Scholars Journal of Physics, Mathematics and Statistics **ISSN 2393-8056 (Print)**
Sch. J. Phys. Math. Stat. 2015; 2(2B):233-248 **ISSN 2393-8064 (Online)**

Malmquist productivity index in two-stage production systems with interval data

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Abstract: The customary two-stage data envelopment analysis model, measures overall performance of a system which includes two stages in a specific period of time. In these models, some variables (the variables between two stages) are ignored. This research develops the two-stage data envelopment analysis model in several time periods which makes possible the periodical and overall performance calculation in each stage as the weighted mean of the previous stage. Therefore, the performance of a two-stage system in one period is decomposed to performance of each period. This analysis is originated from identification of a unit under inefficient assessment. Concerning the model, in order to calculate the performance, also the common local weight of productivity index between two periods can be utilized and its performance is obtained by multiplying two sub-stages and the advances and retreats of a system can be discussed along two or several time periods. Considering the assumption of inaccurate data (interval data), cause the existing problem condition to be solved optimistically and pessimistically and the performance is achieved for the system.
Keywords: data envelopment analysis, two-stage system, Malmquist productivity index, interval data

INTRODUCTION

Two-stage production systems are often systems which have two- stage calculation process which are placed serially and in tandem .in fact, two-stage production systems which have two connected stages serially in a way that input of first stage produce the output which are called intermediate products, then these intermediate products are considered input in second stage and produce the final products of the system.

Figure 1 displays the structure of a two-stage output system x_i $(i=1,...,m)$, z_f $(f=1,...,g)$ and y_r $(r=1,...,s)$ which are respectively the inputs, intermediate products, and outputs.

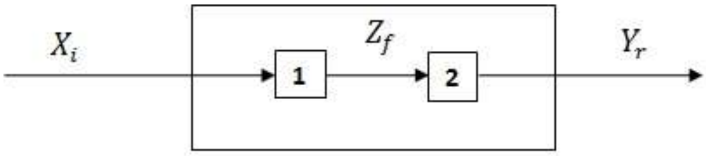

Fig-1: Two-stage productive system

If the inner structure is ignored in this system, Charnes and Cooper [1] model is resulted which calculates the performance for n units under assessment with total system input x_i $(i=1,...,m)$ and the produced output by system y_r. The suggested model is as follows:

$$
\begin{aligned}
& E_k^{CCR} = \max \sum_{r=1}^{s} u_r Y_{rk} \\
& s.t. \\
& \sum_{i=1}^{m} v_i X_{ik} = 1 \qquad\qquad\qquad\qquad (1) \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \qquad j = 1,\dots,n \\
& u_r, v_i \ge \varepsilon \qquad r = 1,\dots,s \quad i = 1,\dots,m
\end{aligned}
$$

where u_r, v_i the corresponding are weights of inputs and outputs and ε is a very small non-Archimedean number which is smaller than any positive real number and is considered to prevent ignoring any effective factor in calculation.
When the performance of decision making unit (DMU) is calculated in best condition, the results are acceptable for all DMUs. On the other hand, without considering the function of local stages, unreasonable results may happen, that is the system may be efficient whereas the two stages are inefficient.

Accordingly, different models are suggested, Fare and Grosskopf [2] proposed one of these models which local function is considered in this model as network structure while calculating system efficiency. In addition, Kao [3] suggested models that the connections and communications between local stages are categorized in it.

New studies have focused on connection models because these models make system assessment and stage performance possible in any time and there is a mathematical relationship between them [4, 5]. All the models suggested so far, aim at calculation efficiency of a DMU in a specific period of time. For example, Kao and Hwang [6] calculated non-life insurance during two years.

The following section provides efficiency and productivity evaluation two-stage DMUs. Interval data are considered in two-stage units to measure interval efficiency and estimate productivity, in section 3. Conclusions and remarks are presented in the last section.

Calculating efficiency

We assume assessing the performance of n unit under assessment which has two-stage structure in a q time period. Assume that x_{ij}^{p} defines inputs, z_{fj}^{p} the intermediate products, and y_{rj}^{p} are the jth outputs of DMU in time period of p. accordingly all the related data of a DMU can be displayed as follows:

$$
Y_{rj} = \sum_{p=1}^{q} y_{rj}^{p} \text{ و } Z_{ij} = \sum_{p=1}^{q} z_{fj}^{p} \text{ ،} X_{ij} = \sum_{p=1}^{q} x_{ij}^{p}
$$

Figure 2 displays the structure of such a system which includes several periods and each period has two-stage structure.

