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Efficiency Evaluation of the Two-Stage Units Using Two-Objective Linear Fractional Programming

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Abstract: Data envelopment analysis (DEA)is a method that evaluates the efficiency of decision making units by the linear programming problems. The decision making units have multiple input and outputs. Some of the decision making units can have a two-stage structure that use the first stage inputs to create the outputs that are considered the inputs in the second stage. Then the second stage uses the outputs of the first stage to produce its outputs. The out puts of the first stage are called the mean value. The relationship between the data envelopment analysis and the multi objective linear fractional programming provides a strong tool for the decision maker to provide an appropriate model for the inefficient units based on the existing limitations. In this thesis we present a new model based on the two-objective linear fractional programming that besides being linear it would be possible to use the obtained results in recognizing the efficiency or inefficiency of the two stage decision making units.
Keywords: data envelopment analysis, two-stage decision making units, multi-objective linear fractional programming, efficiency test.

INTRODUCTION

The data envelopment analysis is an approach to measure the relative efficiency of the homogenous decision making units that have several inputs and outputs. The decision making units do not have a simple structure. There are decision making units in the real world the production process of which can be regarded as a two-stage process. The DMUs can have a two-stage structure in some cases, such as banks, production units, universities, businesses, etc... . A DMU in such situations expresses a two stage process and the mean values are between the two stages. The first stage inputs produce the outputs used as the inputs in the second stage. Hence the outputs of the first stage are called the mean values. Then the second stage used these values to produce the outputs for the second stage [1].

Chen et al. [2] based on CCR and constant returns to scale presented models to calculate the efficiency of the first and second stage. By considering the objective function as a convex combination of the objective functions of the first and second stages and using the offered constraints they provided the final structure of the collective model that leads to the calculation of the efficiency. Kao and Hwang [3] offered models to calculate the efficiency of the first and second stages and efficiency of the whole system according to the CCR model and constant returns to scale. They expressed the multiplicative model by expressing the objective function as the multiplication of the objective functions of the first and second stage efficiency models and using the defined constraints on the first and second stage models and the model performance. Chiou et al. [4] presented the aggregated model of data envelopment analysis to measure the decision making units with a two stage internal network structure with several inputs and outputs and dosage and claimed that any optimal solution is determined by a global optimum solution not a local optimum. Lim and Zhu [5] showed that the DEA model introduced Chiou et al. [4] is a non-convex optimum solution and rejected their claim that any optimal solution is determined by a global optimum solution not a local optimum.

The concept of multi objective functions was first introduced by Kahn and tucker using the vector optimization concept and then performed in many studies concerned with the expansion of decision making models with multiple objectives. Kornbluth [6] was the first one who argued that the DEA model can be considered as a multiple objective linear fractional problem. This has been studied by many researchers such as Ebrahimnejad and Hosseinzadeh Lotfi [7], Joro et al. [8] and Wong et al. [9] and they presented strategies for the expansion and application of this relation [10]. Given that the majority of models for evaluating the efficiency of two component decision making units have a non-linear nature, so this paper is after finding linear models to test and calculate the efficiency of the two stage DMUs considering the evaluation of the two-stage units model as a multi objective linear fractional programming model.

This article includes the following sections: in the following section the linear fractional programming is discussed. In continuation, multi-objective linear fractional model is represented. Choo and Atkins [11]is then used to solve the two-objective linear fractional programming problem. Efficiency evaluation of the DMUs is analyzed by two-objective linear fractional programming. Finally the presented method is discussed using a numerical example.

Linear fractional programming

Working on fractional programming started since the beginning of 1960s and widely extended in early 1970s. The fractional programming optimizes the ratio of the two functions. Linear fractional programming is a special form of fractional programming in which the numerator and denominator of the target are two linear functions. The problems are applied in financial, economical and healthcare programming. The general form of a linear fractional programming problem is as follows:

$$\text{Max } \frac{cx+\alpha}{dx+\beta},$$
$$st \quad x \in S = \{x \in R^n | Ax \le b, x \ge 0\}, \tag{1}$$

Where *c* and *d* are the row vectors with *n* components and *b* is a column vector with *m* components and an $m \times n$ matrix and α and β are scalars and $dx+\beta>0,\ x \in S$.

