

Global Optimization and Filled Function Method

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Abstract: Traditional nonlinear programming method is only used to educe the local optimization but fails to successfully solve global optimization problem. Filled function method combines itself with local minimization algorithm which has been applied maturely, so it is popular with workers. The knowledge of the background and development of filled function method can determine the future research direction, but many problems require being further researched if the filled function method is developed into a mature global optimization method [1-3].

Keywords: Global optimization; Filled function.

INTRODUCTION

This paper describes the basic situations of global optimization problem, and there have been many new theories and algorithms about the global optimization in the last several decades of years so that the filled function method becomes one of the effective and deterministic algorithms in solving the global optimization problem.

Overall of global optimization

Generally, the optimization problem aims to solve the following minimal value problem:

General expression [4-6]:

$$\begin{cases} \min f(x) \\ s.t. x \in X \end{cases} \quad (1)$$

x shows decision-making variable and $x \in R^n$, $f(x)$ shows target function and $X \subseteq R^n$ shows constraint set.

Constrained optimization problem is usually as follows:

$$\begin{cases} \min f(x) \\ s.t. g_i(x) = 0, i \in E \\ g_i(x) \geq 0, i \in I \end{cases} \quad (2)$$

E shows index set of equation constraint, I shows index set constrained by inequality and $g_i(x)$ shows constraint function.

When both target function and constraint function are linear functions, formula (2) is defined as linear programming, or else nonlinear programming.

Definition 1.1: point $x^* \in X$ shows a local minimal point, if $\exists \varepsilon > 0$ and $\|x - x^*\| \leq \varepsilon$, then $f(x^*) \leq f(x)$, $f(x^*)$ shows local minimal value.

Definition 1.2: point $x^* \in X$ shows a global minimal point, if $x \in X$, then $f(x^*) \leq f(x)$, $f(x^*)$ shows global minimal value.

Up to the present, the optimization theory solving local minimal points has been mature and perfected continuously while global optimization algorithm is difficult somewhat due to global requirements; besides, there is still a lack of effective determination rules in determining whether a local optimal solution is its global optimal solution.

Relevant concepts and main thoughts of filled function method

Filled function method is one of the effective algorithms for solving the global optimization problems, this algorithm aims to construct an auxiliary function so as to find out the next better local minimal point from the current local minimal point of the target function.

It is usually assumed that $f(x)$ is continuously differentiable and meets the following on R^n [7-9]:

Assumption 1 : $f(x)$ meets the compulsory conditions:

$$\|x\| \rightarrow +\infty, \quad f(x) \rightarrow +\infty$$

From the above, \exists bounded closed set Ω makes that all global minimal points of $f(x)$ on R^n are within Ω , it is assumed that Ω is knowable, so formula (1) is equivalent to the following:

$$\begin{cases} \min f(x) \\ s.t. x \in \Omega \end{cases} \quad (3)$$

Assumption 2 : $f(x)$ has limited local minimal points on Ω .

From the above, each local minimal point of $f(x)$ on Ω is sure to be an isolated local minimal point.

Definition 2.1: point x_1^* is an isolated minimal point of $f(x)$, the basin domain of which is B_1^* which is a connected domain, $x_1^* \in B_1^*$, the steepest descent path of $f(x)$ starting from either point of B_1^* tends to x_1^* , but the steepest descent path of $f(x)$ starting from the point beyond B_1^* does not tend to x_1^* .

It would assume that some local minimal point x_1^* of $f(x)$ has been found and the following shows the definition of filled function:

Definition 2.2: $p(x)$ shows filled function of $f(x)$ on x_1^* , if $p(x)$ meets:

- (1) x_1^* shows the local maximal point of $p(x)$, and the basin domain B_1^* of $f(x)$ on x_1^* is part of the peak of $p(x)$;
- (2) In all the areas of $f(x)$ beyond the basin domain lower than B_1^* , $p(x)$ has no any minimal point;
- (3) If x_1^* is a global minimal point, there must be a smaller minimal point x_2^* , the basin domain of which is B_2^* , if $\forall x \in B_2^*$, then $x' \in B_2^*$, so that there must be local minimal points of $p(x)$ on the connecting line between x' and x_1^* .

