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On The Non-Homegeneous Biquadratic Equation With Four Unknowns $x^3+y^3+z^3=128(x+y)w^3$

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Abstract: The non-homogeneous biquadratic equation with four unknowns given by $x^3+y^3+z^3=128(x+y)W^3$ is considered and analysed for its non-zero distinct integer solution, Introducing the linear transformation $x=u+v; y=u-v; z=2u$ and employing the method of factorization different patterns of non-zero distinct integer solutions to the above biquadratic equation are obtained. A few interesting relations between solutions and special numbers namely Centered Polygonal numbers, Polygonal numbers, ,Mersenne number are exhibited..
Keywords: Non-Homogeneous biquadratic equation, Integral solutions, Special numbers.
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NOTATIONS:

Special Numbers	**Notations**
Regular Polygonal number	$t_{m,n}$
Pyramidal Number	P_n^m
Centered Polygonal number	$ct_{m,n}$
Centered Pyramidal number	CP_n^m
Mersenne number	M_n

INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular biquadratic Diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-3]. In this context one may refer [4-10] for various problems on the biquadratic Diophantine equations. However, often we come across non-homogeneous biquadratic equations and as such one may require its integral solution in its most general form. This paper concern with the non-homogeneous biquadratic equation with four unknowns $x^3+y^3+z^3=128(x+y)W^3$ for determining its infinitely many non-zero integral solutions. Also a few interesting properties among the solutions are presented.

METHOD OF ANALYSIS

The non-homogeneous biquadratic diophantine equation with four unknowns to be solved for getting non-zero integral solution is

$$x^3+y^3+z^3=128(x+y)W^3 \tag{1}$$

On substituting the linear transformations

$$x=u+v; y=u-v; z=2u \tag{2}$$

(1) reduces to $5u^2+3v^2=128W^3$ (3)

Again, introducing the linear transformation

$$u=X+3T, v=X-5T \tag{4}$$

in (3), it is written as

$$X^2 + 15T^2 = 16W^3 \tag{5}$$

Assume $W(a,b) = a^2 + 15b^2 = (a + i\sqrt{15}b)(a - i\sqrt{15}b)$ (6)

Write 16 as, $16 = (1 + i\sqrt{15})(1 - i\sqrt{15})$ (7)

Substituting (6) and (7) in (5) and using the method of factorization, define

$$(1 + i\sqrt{15})(a + i\sqrt{15}b)^3 = X + i\sqrt{15}T$$

Equating the real and imaginary parts we get,

$$\left.\begin{aligned} X(a,b) &= a^3 - 45ab^2 - 45a^2b + 225b^3 \\ T(a,b) &= a^3 - 45ab^2 + 3a^2b - 15b^3 \end{aligned}\right\} \tag{8}$$

Substituting (8) in (4) the values of u and v are given by

$$\left.\begin{aligned} u(a,b) &= 4a^3 - 180ab^2 - 36a^2b + 180b^3 \\ v(a,b) &= -4a^3 + 180ab^2 - 60a^2b + 300b^3 \end{aligned}\right\}$$

Substituting the values of u, v in (2) the corresponding non-zero integral solutions satisfying (1) are obtained as

$$x(a,b) = -96a^2b + 480b^3$$
$$y(a,b) = 8a^3 - 360ab^2 + 24a^2b - 120b^3$$
$$z(a,b) = 8a^3 - 360ab^2 - 72a^2b + 360b^3$$
$$W(a,b) = a^2 + 15b^2$$

Properties:

1. $32(a+1)W(a,b) - x(a,b) = 256P_a^5$
2. $3y(a,1) + z(a,1) - 12CP_a^6 \equiv 0 \pmod{355}$
3. $32W(2^a,1) - x(2^a,1) - 128M_{2a} = 128$
4. $3\left[32W(a,1) - x(a,1)\right]$ is a nasty number.
5. $z(a,1) + 16(4P_a^{10} + t_{33,a}) \equiv 0 \pmod 8$