Multi-objective linear fractional programming

Work on multi-objective linear fractional programming started since the early 1980s with discussing the strong and weak effective points of the problems of multi-objective linear fractional programming. Consider the following multi-objective linear fractional programming program:

$$\text{Max} \quad z_k(x) = \frac{N_k(x)}{D_k(x)}, \qquad k = 1,\dots,k,$$
$$st. \quad x \in S \tag{2}$$

where $N_k(x) = c_k x + \alpha_k$, $D_k(x) = d_k x + \beta_k$, $c_k, d_k \in R^n$ and α_k, β_k are Scalar and *S* is a non-empty and bounded polyhedron and $\forall x \in S$, $D_k(x) > 0$ is a multi-objective programming and a special form of multi-objective linear fractional programming that $d_k = 0$, $k = 1,\dots,K$.

In the next section the two-objective linear fractional programming method of Choo and Atkins [11] is analyzed.

A Method to solve two-objective linear fractional programming

Definition1: Two-objective linear fractional programming method can be expressed as follows:

$$\text{Max} \quad F(x) = (f_1(x), f_2(x)),$$
$$st. \quad x \in S \tag{3}$$

Where *A* is an $m \times n$ matrix and *b* is a column vector with *m* components and f_1 and f_2 are linear fractional functions defined as follows:

$$f_i(\mathrm{x}) = \frac{c_i^T x + r_i}{d_i^T x + t_i}, \qquad i = 1, 2,$$

where $c_i, d_i \in R^n$ and r_i, t_i are scalar and *S* is a non-empty and bounded polyhedron and $\forall x \in S$, $d_i^T x + t_i > 0$

Definition 2: Suppose x^2 , x^1 are two possible points in *S.* we call x^1 dominant x^2 whenever:

$$f_1(x^1) < f_1(x^2) \quad , \quad f_2(x^1) \le f_2(x^2)$$

or

$$f_1(x^1) \le f_1(x^2) \quad , \quad f_2(x^1) < f_2(x^2)$$

A point which is not dominant over any other possible point is called Pareto efficient. We signify the set of Pareto efficient points of the problem (3) with E.

Lemma1: For any two possible points x^2 , x^1 in *S*:

$$(i) f_i(x^1) \le f_i(x^2), \quad iff \quad (x^2 - x^1)\nabla f_i(x^1) \ge 0,$$

$(ii)\, f_i\left(x^1\right) < f_i\left(x^2\right), \quad iff \qquad \left(x^2 - x^1\right)\nabla f_i\left(x^1\right) > 0.$

Note1: From Lemma 1 we have f_i is monotonic along any segment connecting x^2 , x^1 in *S*, if $f_i\left(x^1\right) < f_i\left(x^2\right)$,

Then

$f_i\left(x^1\right) < f_i(\lambda x^1 + \left(1-\lambda\right)x^2\right) < f_i\left(x^2\right),$

For any $\lambda \in (0,1)$. Also f_i on *S* is both quasi-convex and quasi-concave.

The characteristics of Pareto efficient points

Suppose $p_1, \ldots, p_m$ are the row vectors of the matrix *A* thus an *S* procedure is defined as follows.

Definition3: The subset *T* of *S* is called an *S* procedure when there is $J \subset \{1, \ldots, m\}$, so that *T* includes points that are true in the following conditions:

$$p_i^T x = b_i, \quad \forall i \in J, \tag{4}$$

$$p_i^T x < b_i, \quad \forall i \notin J. \tag{5}$$

T is called the *J* corresponding procedure.
The following theorem characterizes the Pareto efficient points in *T*.

Theorem1: An x^* point is Pareto efficient points in the *T* procedure corresponding $\{1, \ldots, k\}$ if and only if $x^* \in T$ and there are real numbers

$a_i \geq 0, \ i = 1, \ldots, k, \ \ w_1 \geq 1, w_2 \geq 1,$

So that

$$a_1 p_1 + \ldots + a_k p_k = w_1 \nabla f_1\left(x^*\right) + w_2 \nabla f_2\left(x^*\right). \tag{6}$$

Note2: For a multi-objective linear fractional programming with *l* objectives the equation (6) of the Theorem 1is replaced by

$$a_1 p_1 + \ldots + a_k p_k = w_1 \nabla f_1\left(x^*\right) + \ldots + w_l \nabla f_l\left(x^*\right). \tag{7}$$

In the next section we show that for $l = 2$ equation (6) is equivalent to a system of linear equations and inequalities constraints.