Due to the continuous change of minimal points, filled functions constructed on different minimal points are different somewhat, so the expression of the filled function $p(x)$ is not exclusive. Because the function $p(x)$ "fills up" the basin domain higher than B_1^* , $p(x)$ is visualized as filled function[10-11].

Theorem 2.1: x_1^* is assumed as a local minimal point of $f(x)$,

$$p(x, r, \rho) = \frac{1}{r + f(x)} \exp\left(-\frac{\|x - x_1^*\|^2}{\rho^2}\right),$$

Where, r, ρ shows parameters and meet the following

$$r + f(x_1^*) > 0$$

Then x_1^* is a local maximal point of $p(x, r, \rho)$.

Theorem 2.2: it is assumed that x meets:

$$r + f(x) > r + f(x_1^*) > 0$$

The direction d meets

$$d^T (x - x_1^*) > 0$$

And $d^T \nabla f(x) \geq 0$, then

$$0 < \frac{\rho^2}{r + f(x)} < \frac{2d^T(x - x_1^*)}{-d^T \nabla f(x)}$$

d is the descent direction of $p(x, r, \rho)$ at point x . Especially, if

$$0 < \frac{\rho^2}{r + f(x_1^*)} < \frac{2D}{L}$$

D is not a negative,

$$D = \min_{x \notin S_1^*} \|x - x_1^*\|$$
$$L = \sup_{x \in X} \|\nabla f(x)\|$$

$d = (x - x_1^*)$ shows the descent direction of $p(x, r, \rho)$, so points of $\forall f(x) > f(x_1^*)$ are not steady ones of $p(x, r, \rho)$.

Theorem 2.3: it is assumed that the direction d meets the following

$$d^T(x - x_1^*) > 0, \quad x \notin S_1^*$$

$f(x)$ and r, ρ meet the inequality

$$0 < r + f(x) < -\frac{\rho^2 d^T \nabla f(x)}{2d^T(x - x_1^*)}$$

$p(x, r, \rho)$ ascends at x along the direction d . If $f(x)$ descends, then

$$r + f(x) \rightarrow 0^+$$

Accordingly,

$$p(x, r, \rho) \rightarrow +\infty$$

If $f(x)$ descends, then

$$r + f(x) < 0$$

Accordingly,

$$p(x, r, \rho) < 0$$

Theorem 2.4: if r, ρ make the following inequality available

$$0 < \frac{\rho^2}{r + f(x^*)} < \frac{2D}{L}$$

Where, x^* shows a global minimal point of $f(x)$ on X , then $p(x, r, \rho)$ will descends along the direction d , so that $p(x, r, \rho)$ has no any minimal point in the direction along $x - x_1^*$.

Summary of the basic thought of filled function method: above all, the target function is minimized to educe any local minimal point x_1^* of $f(x)$, find out the next lower local minimal point x_2^* of $f(x)$ by starting from x_1^* , namely $x_2^* \neq x_1^*$ and $f(x_2^*) \leq f(x_1^*)$.

Filled function method is composed of two stages including minimizing stage filling stage. A typical algorithm can be used to find out the local minimal point x_1^* in the minimizing stage, while a filled function is defined on the basis of x_1^* in the filling stage, and this function is utilized to find out x_2^* and repeat this process[12].

Brief summary

The advantage of filled function method is to solve the global minimal points in virtue of mature local optimization method. The search of the optimal solution mainly rests on the selection of filled function, the filled function form we construct shall be as simple as possible and parameters shall be as few as possible so as to simplify steps and save parameter-regulating time and improve the algorithm efficiency.

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