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The exact solutions to the generalized Ginzburg-Landau equation with high-order nonlinear term

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Abstract: We study the exact solutions to the generalized Ginzburg-Landau equation with high-order nonlinear term in this paper, After the travelling wave transformation, the equation is reduced to an integrable ordinary differential equation under the simple transform. And then using a complete discrimination system for polynomial, we obtain the classifications of all single travelling wave solutions to the generalized Ginzburg-Landau equation with high-order nonlinear term.
Keywords: Ginzburg-Landau equation, the travelling wave transformation, complete discrimination system for polynomial, exact solutions**.**

INTRODUCTION

The essence of the complete discrimination system for polynomial[1-4] is turning the equation unanswered into the elementary integral form. By discussing the classification of a polynomial's roots, furthermore getting its all possible exact travelling wave solutions. This method is proposed by Liu Chengshi, which is a innovation and simple method following the homogeneous balance method[5], varied elliptic function expansion method[6], varied hyperbolic function expansion method[7], sine-cosine method[8] and nonlinear transformation method[9] and so on. Professor Liu's method obtain the more plentiful exact solutions for nonlinear evolution equations. The author make use of the complete discrimination system for polynomial, to solve the exact solutions to the generalized Ginzburg-Landau equation[10] with high-order nonlinear term. we received the unprecedented solutions by using the other method. In the present paper, we consider the generalized Ginzburg-Landau equation :

$$iu_t + b_1 u_{xx} + b_2 |u|^{2p} u + b_3 |u|^{4p} u + ib_4 \left(|u|^{2p} u\right)_x + ib_5 \left(|u|^{2p}\right)_x = 0,\ p > 0, \tag{1}$$

we will give the classification of traveling wave solutions to the generalized Ginzburg-Landau equation.

CLASSIFICATION OF EXACT TRAVELLING WAVE SOLUTIONS OF GINZBURG-LANDAU EQUATION

In order to obtain the traveling wave solutions, we do the travelling wave transformation,

$$u(x,t) = e^{i(x-t)} a(\xi),\ \xi = x - \omega t \tag{2}$$

Where ω is the Traveling wave parameter.

Equation(1)is reduced to the following equation:

$$\begin{aligned} &b_1 a''(\xi) + (2b_1 - \omega) i a'(\xi) + (1 - b_1) a(\xi) + (b_2 - b_4) a^{2p+1}(\xi) \\ &+ [ib_4 (2p+1) + ib_5 2p] a^{2p}(\xi) a'(\xi) + b_3 a^{4p+1}(\xi) = 0. \end{aligned} \tag{3}$$

Let $\omega = 2b_1,\ p = \dfrac{b_4}{b_5 - 2b_4}$, equation (3) as expressed below

$$a''(\xi) + \frac{1-b_1}{b_1} a(\xi) + \frac{b_2 - b_4}{b_1} a^{2p+1}(\xi) + \frac{b_3}{b_1} a^{4p+1}(\xi) = 0, \tag{4}$$

Denoting $\dfrac{1-b_1}{b_1} = l, \dfrac{b_2 - b_4}{b_1} = m, \dfrac{b_3}{b_1} = n,$ equation (4) is reduced to

$$a''(\xi) + la(\xi) + ma^{2p+1}(\xi) + na^{4p+1}(\xi) = 0. \tag{5}$$

Integrating Eq.(5) once and doing some transformation $a(\xi) = \pm (U(\xi))^{\frac{-1}{2p}}$, we have

$$U_\xi^{\ 2} = -4p^2 lU^2 - \frac{4p^2 m}{p+1} U - \frac{4p^2 n}{2p+1} + 8p^2 cU^{2+\frac{1}{p}}, \tag{6}$$

where c is an integral constant.

In the present paper, we consider $c = 0$, the Eq.(6) is reduced to

$$U_\xi^{\ 2} = -4p^2 lU^2 - \frac{4p^2 m}{p+1} U - \frac{4p^2 n}{2p+1}, \tag{7}$$

$$\pm(\xi - \xi_0) = \frac{1}{|2p|} \int \frac{dU}{\sqrt{-lU^2 - \frac{m}{p+1}U - \frac{n}{2p+1}}}, \tag{8}$$

where we mark $-l = a_2, -\dfrac{m}{p+1} = a_{1,}\ -\dfrac{n}{2p+1} = a_{0.}$

$$\pm|2p|(\xi - \xi_0) = \int \frac{dU}{\sqrt{a_2 U^2 + a_1 U + a_0}}, \tag{9}$$

we sign $F(U) = a_2 U^2 + a_1 U + a_0$.

Now under the complete discrimination system for polynomial ,we give the classification of all travelling solutions to Ginzburg-Landau equation. we have the following results. Then we need to classify the roots of a polynomial according to its coecients as following:

Case 1. $\Delta = 0$.

$F(U)$ has two real equal roots, $F(U) = a_2\left(U + \dfrac{a_1}{2a_2}\right)^2$.

If $a_2 > 0$, we get the corresponding solution of Eq.(9)

$$U_1 = \pm \exp\left[\pm 2\sqrt{a_2}\,|p|(\xi - \xi_0)\right] - \frac{a_1}{2a_2}, (a_2 > 0) \tag{10}$$

Case 2. $\Delta > 0$.

$F(U)$ has two real roots unequal. $F(U) = a_2\left(U + \frac{a_1}{2a_2}\right)^2 - \frac{a_1^{\,2} - 4a_2a_0}{8a_2^{\,2}}$.

Then the corresponding solutions are:

$$U_2 = \pm\frac{1}{2}\exp\left[\pm 2\sqrt{a_2}\,|p|(\xi - \xi_0)\right] + \frac{a_1^{\,2} - 4a_2a_0}{8a_2^{\,2}}\exp\left[\pm 2\sqrt{a_2}\,|p|(\xi - \xi_0)\right] \mu \frac{a_1}{2a_2}, (a_2 > 0) \tag{11}$$

$$U_3 = -\frac{1}{2a_2}\left\{\pm\sqrt{a_1^{\,2} - 4a_2a_0}\,\sin\left[\pm 2\sqrt{a_2}\,|p|(\xi - \xi_0)\right] + a_1\right\}, (a_2 < 0) \tag{12}$$

Case 3. $\Delta < 0$.

Now $F(U)$ has no real roots.

When $a_2 > 0$, we get the corresponding solution of Eq.(9) the same

as **Case 2**.

$$U_4 = \pm\frac{1}{2}\exp\left[\pm 2\sqrt{a_2}\,|p|(\xi - \xi_0)\right] + \frac{a_1^{\,2} - 4a_2a_0}{8a_2^{\,2}}\exp\left[\pm 2\sqrt{a_2}\,|p|(\xi - \xi_0)\right] \mu \frac{a_1}{2a_2}, (a_2 > 0) \tag{13}$$

CONCLUSIONS

In summary, the generalized Ginzburg-Landau equation with high-order nonlinear term was reduced to an elementary integral form, furthermore using the complete discrimination system for polynomial to classify the roots. we obtained all possible exact travelling waves solutions. This method is simple but powerfully applicable to many nonlinear evolution equations of Mathematical Physics.

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