The Linear feature of the Pareto efficient points set

For each $x \in S$ the gradient vector $\nabla f_i\left(x\right)$ has a direction like the vector $\left(d_i^T x + t_i\right)c_i - (c_i^T x + r_i)d_i$ because $d_i^T x + t_i > 0$ Thus, equation(4) in Theorem 1 can be replaced by

$$a_1 p_1 + \ldots + a_k p_k = w_1\left(d_1^T x^* + t_1\right)c_1 - w_1\left(c_1^T x^* + r_1\right)d_1 + w_2\left(d_2^T x^* + t_2\right)c_2 - w_2\left(c_2^T x^* + r_2\right)d_2. \tag{8}$$

Equation (8) is equivalent to

$$a_1 p_1 + \ldots + a_k p_k = q_1 c_1 - h_1 d_1 + q_2 c_2 - h_2 d_2, \tag{9-1}$$

$$f_i\left(\mathrm{x}^*\right) = \frac{h_i}{q_i}, \ i = 1, 2, \quad q_i > 0. \tag{9-2}$$

Since the equation (9-1) can be multiplied by any positive real number, then

$f_1\left(\mathrm{x}^*\right) = \frac{h_1}{q_1},$

Can be replaced by

$c_1^T x^* + r_1 = h_1, \qquad\qquad d_1^T x^* + t_1 = q_1,$

With inner multiplication of x^* on both sides of the equation (9-1), we have:

$b_1 a_1 + \ldots + b_k a_k = q_1 c_1^{\mathrm{T}} x^* - h_1 d_1^T x^* + q_2 c_2^T x^* - h_2 d_2^T x^*.$

Now

$f_2\left(\mathrm{x}^*\right) = \frac{h_2}{q_2} \quad iff \quad h_2 d_2^T x^* + h_2 t_2 = q_2 c_2^T x^* + q_2 r_2,$

$$iff \quad h_2 t_2 - q_2 r_2 = q_2 c_2^T x^* - h_2 d_2^T x^*,$$

$$iff \quad h_2 t_2 - q_2 r_2 - q_1\left(c_1^T x^* + r_1\right) = q_2 c_2^T x^* - h_2 d_2^T x^* - h_1\left(d_1^T x^* + t_1\right),$$

$$iff \quad h_2 t_2 - q_2 r_2 - q_1 r_1 + h_1 t_1 = q_1 c_1^T x^* - h_1 d_1^T x^* + q_2 c_2^T x^* - h_2 d_2^T x^*,$$

$$iff \quad h_1 t_1 + h_2 t_2 - q_1 r_1 - q_2 r_2 = b_1 a_1 + \ldots + b_k a_k .$$

Thus, equation (6) in Theorem 1 is equivalent to the system

$$\begin{aligned}
& a_1 p_1 + \ldots + a_k p_k = q_1 c_1 - h_1 d_1 + q_2 c_2 - h_2 d_2 \ , \\
& c_1^T x^* + r_1 = h_1, \\
& d_1^T x^* + t_1 = q_1, \\
& t_1 h_1 + t_2 h_2 - r_1 q_1 - r_2 q_2 = b_1 a_1 + \ldots + b_k a_k , \\
& q_i > 0, i = 1, 2.
\end{aligned}$$

Lemma2: Point $x^* \in T$ is Pareto efficient if and only if there are real numbers $a_1, \ldots, a_k, h_1, h_2, q_1, q_2$, so that

$$\begin{aligned}
& a_1 p_1 + \ldots + a_k p_k = q_1 c_1 - h_1 d_1 + q_2 c_2 - h_2 d_2, \\
& d_1^T x^* + t_1 = q_1, \\
& t_1 h_1 + t_2 h_2 - r_1 q_1 - r_2 q_2 = b_1 a_1 + \ldots + b_k a_k , \\
& p_i^T x^* = b_i , \qquad i = 1, \ldots, k , \\
& p_i^T x^* < b_i , \qquad i = k + 1, \ldots, m , \\
& a_i \geq 0 \ , \ i = 1, \ldots, k , \qquad q_i > 0, i = 1, 2.
\end{aligned} \tag{10}$$

From Lemma 2 it can be seen that the Pareto efficient points in *T* are determined by the linear and non-linear equations system. Therefore, we conclude the following theorem.

Result 1: The set of all Pareto efficient points is a convex set.

Theorem 2: The intersection of *S* and *T* is a set of linear constraints and *E* is the finite number of sets of linear constraints.

