure is a function of the species abundances in the community, which are usually unknown quantities. Accordingly, the abundances are estimated basis on suitable sampling strategies that can handle the problems arising when working with ecological communities as well as ensuring good statistical properties.

Bowman *et al.*, (1971) showed that the distribution of Shannon's index for the broken-stick model is asymptotically normal for general species relative abundance. When the species relative abundances are equiprobable, the distribution can be modelled as chi-squared. Lyons and Hutcheson (1979) derived the first four moments of Simpson's index of diversity and examined the distribution of the index when the relative abundances follow a geometric series. Bhargava and Uppuluri (1976, 1977) derived the mean and variance of Gini's index; the complement of Simpson's index.

Shamia (1997) suggested a generalized diversity index, $H(\alpha,\beta)$, due to Good's index (1953) and discussed its optimal bounds of (α,β) for ecological applications. Further results, such as large-sample normality and related issues, are contained in Backowski *et al.*, (1998).

Baczkowski *et al.*, (2000) derived the *r*th moments of both a general species abundance and equiprobable case and examined the distribution of the index, $H(\alpha,\beta)$, for fitting a Pearson system of curves.

Skewness and kurtosis reveal significant insights about the nature of distributions. This study aimed to provide a methodology for deriving the four central moments of the estimator for a diversity index belonging to what is known as the improved index, $N(\alpha,\beta)$ to study the power or effectiveness of the ecological models from a statistical standpoint.

1.1 *Generalized diversity index in ecology*

The use of families of indices has a long history in ecology, see Lövei (2005) and Tóthmérész (1995). In the 1970s of the last century, Hill (1973) proposed a family of diversity indices that may be interpreted as mean numbers of species, see Jost (2006). These numbers can be written in an entropic form as:

$$N_\alpha = \left\{\sum_{i=1}^{S}\pi_i^\alpha\right\}^{1/(1-\alpha)} \qquad ;\alpha \ge 0, \alpha \ne 1.$$

The parameter α determines the sensitivity of the measure to the relative frequencies. When α=0, the abundances of individual species do not contribute to the sum in the above equation. Rather, only presences are counted, so that N_0 is simply species richness. For α=1, the above equation is undefined, but its limit as α tends to one is the exponential of the familiar Shannon index, referred to here as Shannon diversity:

$$N_1 = Exp\left\{-\sum_{i=1}^{S}\pi_i\{log(\pi_i)\}\right\}.$$

The variable N_1 weighs species in proportion to their frequency. When α=2, N_2 yields Simpson diversity, the inverse of the Simpson concentration is as follows:

$$N_2 = \left\{1\Big/\sum_{i=1}^{S}\pi_i^2\right\}.$$

This places more weight on the frequencies of abundant species and discounts rare species. Investigators using Hill numbers should report, at least, the diversity of all species (α=0), of ‘‘typical’’ species (α=1), and dominant species (α=2).

Patil and Taillie (1979) proposed a parametric diversity index family β. In (1982) they called it the diversity index of degree β, in the form

$$\Delta_\beta = \frac{1}{\beta}\left\{1-\sum_{i-1}^{S}\pi_i^{\beta+1}\right\} ;\beta \ge -1, \beta \ne 1.$$

This equation interpolates the well-known Simpson index for β=1, and when (β+1) tends to one, it interpolates the well-known Shannon-Wiener index.

From this family, a critical discussion of these indices was made by Jost (2006). A new index, S_q, was introduced as a unified way to measure ecological diversity and species rarity as special diversity and evenness indices that balance commonness and rareness, a practice still unemployed by ecologists. It is based on Patil and Tallies (1979, 1982) indices and the corresponding evenness.

$$S_q = \frac{1}{(1-q)}\left\{1-\sum_{i-1}^{S}\pi_i^q\right\}.$$

In general, a desirable measure of diversity has to take all the relevant aspects that characterize ecological systems into account, from richness to species dominance.

Good (1953) outlined a generalized diversity index that incorporates richness and evenness into a single value that generally increases with both richness and evenness.

$$H(\alpha,\beta) = \sum_{i=1}^{S}\pi_i^\alpha\{-log(\pi_i)\}^\beta.$$

Where π_i is the relative abundance of taxon-*i* (species-*i*), such that $0\le\pi i\le1$, $\sum\pi_i=1$, and S is the total number of species present in the community, and (α,β) define structures of the algorithm. It was defined for non-negative integer values of α and β which included as special cases for both $H(1,1)$, Shannon’s (1948) index, and $H(2,0)$, related to Simpson’s (1949) index.

In practice, when the abundance data are available, the relative abundance can be estimated by using the maximum likelihood estimator $\hat{\pi}_i = p_i = (n_i/n)$.

Within this framework, Shamia (1997) proposed a further generalization of $H(\alpha,\beta)$**,** defined for real values of (α,β), and discussed its optimal bounds**.** For 0 <α≤0.3267 the valid region for rare species is given by 0≤β≤4α(1-α), while for 0.3267<α≤1.000 the valid region for β satisfies 0≤β≤0.153426+0.693147α+½√(0.094159+2.772589α). For $H(\alpha,\beta)$ it consists of the region α≥1 and β≤0 for common species. The case with (α,β) equal to (1,0) is excluded. Baczkowski *et al.*, (1998) determine the range of values (α,β) for which $H(\alpha,\beta)$ satisfies two key properties of Pielou (1975, p.7).

A short review of diversity measures is given, then the rest of this paper is organized as follows: a brief discussion of the Subject and Method are provided in Section-2 whereas in Section-3 we deal with Methodology for Deriving Moments of $N(\alpha,\beta)$; say *IGDI,* including the Results for General Abundance Distributions. Section-4 presents the Simulation and Empirical evaluation of the measures. Finally, Section-5 contains the Summary and Conclusions.

2. SUBJECT AND METHOD

A commonly used approach is that of rank-abundance distributions in ecology. Favoured, biologically founded models, *e.g.*, the geometric series model, the broken-stick model (see; Magurran, 1988; Wilson, 1991; Magurran, 2004), and the sequential-breakage model of Sugihara (1980).

In this work, asymptotic moments of the estimator $\hat{N}(\alpha,\beta)$ have been derived which is a developed measure from generalized diversity due to Good (1953) by Shamia (1997) and Baczkowski *et al.*, (1997). It is sufficient to evaluate the skewness and kurtosis, which were examined by analysing simulated communities generated under the broken-stick model due to MacArthur (1957) and the distribution of commonness and rarity is hypothesized to reflect “minimal structure” in biological communities, statistically, it has a link with canonical log-normal; namely sequential-breakage model due to Sugihara (1980).