Also, substituting $X = 4y, T = 4t$ (9)

in (5), it is written as $y^2 + 15t^2 = W^3$ (10)

Assume $W(a,b) = a^2 + 15b^2 = (a + i\sqrt{15}b)(a - i\sqrt{15}b)$ (11)

Write $1 = \dfrac{(1 + i8\sqrt{15})(1 - i8\sqrt{15})}{961}$ (12)

Substituting (11) and (12) in (10), following the above procedure we get the values of y, t are

$$\left.\begin{aligned} y(a,b) &= \frac{1}{31}\left[a^3 - 45ab^2 - 360a^2b + 1800b^3\right] \\ t(a,b) &= \frac{1}{31}\left[8a^3 - 360ab^2 + 3a^2b - 15b^3\right] \end{aligned}\right\} \tag{13}$$

As our aim is to find integer solutions of (1) it is seen that one may choose y and t so that the values of x, y, z, W are integers.

Let $a = 31A, b = 31B$ in (10) and (12), we get

$$X(A,B) = 3844A^3 - 172980AB^2 - 1383840A^2B + 6919200B^3$$
$$T(A,B) = 30752A^3 - 1383840AB^2 + 11532A^2B - 57660B^3$$

Substituting these values in (9), we get

$$u(A,B)=96100A^3-4224500AB^2-1349244A^2B+6746220B^3$$
$$v(A,B)=-149916A^3+6746220AB^2-1441500A^2B+7207500B^3$$

Substituting these values in (2) the non-zero integral distinct points satisfying (1) is given by

$$x(A,B)=-53816A^3+2421720AB^2-2790744A^2B+13953720B^3$$
$$y(A,B)=246016A^3-11070720AB^2+92256A^2B-461280B^3$$
$$z(A,B)=192200A^3-8649000AB^2-2698488A^2B+13492440B^3$$
$$W(A,B)=961A^2+14415B^2$$

Properties:

1. $x(1,1)+y(1,1)+z(1,1)=4674304$
2. $W(A,A-1)-t_{30754,A}\equiv 0(\mathrm{mod}15)$
3. $x(A,B)+y(A,B)-z(A,B)=0$
4. $W(A,A)-2(ct_{961,A}-1)\equiv 0(\mathrm{mod}13454)$
5. $y(A,A)+6CP_A^{11193728}\equiv 0(\mathrm{mod}11193722)$

REMARK:

It is worth to mention that instead of (7) one may have the following representation for 16,

$$16=\frac{(7+i\sqrt{15})(7-i\sqrt{15})}{4}$$

and instead of (12) one may have the following representations for 1

$$1=\begin{cases}\dfrac{(1+i\sqrt{15})(1-i\sqrt{15})}{4}\\ \dfrac{(7+i\sqrt{15})(7-i\sqrt{15})}{64}\\ \dfrac{(7+i4\sqrt{15})(7-i4\sqrt{15})}{289}\\ \dfrac{(7+i12\sqrt{15})(7-i12\sqrt{15})}{2209}\\ \dfrac{(11+i3\sqrt{15})(11-i3\sqrt{15})}{256}\\ \dfrac{(11+i4\sqrt{15})(11-i4\sqrt{15})}{361}\\ \dfrac{(11+i28\sqrt{15})(11-i28\sqrt{15})}{11881}\end{cases}$$

Following the procedure presented above, the other choices of solutions to (1) are obtained.

CONCLUSION

Instead of (4) one may consider the following transformation,

$$u=X-3T,v=X+5T$$

Following the procedure presented above, the other choices of solutions to (1) are obtained. In this paper, the non-homogeneous biquadratic equation with four unknowns given by $x^3+y^3+z^3=128(x+y)W^3$ has been analysed for its patterns of distinct integer solutions. As the cubic diophantine equations(homogeneous-non-homogeneous) are rich in variety due to the definition of diophantine equation, one may consider cubic equation with multivariables(≥4)and search for their non-zero distinct integer solutions along with their corresponding properties.

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