Two-stage data envelopment analysis

Suppose that there are *n* decision-making units that $x_{ij} (i = 1, \ldots, m)$ and $y_{rj} (r = 1, \ldots, s)$ respectively represent the i[th] input and the j[th] output in $DMU_j \ (j = 1, \ldots, n)$. The DEA standard model assuming constant returns to scale for calculating the efficiency of the whole k[th] decision making unit is as follows:

$$\begin{aligned}
& E_k = Max \qquad \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}, \\
& S.t. \qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad j = 1, \ldots, n, \\
& u_r \geq \varepsilon, \qquad\qquad r = 1, \ldots, s, \\
& v_i \geq \varepsilon, \qquad\qquad i = 1, \ldots, m.
\end{aligned} \tag{11}$$

where ε is a small non-Archimedean value and x and y are the inputs and outputs of decision making unit. E_k represents the total efficiency of DMU_k by the DEA standard models. If $E_k = 1$ then DMU_k is efficient and if $E_k < 1$ then DMU_k is inefficient. Now suppose that the manufacturing process consists of a series of two-stage sub processes. Unlike standard manufacturing processes it consists of two processes with mean value (middle data) $(p = 1, \ldots, q) z_{pk}$. In addition, the z_{pk} middle data are the outputs of the first stage that are the input of the second stage without any changes. This is shown in Figure 1.

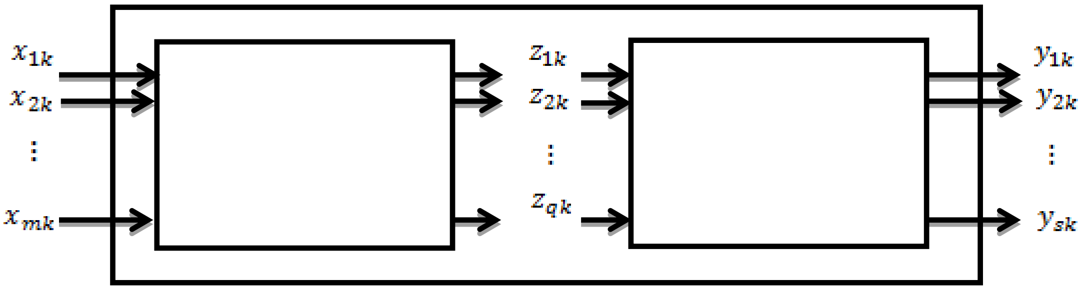

Figure 1: A two-staged process with the x input and y output and the middle input z

Previous studies in the context of a two-stage units of model (11) to measure the overall efficiency of DMU_k and the models (12) and (13) are used to calculate the efficiency of the first and second stages.

$$E_k^1 = Max \quad \frac{\sum_{p=1}^{q} w_p z_{pk}}{\sum_{i=1}^{m} v_i x_{ik}},$$
$$S.t. \quad \frac{\sum_{p=1}^{q} w_p z_{pj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j=1,\dots,n,$$
$$w_p \ge \varepsilon, \qquad p=1,\dots,q,$$
$$v_i \ge \varepsilon, \qquad i=1,\dots,m. \tag{12}$$

$$E_k^2 = Max \quad \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{p=1}^{q} w_p z_{pk}},$$
$$S.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{p=1}^{q} w_p z_{pj}} \le 1, \qquad j=1,\dots,n,$$
$$u_r \ge \varepsilon, \qquad r=1,\dots,s,$$
$$w_p \ge \varepsilon, \qquad p=1,\dots,q. \tag{13}$$

Models (12) and (13) are taken from the model (11). In these models, the efficiency of the system and two sub stages are considered independent of each other. Therefore we need a model that could establish logical relation between the whole system and the processes.

The efficiency evaluation of two-stage units

Consider the vector two-stage DEA model as follows:

$$E_k = \text{Max} \quad \frac{w^T z_k}{v^T x_k} + \frac{u^T y_k}{w^T z_k},$$
$$s.t. \quad u^T y_j \le w^T z_j, \qquad j=1,\dots,n,$$
$$w^T z_j \le v^T x_j, \qquad j=1,\dots,n,$$
$$v \ge 0, w \ge 0, u \ge 0. \tag{14}$$

In fact the two-stage DEA model of a two-objective linear fractional programming problem is as follows:

$$\text{Max} \quad \frac{w^T z_k}{v^T x_k},$$
$$\text{Max} \quad \frac{u^T y_k}{w^T z_k},$$

$$S.t. \qquad \frac{u^T y_j}{w^T z_j} \le 1, \qquad j = 1,\ldots,n,$$

$$\frac{w^T z_j}{v^T x_j} \le 1, \qquad j = 1,\ldots,n,$$

$$v \ge 0, w \ge 0, u \ge 0. \tag{15}$$

By the summing up the targets by a weighted factor 0.5 we can reach the model (14). Now considering

$$q_1 = vx_k, \quad q_2 = wz_k, \; r_1 = 0, \quad r_2 = 0,$$
$$h_1 = wz_k, \quad h_2 = uy_k, \quad t_1 = 0, \quad t_2 = 0,$$
$$c_1 = z_k, \quad d_1 = x_k, \quad c_2 = y_k, \quad d_2 = z_k,$$
$$p_k = (z_k \;\; -x_k \;\; 0), \; p_k^{'} = (z_k \;\; -x_k \;\; 0),$$