It is not possible to recommend a single index as superior to all the others and choosing the appropriate index depends on what sort of question is being asked. Furthermore, studies dealing with the topic of community comparison utilizing diversity indices concluded that no single diversity index adequately summarizes community structure (see Hurlbert, 1971). Thus, a family of indices whose members have varying sensitivities to the rare and common species may be used.

In this context, it is proposed to be conducted some mathematical and applied investigations to the range of values (α,β) of the biodiversity index, $N(\alpha,\beta)$. As well as deriving moments and studying the power or effectiveness of the models in terms of statistical amended.

So, we are dealing with an Improved Generalized Diversity Index (IGDI), $N(\alpha,\beta)$, which is satisfying the key properties due to Pielou (1975, p.7) and other desirable properties due to Rutledge (1979, p.511). High index scores indicate both high species richness and a more equal distribution of individuals among species.

Suppose that the relative abundances in the community of the s species are $\pi_1,\pi_2,\ldots,\pi_s$. Then the improved generalized diversity index of (α,β), which gives more weight to the rare than to common for ($0<\alpha\leq1,\beta\geq0$) otherwise it gives more weight to the common than to the rare for ($\alpha\geq1,\beta\leq0$), is given by

$$N(\alpha,\beta)=\left\{\frac{H(\alpha,\beta)}{(\ln s)^{\beta}}\right\}^{\frac{1}{1-\alpha}}\ ;\ \alpha>0,\alpha\neq1,\beta\epsilon\{-2.5,2.5\}.$$

$$N(1,\beta)=Exp\{\,H(1,\beta)\}^{\,1/\beta};\ \alpha=1,\beta\epsilon\{-2.5,2.5\}.$$

If a sample of size n is available with n_i the number of individuals of the ith species, where $n=\sum_{i}^{s}n_i$, then the relative abundance of species i can be estimated by $p_i=n_i/n$ then the estimator of $N(\alpha,\beta)$ is given by

$$\widehat{N}(\alpha,\beta)=\left\{\frac{h(\alpha,\beta)}{(\ln s)^{\beta}}\right\}^{\frac{1}{1-\alpha}}\ ;\ \alpha>0,\alpha\neq1,\beta\epsilon\{-2.5,2.5\}.$$

$$\widehat{N}(1,\beta)=Exp\{\,h(1,\beta)\}^{\,1/\beta};\ \alpha=1,\beta\epsilon\{-2.5,2.5\}.$$

Where

$$h(\alpha,\beta)=\sum_{i=1}^{s}p_i^{\alpha}\,\{-ln(p_i)\}^{\beta}.$$

The central role of the exponential quantity gives the measure a privileged place as a measure of complexity and diversity in all of the sciences. Since a suitable transformation of $N(\alpha\geq1,\beta\leq0)$, such as inverse or others, would not be required to satisfy the properties. It does not need to be borrowed from information theory but arises naturally from this formalism of number equivalents.

The transformation $N(\alpha,0)$ gives the class of diversity index $N_{\alpha}=\left[\sum\pi_i^{\alpha}\right]^{1/(1-\alpha)}$ for some positive values of α as Hill's (1973) index.

These families of indices have satisfied additional properties such as consistency and monotonicity. In addition, the population indices have some desirable properties such as the addition of a new species increases the measure, and transferring abundance from a species to another one that is strictly less abundant increases the measure. High index scores indicate both high species richness and a more equal distribution of individuals among species. In fact, such transformations express the data in terms of the number of species and thus are more easily interpreted. The value of a traditional heterogeneity index with equal probabilities can be considered a richness measure. This is because these indices are monotone-increasing functions of the number of species s. Furthermore, these families of indices can be used as an equitability measure of species evenness for fixed s species. For more details, see Al-Shareef (2011) and Shamia (2013).

In this paper, the transformation of $H(\alpha,\beta)$ with base (e) logs can be used, since it is more common and more convenient, see Magurran (1988). $N(\alpha,\beta)$ (IGDI) shows the same combined acceptable region as for $H(\alpha,\beta)$, particularly for the *equiprobable* model and *broken-stick* model due to MacArthur (1957). So, it is sufficient to display the essential features of *IGDI*, *N*(α,β), for real arbitrary α and β for which satisfied the requirement properties of a diversity index and determining acceptable regions: Rare Species Region $\boldsymbol{R_1}$ contained in the region $\alpha \in (0,1]$ and $\beta \geq 0$ and Common Species Region $\boldsymbol{R_2}$, given by $\alpha \geq 1$ and $\beta \leq 0$.

Deriving an asymptotic expression for the expectation of $N(\alpha,\beta)$ to terms of order four in relation to $H(\alpha,\beta)$ is given in two cases.

3. Methodology for Deriving Moments of *N*(α,β)

In this section, the moment coefficients of skewness, $\boldsymbol{\beta_1}$, and the kurtosis, $\boldsymbol{\beta_2}$ of $N(\alpha,\beta)$ can be evaluated. The second, third, and fourth central moments of $\hat{N}(\alpha,\beta)$ are required.

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \text{ and } \beta_2 = \frac{\mu_4}{\mu_2^2}.$$

Where $\sqrt{\boldsymbol{\beta_1}}$ is often used as a measure of *skewness* and $\boldsymbol{\beta_2}$ is often used as a measure of *kurtosis*. It should be noted $\sqrt{\boldsymbol{\beta_1}}$ undergoes a significant change within acceptable regions ($\boldsymbol{R_1}$, $\boldsymbol{R_2}$).

To evaluate the moment coefficients of skewness and kurtosis ($\boldsymbol{\beta_1}$,$\boldsymbol{\beta_2}$) of the improved index $N(\alpha,\beta)$, let us consider the first 4-central moments of the index $H(\alpha,\beta)$ due to Shamia (1997) and Baczkowski *et al.*, (2000). The methodology used to derive the central moments of $h(\alpha,\beta)$ is presented in Baczkowski *et al.*, (2000), in which derived the *r*th central moments for both a general species abundance and equiprobable case.