And placement in Lemma 2 we have the following situation:

$$a_k(z_k \;\; -x_k \;\; 0) + a_k^{'}(-z_k \;\; 0 \;\; y_k) = (vx_k)(z_k \;\; 0 \;\; 0) - (wz_k)(0 \;\; x_k \;\; 0) + (wz_k)(0 \;\; 0 \;\; y_k) - (uy_k)(z_k \;\; 0 \;\; 0),$$

$$(z_k \;\; -x_k \;\; 0)\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} a_k + (-z_k \;\; 0 \;\; y_k)\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} a_k^{'} = 0,$$

$$(z_k \;\; -x_k \;\; 0)\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} = 0,$$

$$(-z_k \;\; 0 \;\; y_k)\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} = 0,$$

$$\begin{pmatrix} z_j & -x_j & 0 \\ -z_j & 0 & y_j \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} \le 0, \quad j = 1,\ldots,n, \;\; j \ne k,$$

$$a_k \ge 0,$$
$$a_k^{'} \ge 0,$$
$$v^* x_k > 0,$$
$$w^* z_k > 0,$$

Which is a system of linear equations and inequalities. The objective is to maximize the sum of the deviations in the inactive constraints. Thus we have the following model:

$$\text{Max} \qquad \sum_{\substack{j=1 \\ j \ne k}}^{n} (s_j + s_j^{'}) + p + q,$$

$$s.t \quad a_k(z_k \;\; -x_k \;\; 0) + a_k^{'}(-z_k \;\; 0 \;\; y_k) = (vx_k)(z_k \;\; 0 \;\; 0) - (wz_k)(0 \;\; x_k \;\; 0) + (wz_k)(0 \;\; 0 \;\; y_k) - (uy_k)(z_k \;\; 0 \;\; 0),$$

$$(z_k \;\; -x_k \;\; 0)\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} a_k + (-z_k \;\; 0 \;\; y_k)\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} a_k^{'} = 0,$$

$$\begin{pmatrix} z_k & -x_k & 0 \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} = 0,$$

$$\begin{pmatrix} -z_k & 0 & y_k \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} = 0,$$

$$\begin{pmatrix} z_j & -x_j & 0 \\ -z_j & 0 & y_j \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} + \begin{pmatrix} s_j \\ s'_j \end{pmatrix} = 0, \quad j = 1,\dots,n, \;\; j \neq k,$$

$$a_k \geq 0,$$
$$a'_k \geq 0,$$
$$v^* x_k - p = 0,$$
$$w^* z_k - q = 0,$$
$$v^* \geq 0, u^* \geq 0, w^* \geq 0. \qquad (16)$$

If we could find positive w^*, v^* and u^* that apply in terms of the model then the DMU under evaluation is efficient, otherwise if w^*, v^* and u^* are zero and the model is not applicable or if the w^*, v^* and u^* are positive and the function is zero then the DMU under evaluation is inefficient.

In order to prevent the weight from becoming zero the non-Archimedean ε is used. So that this value as a lower bound for the input and output weights prevents them from being zero. The zero weights mean that these unique inputs and outputs were not effective in the efficiency. In order to resolve this problem the $\varepsilon > 0$ non-Archimedean value is used. Under this condition the nonnegative weights with $v^* \geq \varepsilon, u^* \geq \varepsilon, w^* \geq \varepsilon$ boundaries are replaced.