Really, what is needed are the central moments of $f(x) \equiv \hat{N}(\alpha,\beta)$ about their mean *E*[*f*(*x*)]. Consequently, by using the Taylor series expansion at deriving an asymptotic expression, it can be written.

$$f(x) - E\left[f(x)\right] \cong \{f(x) - f(\mu)\} - \frac{1}{2!}\mu_2 f^{(2)}(\mu) - \frac{1}{3!}\mu_3 f^{(3)}(\mu) - \frac{1}{4!}\mu_4 f^{(4)}(\mu).$$

Where $f^{(r)}(\mu)$; r = 2, 3,4 denotes the *r*th differential of $f^{(r)}(\mu)$ with respect to μ.

Now let $D^r = \{f(x) - f(\mu)\}^r \equiv \left\{\hat{N}(\alpha,\beta) - E\left[\hat{N}(\alpha,\beta)\right]\right\}^r$ and taking the expectation gives,

$$E\left[f(x)\right] \cong \{f(\mu)\} + \frac{1}{2!}\mu_2 f^{(2)}(\mu) + \frac{1}{3!}\mu_3 f^{(3)}(\mu) + \frac{1}{4!}\mu_4 f^{(4)}(\mu),$$

$$E\left[D^2\right] \cong \mu_2\left[f^{(1)}(\mu)\right]^2 + \mu_3 f^{(1)}(\mu) f^{(2)}(\mu) + \mu_4\left\{\frac{1}{3} f^{(1)}(\mu) f^{(3)}(\mu) + \frac{1}{4}\left[f^{(2)}(\mu)\right]^2\right\},$$

$$E\left[D^3\right] \cong \mu_3\left[f^{(1)}(\mu)\right]^3 + \frac{3}{2}\mu_4\left[f^{(1)}(\mu)\right]^2 f^{(2)}(\mu), \text{ and } E\left[D^4\right] \cong \mu_4\left[f^{(1)}(\mu)\right]^4.$$

In this way, D^r can be written in terms of powers μ_r's. To do so let us write that:

$$\mu_r = \sum_{k=1}(C_{rk}/n^k) = (C_{r1}/n) + (C_{r2}/n^2) + (C_{r3}/n^3) + \cdots,$$

Where C_{rk} is the *k*th combination out of "*r*" cases.

It can be shown in the general case that C_{11}, C_{12}, C_{31}, and C_{41} are zeros. To derive the results, it is necessary to only evaluate μ_r up to terms $0(n^{-3})$. The results for C_{rk}*'s* are given in Appendix (I).

3.1. *Results for general abundance distribution*

The results of mean, 2nd, 3rd, and 4th central moments of $\hat{N}(\alpha,\beta)$ are given by

Case –I:

For deriving an asymptotic expression for the expectation of $\hat{N}(\alpha,\beta)$ to terms of the order four at $\alpha \neq 1$,

$$\hat{N}(\alpha,\beta) = \left\{\frac{h(\alpha,\beta)}{(\ln s)^{\beta}}\right\}^{1/(1-\alpha)} = \left\{\frac{h^{1/(1-\alpha)}}{(\ln s)^{\beta/(1-\alpha)}}\right\} = C_0\, h^{1/(1-\alpha)}; \quad C_0 = \left\{\frac{1}{(\ln s)^{\beta/(1-\alpha)}}\right\}, \quad \alpha \neq 1.$$

In addition,

$$C_1 = \frac{1}{(1-\alpha)}, \quad C_2 = \frac{\alpha}{(1-\alpha)}, \quad C_3 = \frac{(2\alpha-1)}{(1-\alpha)}, \quad \text{and} \quad C_4 = \frac{(3\alpha-2)}{(1-\alpha)}\ .$$

$$E\left[\hat{N}(\alpha,\beta)\right]\cong\left\{C_0\left[H+\frac{C_{11}}{n}+\frac{C_{22}}{n^2}\right]^{1/(1-\alpha)}\right\}+\frac{1}{2}\left[\frac{C_{21}}{n}+\frac{C_{22}}{n}\right]C_0C_1C_2\left[H+\frac{C_{11}}{n}+\frac{C_{22}}{n^2}\right]^{(2\alpha-1)/(1-\alpha)}$$

$$+\frac{1}{6}\left[\frac{C_{32}}{n^2}\right]C_0C_1C_2C_3\left[H+\frac{C_{11}}{n}+\frac{C_{22}}{n^2}\right]^{(3\alpha-2)/(1-\alpha)}+\frac{1}{24}\left[\frac{C_{42}}{n^2}\right]C_0C_1C_2C_3C_4\left[H+\frac{C_{11}}{n}+\frac{C_{22}}{n^2}\right]^{(4\alpha-3)/(1-\alpha)}.$$

$$\mu_2\left[\hat{N}(\alpha,\beta)\right]\cong\left[\frac{C_{21}}{n}+\frac{C_{22}}{n^2}\right]C_0^2C_1^2C_2^2\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{2(2\alpha-1)/(1-\alpha)}+\left[\frac{C_{32}}{n^2}\right]C_0^2C_1^2C_2$$

$$\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{(3\alpha-1)/(1-\alpha)}+\left[\frac{C_{42}}{n^2}\right]\left\{\frac{1}{3}C_0^2C_1^2C_2C_3\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{(4\alpha-2)/(1-\alpha)}+\frac{1}{4}C_0^2C_1^2C_2^2\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{2(2\alpha-1)/(1-\alpha)}\right\},$$

$$\mu_3\left[\hat{N}(\alpha,\beta)\right]\cong\left[\frac{C_{32}}{n^2}\right]C_0^3C_1^3\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{3\alpha/(1-\alpha)}+\frac{3}{2}\left[\frac{C_{42}}{n^2}\right]C_0^3C_1^3C_2\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{(4\alpha-1)/(1-\alpha)},$$

and

$$\mu_4\left[\hat{N}(\alpha,\beta)\right]\cong\left[\frac{C_{42}}{n^2}\right]C^4C_1^4\left[H+\frac{C_{11}}{n}+\frac{C_{12}}{n^2}\right]^{4\alpha/(1-\alpha)}.$$

Case –II:

For deriving an asymptotic expression for the expectation of $\hat{N}(\alpha,\beta)$ to terms of the order four at α=1,

$$\hat{N}(1,\beta)=Exp\{h(1,\beta)\}^{1/\beta}=Exp\{h^{1/\beta}\};\qquad\alpha=1.$$

$$E\left[\hat{N}(1,\beta)\right]\cong Exp\left[h^{1/\beta}\right]+\frac{1}{2!}\left[\frac{C_{21}}{n}+\frac{C_{22}}{n^2}\right]\frac{1}{\beta}h^{(1-2\beta)/\beta}\left(h^{1/\beta}-\beta+1\right)Exp\left[h^{1/\beta}\right]+\frac{1}{3!}\left[\frac{C_{32}}{n^2}\right]$$

$$\frac{1}{\beta^3}h^{(1-3\beta)/\beta}\left(h^{2/\beta}+(3-3\beta)h^{1/\beta}+2\beta^2-3\beta+1\right)Exp\left[h^{1/\beta}\right]$$

$$+\frac{1}{4!}\left[\frac{C_{42}}{n^2}\right]\frac{1}{\beta^4}h^{(1-4\beta)/\beta}\left(h^{3/\beta}+(6-6\beta)h^{2/\beta}+(11\beta^2-18\beta+7)h^{1/\beta}\right.$$

$$\left.-\left(6\beta^3+11\beta^2-6\beta+1\right)\right),$$

$$\mu_2\left[\hat{N}(1,\beta)\right]\cong\left[\frac{C_{21}}{n}+\frac{C_{22}}{n^2}\right]\frac{1}{\beta^2}h^{2(1-\beta)/\beta}Exp\left[2h^{1/\beta}\right]+\left[\frac{C_{32}}{n^2}\right]\frac{1}{\beta^3}h^{(2-3\beta)/\beta}\left(h^{1/\beta}-\beta+1\right)Exp\left[2h^{1/\beta}\right]$$

$$+\left[\frac{C_{42}}{n^2}\right]\left(\frac{1}{3\beta^3}h^{(2-3\beta)/\beta}(h^{1/\beta}-\beta+1)Exp\left[2h^{1/\beta}\right]+\frac{1}{4\beta^4}h^{(2-4\beta)/\beta}(h^{1/\beta}-\beta+1)^2Exp\left[2h^{1/\beta}\right]\right),$$

$$\mu_3\left[\hat{N}(1,\beta)\right]\cong\left[\frac{C_{32}}{n^2}\right]\frac{1}{\beta^3}h^{3(1-\beta)/\beta}Exp\left[3h^{1/\beta}\right]+\frac{3}{2}\left[\frac{C_{42}}{n^2}\right]\frac{1}{\beta^3}h\left(h^{1/\beta}-\beta+1\right)Exp\left[3h^{1/\beta}\right],$$

and

$$\mu_4\left[\hat{N}(1,\beta)\right]\cong\left[\frac{C_{42}}{n^2}\right]\frac{1}{\beta^4}h^{4(1-\beta)/\beta}Exp\left[4h^{1/\beta}\right].$$

If the relative abundances of the *S* species are all equal (equiprobable case); $\hat{\pi}_i=p_i=n_i/n=1/S$, then it can be shown C_{11}, C_{12}, C_{21}, C_{31}, C_{32}, C_{41}, and C_{42} are all zeros.

To obtain the central moments of order *r* of bias, it is necessary to evaluate terms of up to $0(1/n^3)$, obtaining, consequently, the moment coefficients of skewness and kurtosis (**β_1,β_2**) of $N(\alpha,\beta)$.

In doing so, it is possible to select an appropriate un-equiprobable case as two key models considered here are the broken-stick model of MacArthur (1957) and a sequential- breakage model of Sugihara (1980) presents a distribution for the generalized diversity index, as shown in the next section.

4. Simulation Behaviour of the Distribution of Measures

The moments of $\hat{N}(\alpha,\beta)$ are evaluated for several different species abundance models. The two key models considered here are the broken-stick model and the sequential-breakage model. The simulation is set to an initial sample size of 100 with ten species from the population.

Figure-1 gives the population proportions plot for the broken-stick model and the sequential-breakage model in the cases *s*=10 respectively. Do this several times, once you see how this works, you can speed things up by taking 1000 as a large community.

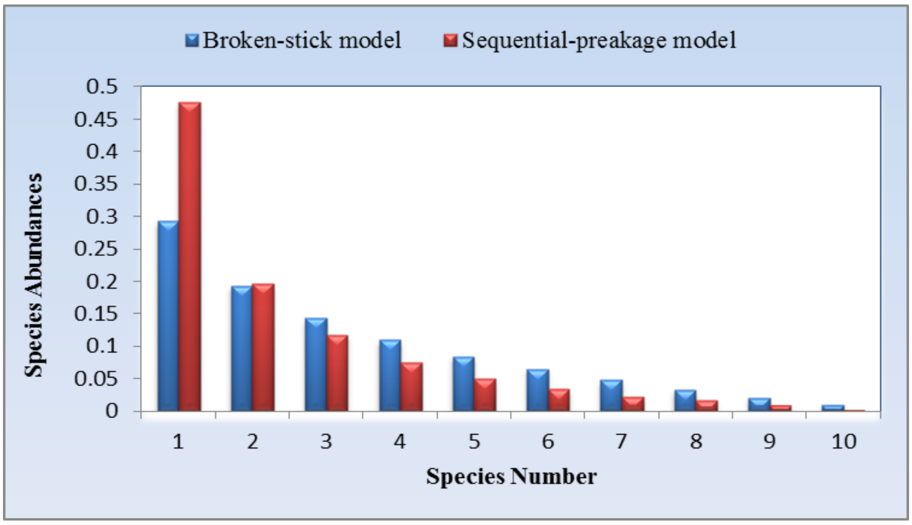


Figure 1: Comparisons between the pattern of the broken-stick and sequential-breakage models of 10-species communities

It is clear that the sequential-breakage model trend to predict the rarest species compared with the broken-stick model. However, the broken-stick model is the most common species abundance distribution model in biological diversity due to treatments.

Tabulate the moments of $\hat{N}(\alpha,\beta)$ for a range of α and β values for the broken-stick model and the sequential-breakage model with *s*=10 and sample size *n*=1000 produced, including the sample mean of $\hat{N}(\alpha,\beta)$, the central moments μ_r for *r*=2;3;4, and the sample skewness β_1 and kurtosis β_2 of the improved index, by the ***R***-programs, are given below.