Therefore, considering the $\varepsilon = 10^{-6}$ condition we rewrite the model:

$$\text{Max} \quad \sum_{\substack{j=1 \\ j\neq k}}^{n} (s_j + s'_j) + p + q,$$

$$s.t \quad a_k \begin{pmatrix} z_k & -x_k & 0 \end{pmatrix} + a'_k \begin{pmatrix} -z_k & 0 & y_k \end{pmatrix} = (vx_k)\begin{pmatrix} z_k & 0 & 0 \end{pmatrix} - (wz_k)\begin{pmatrix} 0 & x_k & 0 \end{pmatrix} + (wz_k)\begin{pmatrix} 0 & 0 & y_k \end{pmatrix} - (uy_k)\begin{pmatrix} z_k & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} z_k & -x_k & 0 \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} a_k + \begin{pmatrix} -z_k & 0 & y_k \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} a'_k = 0,$$

$$\begin{pmatrix} z_k & -x_k & 0 \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} = 0,$$

$$\begin{pmatrix} -z_k & 0 & y_k \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} = 0,$$

$$\begin{pmatrix} z_j & -x_j & 0 \\ -z_j & 0 & y_j \end{pmatrix}\begin{pmatrix} w^* \\ v^* \\ u^* \end{pmatrix} + \begin{pmatrix} s_j \\ s'_j \end{pmatrix} = 0, \quad j = 1,\dots,n, \;\; j \neq k,$$

$$a_k \geq 0,$$
$$a_k^{'} \geq 0,$$
$$v^{*} x_k - p = 0,$$
$$w^{*} z_k - q = 0,$$
$$v^{*} \geq \varepsilon, u^{*} \geq \varepsilon, w^{*} \geq \varepsilon \qquad (17)$$

Now, if we find w^{*}, v^{*} and u^{*} that apply in the active constraints, that w^{*}, v^{*} and u^{*} is effective then the DMU under evaluation is efficient. However, if the model does not apply which means that it does not find w^{*}, v^{*} and u^{*} in which the efficiency of the first and second stage is 1, then the DMU under consideration is inefficient. Models (16) and (17) only analyze the efficiency of the DMU under consideration.

Example

Consider the example presented in Chapter 3. We study the accuracy of the proposed model on the provided example. The results of the proposed models regarding ε in Table 1 and regardless of ε are expressed as follows. We solved model (16) for 24 units, the results obtained from solving the model shows that for all DMUs the objective function value is zero, so none of the DMUs in this method using this method are efficient.

Table 1: Results of model (16) for 24 participants

Unit	Results of model	**Infeasibility**
1	0.310E-04	0.441E-06
2	0.124E-04	0.825E-06
3	0.497E-04	0.100E-05
4	0.493E-04	0.826E-06
5	0.495E-04	0.912E-06
6	0.261E-04	0.617E-07
7	0.658E-05	0.972E-07
8	0.162E-04	0.100E-05
9	0.242E-04	0.580E-06
10	0.545E-05	0.179E-05
11	0.318E-04	0.100E-05
12	0.296E-04	0.869E-06
13	0.106E-04	0.100E-06
14	0.851E-05	0.399E-05
15	0.151E-04	0/384E-06
16	0.366E-04	0.979E-06
17	0.186E-04	0.100E-05
18	0.612E-05	0.982E-06
19	0.125E-03	0.100E-05
20	0.202E-03	0.855E-06
21	0.433E-04	0.679E-06
22	0.376E-03	0.822E-06
23	0.568E-04	0.100E-05
24	0.935E-04	0.100E-05

As can be seen by considering the ε condition all DMUs are positive but since they have infeasible value thus none of the DMUs are efficient.

CONCLUSION

The main problem of the models presented to calculate the two-stage units efficiency is that due to using fractional models it is not always possible to calculate the two-stage DMU models relative efficiency and models calculate the absolute efficiency. Their main problems in analyzing the DMUs such as producing the function boundary and the lack of determination of the model units for the inefficient units are among the results of using the fractional models. So a model that can recognize the efficiency of the two stage DMUs using the tow objective linear fractional programming solution is presented. In this thesis, a discussion about DEA, some DEA models and basic definitions was presented. According to the two-stage DMUs it is necessary to use the two stage DEA model to evaluate the efficiency of these units. So in the chapter 3 we discussed the DEA models through introducing some models and providing examples. In chapter four which is the most important part of this research first we introduced the multi-objective linear fractional programming and then we used this model in two- stage DEA by introducing the two-objective linear fractional programming.

At the end of this thesis a model was presented to two- stage DEA using two-objective linear fractional programming. The model was tested on an example. This model does not offer the efficiency value but it only evaluated the efficiency of the units. In this model if the target function is positive, the unit under evaluation is efficient otherwise it is not. The main advantage of this model over the previous two-stage models is its linearity.

Since many organizations and institutions such as banks, social security institutions or … have two stage structures so the proposed model can be used to test their efficiency. The future studies can present a model that can calculate the value of efficiency. The discussion of the multiplicative model with variable returns to scale can be studied in future research.

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