Tables (1.a) and (1.b) show asymptotic results, for a selection of values (α,β) in rare species region R_1, for the mean of $N(\alpha,\beta)$, 2nd, 3rd, 4th moments, the coefficients of skewness β_1 and kurtosis β_2 of $\hat{N}(\alpha,\beta)$ index for the case of *s*=10 species and sample size *n*=1000 using the two key models respectively. Again, Tables (2.a) and (2.b) show asymptotic results, for a selection of values (α,β) in common species region R_2.

Table 1.a: ***Values of N(α,β) and the moments of $\hat{N}(\alpha,\beta)$ with Broken-Stick Model in the acceptable region R_1***

(α,β)	$N(\alpha,\beta)$	$E(N)$	$\mu_2\times10^5$	$\mu_3\times10^5$	$\mu_4\times10^5$	$\sqrt{\beta}_1$	β_2
(0.25,0.25)	9.137	9.137	558.286	-9.746	8.236	-0.234	2.642
(0.25,0.50)	9.317	9.317	416.312	-8.234	4.628	-0.307	2.670
(0.50,0.00)	8.305	8.305	1457.107	-18.729	59.463	-0.106	2.801
(0.50,0.50)	7.954	7.954	2200.467	-40.631	139.224	-0.124	2.875
(0.50,1.00)	8.160	8.161	2217.497	-68.345	141.952	-0.207	2.887
(0.75,0.00)	7.682	7.683	2144.279	-12.869	133.512	-0.041	2.904
(0.75,0.50)	6.053	6.056	5109.437	18.721	798.676	0.016	3.059
(0.75,1.00)	5.438	5.443	6503.350	54.578	1297.028	0.033	3.067
(1.00,0.50)	6.760	6.762	3073.760	16.614	314.063	0.031	3.324
(1.00,1.00)	7.173	7.174	2563.937	1.612	210.549	0.004	3.203

Table 1.b: ***Values of N(α,β) and the moments of*** $\hat{N}(\alpha,\beta)$ ***with Sequential-Breakage Model in the acceptable region R₁***

(α,β)	$N(\alpha,\beta)$	$E(N)$	$\mu_2\times10^5$	$\mu_3\times10^5$	$\mu_4\times10^5$	$\sqrt{\beta_1}$	β_2
(0.25,0.25)	8.171	8.171	1757.881	-68.005	60.699	-0.292	1.964
(0.25,0.50)	8.520	8.520	1707.598	-85.212	51.188	-0.382	1.755
(0.50,0.00)	6.667	6.668	2367.904	-24.866	148.138	-0.068	2.642
(0.50,0.50)	5.977	5.978	3758.986	-66.300	366.048	-0.091	2.591
(0.50,1.00)	6.372	6.374	4903.644	-230.058	552.512	-0.212	2.298
(0.75,0.00)	5.660	5.661	2504.074	10.639	181.557	0.027	2.895
(0.75,0.50)	3.148	3.153	3902.772	105.233	457.144	0.136	3.001
(0.75,1.00)	2.537	2.543	3924.522	129.943	452.050	0.167	2.935
(1.00,0.50)	4.283	4.287	2591.549	57.360	212.445	0.137	3.163
(1.00,1.00)	4.929	4.931	2399.898	31.984	180.674	0.086	3.137

Table 2.a: ***Values of N(α,β) and the moments of*** $\hat{N}(\alpha,\beta)$ ***with Broken-Stick Model in the acceptable region R₂***

(α,β)	$N(\alpha,\beta)$	$E(N)$	$\mu_2\times10^5$	$\mu_3\times10^5$	$\mu_4\times10^5$	$\sqrt{\beta_1}$	β_2
(1.00,-1.50)	5.534	5.546	5535.600	173.641	887.811	0.133	.2897
(1.00,-0.50)	6.067	6.074	4593.576	68.057	587.565	0.069	2.785
(1.25,-1.00)	1.867	1.887	6017.829	625.246	1010.387	0.424	2.790
(1.25,0.00)	6.752	6.755	3086.186	19.871	283.757	0.037	2.979
(1.50,-1.50)	2.028	2.052	6578.299	724.196	1179.837	0.429	2.726
(1.50,-0.50)	4.544	4.555	6573.429	249.153	1226.426	0.148	2.838
(1.75,-1.00)	3.584	3.602	7520.563	501.897	1541.353	0.243	2.725
(1.75,0.00)	6.108	6.113	3727.356	64.460	415.064	0.090	2.988
(2.00,0.00)	5.858	5.865	3980.723	91.006	472.719	0.115	2.983
(2.50,0.00)	5.460	5.470	4386.709	152.543	570.586	0.166	2.965

Table 2.b: ***Values of N(α,β) and the moments of*** $\hat{N}(\alpha,\beta)$ ***with Sequential-Breakage Model in the acceptable region R₂***

(α,β)	$N(\alpha,\beta)$	$E(N)$	$\mu_2\times10^5$	$\mu_3\times10^5$	$\mu_4\times10^5$	$\sqrt{\beta_1}$	β_2
(1.00,-1.50)	2.887	2.899	2198.216	134.450	138.218	0.413	2.860
(1.00,-0.50)	3.381	3.389	2677.706	114.564	208.398	0.068	2.906
(1.25,-1.00)	0.169	0.175	162.200	5.728	0.662	0.877	2.515
(1.25,0.00)	4.392	4.396	2357.836	45.198	166.414	0.125	2.993
(1.50,-1.50)	0.246	0.253	245.379	10.017	1.515	0.824	2.516
(1.50,-0.50)	1.716	1.726	2193.268	115.437	138.543	0.355	2.880
(1.75,-1.00)	1.040	1.051	1322.150	78.286	48.264	0.515	2.761
(1.75,0.00)	3.690	3.695	2086.529	58.735	130.071	0.195	2.988
(2.00,0.00)	3.457	3.463	1953.581	61.814	113.644	0.226	2.978
(2.50,0.00)	3.129	3.136	1718.313	63.991	87.127	0.284	2.951

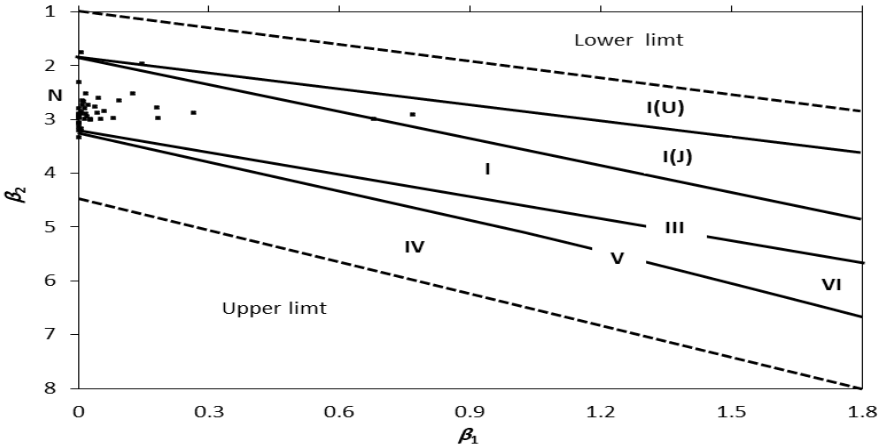
From Tables (1.a) and (1.b), it can be seen that the values of $N(\alpha,\beta)$ and their expected in acceptable regions are close to each other for both models. However, for the broken-stick model, they are greater slightly than those for the sequential-breakage model for all selective values of (α,β) within the two regions.

Statistically, it is known that if the coefficient of skewness lies between -0.50 and 0.50 then the distribution is fairly symmetrical. Consequently, it can be noted that in region ***R₁*** when $\alpha\leq0.50$ and $\beta\leq1.00$ the coefficients of skewness give small negative values (close to zero), and for the kurtosis are close to three. Similarly, the same behaviour occurs when $\alpha>0.50$ and $\beta\geq0.50$. Kurtosis between 1.80 and 3.00 is considered acceptable to prove normal distribution. Then the distribution is approximately symmetrical and nearly mesokurtic**.** This suggests a Gaussian distribution, when using both key models.

Again, the same conclusion can be drawn the distribution is nearly normal, in region ***R₂*** when $\alpha\geq1.00$ and $\beta\leq0.00$, specifically, for the case of the broken-stick model as the information shown in Tables (2.a) and (2.b).

It is noted for values of 2nd, 3rd, and 4th central moments of $\hat{N}(\alpha,\beta)$ in both regions (***R₁***, ***R₂***) for the most values of (α,β) are very small.

The Pearson system of classifying distributions is based on the two parameters $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$, see Pearson and Merrington (1951). In the acceptable regions using both key models, it can be seen that $(\boldsymbol{\beta_1},\boldsymbol{\beta_2})\approx(0,3)$, corresponds to a Gaussian distribution for $(\alpha,\beta)\approx(1,1)$ including the Shannon index. Again, it is valid for $(\alpha,\beta)\approx(\alpha,0)$ including Simpson and Hill indices. For small values of $(\alpha,\beta)\approx(0.25,0.25)$ and $(\alpha,\beta)\approx(0.25,0.50)$ in region $\boldsymbol{R_1}$ for the case of broken-stick model, the distribution of *IGDI* is the beta distribution of the first kind (Type I), in which values of (α,β) satisfy $2\boldsymbol{\beta_2}-3\boldsymbol{\beta_1}-6<0$, and as well as for the case of the sequential-breakage model.



N : Normal distribution	**I(U)** : Beta (U-shaped	**I(J)** : Beta (J-shaped	**I** : Beta (type-I)
III : Gamma (Chi-square)	**IV** : Cinderella distrib.	**V** : Inverse Gamma	**VI** : Beta prime

Figure 2: The (β_1,β_2) values for $\hat{N}(\alpha,\beta)$ in the pattern of broken-stick andsequential-breakage models of 10-species communities

While we have only presented results for *IGDI*, it is possible to consider alternative values of species number (s) and sample sizes (n) in both regions ($\boldsymbol{R_1}$, $\boldsymbol{R_2}$) using the two key models or alternative models for this index as suitable diversity indices for different values of (α,β). For example, for the case s=10 and n=100, 500. In general, the results indicate platykurtic distribution specifically when n=100 for the most values of (α,β). This is not pursued here.

5. SUMMARY AND CONCLUSIONS

The present study derived the central moments of an improved generalized diversity index that is widely applied in ecology, which is including $N(1,1)$ for the Shannon index, $N(\alpha,0)$ for the Hill index, and $N(2,0)$ for the Simpson index. The contribution of this paper about this index, *IGDI*, provided consistency and asymptotic normality of estimators under conditions.

Accordingly, it is recommended for sampled species' data, especially when large species communities are involved using the broken-stick and the sequential-breakage models. Other models that also assume ecological equivalence between species make predictions similar to those we used is the Poisson lognormal model that invokes standard statistical distributions. In this paper, we present the theoretical background by which we can understand why these neutral generative models are so successful.

Briefly, we may draw the following conclusions:

1. This index, *IGDI* measuring the diversity of large communities should be a function of species abundances (π_i), satisfying that $N(\alpha,\beta)\equiv s$ in the equiprobable case, and $N(\alpha,\beta)<s$ otherwise. In addition, no need to use any transformation of this index to satisfy the main two key properties.
2. The second point deserves some elaboration. When one investigates any species abundance distribution, previous studies suggested that it was ubiquitous and observed that many species are rare and just a few are common.
3. The new asymptotically unbiased estimators generally have reasonably stable sample paths, making the choice of optimal values of (α,β) within the acceptable regions less troublesome.
4. These results of the distribution of *IGDI*, $N(\alpha,\beta)$, hold if other species abundance models are used.
5. In general, the behaviour of the distribution gives a marginally better approximation to normality. Shannon index, $N(1,1)$; Hill index, $N(\alpha,0)$; and Simpson index, $N(2,0)$ show corresponding values of $(\boldsymbol{\beta_1},\boldsymbol{\beta_2})\approx(0,3)$ within the acceptable regions.

To sum up, the simulation results demonstrated that the $0(n^{-3})$ approximation to the moments of $N(\alpha,\beta)$ is acceptable providing that the species abundances using the broken-stick model are close to the sequential-breakage model.

Generally, the distribution of $\hat{N}(\alpha,\beta)$ is approximately normally distributed for the *Broken-Stick Model* as well for the *Sequential-Breakage Model* in both acceptable regions.

It is hoped that the findings of this work will be useful for practitioners in various fields of theoretical and applied sciences in ecology.

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Appendix (I): Expressions of "C_{rk}"

$$C_{21}=\beta^2H(2\alpha-1,2\beta-2)-2\alpha\beta H(2\alpha-1,2\beta-1)+\alpha^2H(2\alpha-1,2\beta)-\beta^2H(\alpha,\beta-1)^2+2\alpha\beta H(\alpha,\beta-1)H(\alpha,\beta)-\alpha^2H(\alpha,\beta)^2.$$

$$C_{22}=\beta^3H(\alpha,\beta-2)^2+1/2\beta^2H(\alpha,\beta-2)^2-5\alpha\beta^3H(\alpha,\beta-2)H(\alpha,\beta-1)+5\alpha\beta^2H(\alpha,\beta-2)H(\alpha,\beta-1)+2\beta^3H(\alpha,\beta-2)H(\alpha,\beta-1)-2\beta^2H(\alpha,\beta-2)H(\alpha,\beta-1)+4\alpha^2\beta^2H(\alpha,\beta-2)H(\alpha,\beta)-4\alpha^2\beta H(\alpha,\beta-2)H(\alpha,\beta)-2\alpha\beta^2H(\alpha,\beta-2)H(\alpha,\beta)+2\alpha\beta H(\alpha,\beta-2)H(\alpha,\beta)+5\alpha^2\beta^2H(\alpha,\beta-1)^2-4\alpha\beta^2H(\alpha,\beta-1)^2+1/2\beta^2H(\alpha,\beta-1)^2-6\alpha^3\beta H(\alpha,\beta-1)+6\alpha^2\beta H(\alpha,\beta-1)H(\alpha,\beta)-\alpha\beta H(\alpha,\beta-1)H(\alpha,\beta)+3/2\alpha^4H(\alpha,\beta)^2-2\alpha^3\beta H(\alpha,\beta)^2+1/2\alpha^2H(\alpha,\beta)^2-2\beta^3H(\alpha-1,\beta-2)H(\alpha,\beta-1)+2\beta^2H(\alpha-1,\beta-2)H(\alpha,\beta-1)-3\alpha^2\beta^2H(\alpha-1,\beta-2)H(\alpha,\beta)+3\alpha^2\beta H(\alpha-1,\beta-2)H(\alpha,\beta)+2\alpha\beta^2H(\alpha-1,\beta-2)H(\alpha,\beta)-2\alpha\beta H(\alpha-1,\beta-2)H(\alpha,\beta)-3\alpha^2\beta^2H(\alpha-1,\beta-1)H(\alpha,\beta-1)+4\alpha\beta^2H(\alpha-1,\beta-1)H(\alpha,\beta-1)-\beta^2H(\alpha-1,\beta-1)H(\alpha,\beta-1)+3\alpha^3\beta H(\alpha-1,\beta-1)H(\alpha,\beta)-4\alpha^2\beta H(\alpha-1,\beta-1)H(\alpha,\beta)+\alpha\beta H(\alpha-1,\beta-1)H(\alpha,\beta)+\alpha^3\beta H(\alpha-1,\beta)H(\alpha,\beta-1)-2H(\alpha-1,\beta)H(\alpha,\beta-1)\alpha^2\beta+\alpha\beta H(\alpha-1,\beta)H(\alpha,\beta-1)-\alpha^4H(\alpha-1,\beta)\ H(\alpha,\beta)+2\alpha^3H(\alpha-1,\beta)H(\alpha,\beta)-\ \alpha^2H(\alpha-1,\beta)\ H(\alpha,\beta)+\beta^4H(\alpha,\beta-3)H(\alpha,\beta-1)-3\beta^3H(\alpha,\beta-3)H(\alpha,\beta-1)+2\beta^2H(\alpha,\beta-3)H(\alpha,\beta-1)-\alpha\beta^3H(\alpha,\beta-3)H(\alpha,\beta)\ +3H(\alpha,\beta-3)H(\alpha,\beta)\alpha\beta^2-2\alpha\beta H(\alpha,\beta-3)H(\alpha,\beta)+1/2\beta^4H(\alpha,\beta-2)^2-10\alpha\beta^2H(2\alpha-1,2\beta-3)+2\alpha\beta H(2\alpha-1,2\beta-3)-3\beta^3H(2\alpha-1,2\beta-3)+3\beta^2H(2\alpha-1,2\beta-3)-12\alpha^2\beta^2H(2\alpha-1,2\beta-2)+5\alpha^2\beta H(2\alpha-1,2\beta-2)+9\alpha\beta^2H(2\alpha-1,2\beta-2)-3\alpha\beta H(2\alpha-1,2\beta-2)-\beta^2H(2\alpha-1,2\beta-2)+8\alpha^3\beta H(2\alpha-1,2\beta-1)-9\alpha^2\beta H(2\alpha-1,2\beta-1)+2H(2\alpha-1,2\beta-1)\alpha\beta-2\alpha^4H(2\alpha-1,2\beta)+3\alpha^3H(2\alpha-1,2\beta)-\alpha^2H(2\alpha-1,2\beta)-\beta^4H(\alpha-1,\beta-3)H(\alpha,\beta-1)+3\beta^3H(\alpha-1,\beta-3)H(\alpha,\beta-1)-2\beta^2H(\alpha-1,\beta-3)H(\alpha,\beta-1)+\alpha\beta^3H(\alpha-1,\beta-3)H(\alpha,\beta)-3\alpha\beta^2H(\alpha-1,\beta-3)H(\alpha,\beta)+2\alpha\beta H(\alpha-1,\beta-3)H(\alpha,\beta)+3\alpha\beta^3H(\alpha-1,\beta-2)H(\alpha,\beta-1)-3\alpha\beta^2H(\alpha-1,\beta-2)H(\alpha,\beta-1)+3/2\beta^4H(2\alpha-2,2\beta-4)-4\beta^3H(2\alpha-2,2\beta-4)+5/2\beta^2H(2\alpha-2,2\beta-4)-6\alpha\beta^2\ H(2\alpha-2,2\beta-3)+8\alpha\beta^2H(2\alpha-2,2\beta-3)-2\alpha\beta\ H(2\alpha-2,2\beta-3)+3\beta^3H(2\alpha-2,2\beta-3)-3\beta^2H(2\alpha-2,2\beta-3)+9\alpha^2\beta^2H(2\alpha-2,\ 2\beta-2)-4\alpha^2\beta H(2\alpha-2,2\beta-2)-9\alpha\beta^2H(2\alpha-2,2\beta-2)+3\alpha\beta H(2\alpha-2,2\beta-2)+3/2\beta^2H(2\alpha-2,2\beta-2)-6\alpha^3\beta H(2\alpha-2,2\beta-1)\ +9\alpha^2\beta H(2\alpha-2,2\beta-1)-3\alpha\beta H(2\alpha-2,2\beta-1)+3/2\alpha^4H(2\alpha-2,2\beta)-3\alpha^3H(2\alpha-2,2\beta)+3/2H(2\alpha-2,2\beta)\alpha^2-2\beta^4H(2\alpha-1,\ 2\beta-4)+5\beta^3H(2\alpha-1,2\beta-4)-3\beta^2H(2\alpha-1,2\beta-4)+8\alpha\beta^3\ H(2\alpha-1,2\beta-3).$$

$$C_{32}=3\alpha\beta^2H(2\alpha-1,2\beta-2)H(\alpha,\beta)-18\alpha^2\beta^2H(2\alpha-1,2\beta-1)H(\alpha,\beta-1)+6\alpha\beta^2H(2\alpha-1,2\beta-1)H(\alpha,\beta-1)+18\alpha^3\beta\ H(2\alpha-1,2\beta-1)H(\alpha,\beta)-6\alpha^2\beta H(2\alpha-1,2\beta-1)H(\alpha,\beta)+6\alpha^3\beta H(2\alpha-1,2\beta)H(\alpha,\beta-1)-3\alpha^2\beta H(2\alpha-1,2\beta)H(\alpha,\beta-1)-6\alpha^4H(2\alpha-1,2\beta)H(\alpha,\beta)+3\alpha^3H(2\alpha-1,2\beta)H(\alpha,\beta)+3\beta^4H(\alpha,\beta-2)H(\alpha,\beta-1)^2-3\beta^3H(\alpha,\beta-2)H(\alpha,\beta-1)^2-6H(\alpha,\beta-2)H(\alpha,\beta-1)H(\alpha,\beta)\alpha\beta^3+6\alpha\beta^2H(\alpha,\beta-2)H(\alpha,\beta-1)H(\alpha,\beta)+3\alpha^2\beta^2H(\alpha,\beta-2)H(\alpha,\beta)^2-3H(\alpha,\beta-2)H(\alpha,\beta)^2\alpha^2\beta-6\alpha\beta^3H(\alpha,\beta-1)^3\ +\beta^3H(\alpha,\beta-1)^3+15\alpha^2\beta^2H(\alpha,\beta-1)^2H(\alpha,\beta)-3\alpha\beta^2H(\alpha,\beta-1)^2H(\alpha,\beta)-12\alpha^3\beta\ H(\alpha,\beta-1)H(\alpha,\beta)^2+3\alpha^2\beta\ H(\alpha,\beta-1)H(\alpha,\beta)^2+3\alpha^4H(\alpha,\beta)^3-\alpha^3H(\alpha,\beta)^3+3\beta^4H(3\alpha-2,3\beta-4)-3\beta^3H(3\alpha-2,3\beta-4)\ -12\alpha\beta^3H(3\alpha-2,3\beta-3)+6\alpha\beta^2H(3\alpha-2,\ 3\beta-3)+2\beta^3H(3\alpha-2,3\beta-3)+18\alpha^2\beta^2H(3\alpha-2,3\beta-2)-3\alpha^2\beta H(3\alpha-2,3\beta-2)-6\alpha\beta^2H(3\alpha-2,3\beta-2)-12\alpha^3\beta H(3\alpha-2,3\beta-1)+6\alpha^2\beta H(3\alpha-2,3\beta-1)+3\alpha^4H(3\alpha-2,3\beta)-2\alpha^3H(3\alpha-2,3\beta)-6\beta^4H(2\alpha-1,2\beta-3)H(\alpha,\beta-1)+6\beta^3H(2\alpha-1,2\beta-3)H(\alpha,\beta-1)+6\alpha\beta^3H(2\alpha-1,2\beta-3)H(\alpha,\beta)-6\alpha\beta^2H(2\alpha-1,2\beta-3)H(\alpha,\beta)+18\alpha\beta^3H(2\alpha-1,2\beta-2)H(\alpha,\beta-1)-6\alpha\beta^2H(2\alpha-1,2\beta-2)H(\alpha,\beta-1)-3\beta^3H(2\alpha-1,2\beta-2)H(\alpha,\beta-1)-18\alpha^2\beta^2H(2\alpha-1,2\beta-2)H(\alpha,\beta)+6\alpha^2\beta H(2\alpha-1,2\beta-2)H(\alpha,\beta).$$

$$C_{42}=3(-4\alpha^3\beta H(\alpha,\beta-1)H(\alpha,\beta)^3+\alpha^4H(\alpha,\beta)^4+\beta^4H(2\alpha-1,2\beta-2)^2-4\alpha\beta^3H(2\alpha-1,2\beta-2)H(2\alpha-1,2\beta-1)+2\alpha^2\beta^2H(2\alpha-1,2\beta-2)H(2\alpha-1,2\beta)-2\beta^4H(2\alpha-1,2\beta-2)H(\alpha,\beta-1)^2+4\alpha\beta^3H(2\alpha-1,2\beta-2)H(\alpha,\beta-1)H(\alpha,\beta)-2\alpha^2\beta^2\ H(2\alpha-1,2\beta-2)\ H(\alpha,\beta)^2+4\alpha^2\beta^2H(2\alpha-1,2\beta-1)^2-4\alpha^3\beta H(2\alpha-1,2\beta-1)H(2\alpha-1,2\beta)+4\alpha\beta^3H(2\alpha-1,2\beta-1)\ H(\alpha,\ \beta-1)^2-8\alpha^2\beta^2H(2\alpha-1,\ 2\beta-1)H(\alpha,\beta-1)H(\alpha,\beta)+4\alpha^3\beta H(2\alpha-1,2\beta-1)H(\alpha,\beta)^2+\alpha^4H(2\alpha-1,2\beta)^2-2\alpha^2\beta^2H(2\alpha-1,2\beta)H(\alpha,\beta-1)^2+4\alpha^3\beta H(2\alpha-1,2\beta)H(\alpha,\beta-1)H(\alpha,\beta)-2\alpha^4H(2\alpha-1,2\beta)H(\alpha,\beta)^2+\beta^4H(\alpha,\beta-1)^4-4\alpha\beta^3H(\alpha,\beta-1)^3H(\alpha,\beta)\ +6\alpha^2\beta^2H(\alpha,\beta-1)^2H(\alpha,\beta)^2).$$

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