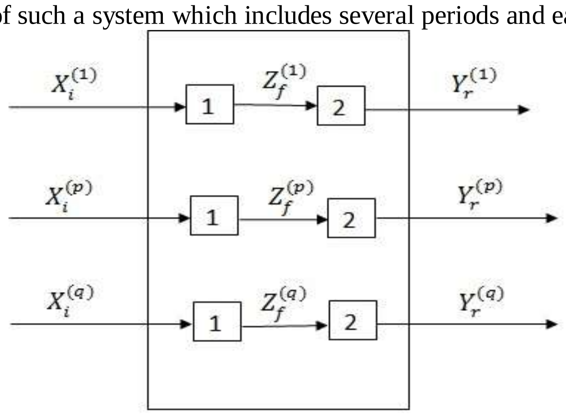

Fig-2: Two-stage productive system in several periods

Kao and Hwang [5] model to assess the total system performance and the k_{th} stage of DMU which has a two-stage system is like model (1) considering the common definitions in data envelopment analysis (DEA), and the function is calculated with necessary constraints that is, sum of output is smaller or equals sum of inputs.

$$
\begin{aligned}
&\hat{E}_k^s = \max \sum_{r=1}^{s} u_r Y_{rk} \\
&s.t. \\
&\sum_{i=1}^{m} v_i X_{ik} = 1 \\
&\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \qquad j = 1,\dots,n \qquad (2) \\
&\sum_{f=1}^{g} w_f Z_{fj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \qquad j = 1,\dots,n \\
&\sum_{r=1}^{s} u_r Y_{rj} - \sum_{f=1}^{g} w_f Z_{fj} \le 0 \qquad j = 1,\dots,n \\
&u_r, v_i, w_f \ge \varepsilon \qquad r = 1,\dots,s, \quad i = 1,\dots,m, \quad f = 1,\dots,g
\end{aligned}
$$

On the other hand, sum of the third and fourth constraints equals the second constraint and the second set of constraints is redundant and can be deleted. The optimum answer (u_r^*, v_i^*, w_f^*) obtains and calculates the system performance $\hat{E}_k^s$ and two performance of two stages $\hat{E}_k^{\mathrm{II}}$ and $\hat{E}_k^{\mathrm{I}}$ as follows:

$$
\begin{aligned}
&\hat{E}_k^s = \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{i=1}^{m} v_i X_{ik} \\
&\hat{E}_k^{\mathrm{I}} = \sum_{f=1}^{g} w_f Z_{fk} \Big/ \sum_{i=1}^{m} v_i X_{ik} \qquad (3) \\
&\hat{E}_k^{\mathrm{II}} = \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{f=1}^{g} w_f Z_{fk}
\end{aligned}
$$

It's clear that system performance is calculated by multiplying two stages as $E_k^s = E_k^{\mathrm{I}} \times E_k^{\mathrm{II}}$.

Considering total data (all inputs, intermediate products, and outputs) in all q periods, we can also obtain total and stage performance. As intuitive, they are sum of separate period's performance and knowledge of wrap mechanism on periods is desirable, as it is effective on total function of the period. Therefore the function of each period must include the reason of total performance. The function of each period is calculated by

$$
\sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0, \quad \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0, \quad \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0
$$

Therefore by adding three sets of constraints to model (2) we get:

$$\hat{E}_k^s = \max \sum_{r=1}^{s} u_r Y_{rk}$$

$$s.t.$$

$$\sum_{i=1}^{m} v_i X_{ik} = 1$$

$$u_r, v_i, w_f \geq \varepsilon \qquad r = 1,\dots,s, \qquad i = 1,\dots,m, \qquad f = 1,\dots,g$$

$$\left.\begin{array}{ll} \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 & j = 1,\dots.,\mathrm{n} \\ \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \leq 0 & \mathrm{p} = 1,\dots,\mathrm{q}, \quad j = 1,\dots.,\mathrm{n} \end{array}\right. \qquad (A)$$

$$\left.\begin{array}{ll} \sum_{f=1}^{g} w_f Z_{fj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 & j = 1,\dots.,\mathrm{n} \\ \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \leq 0 & \mathrm{p} = 1,\dots,\mathrm{q}, \quad j = 1,\dots.,\mathrm{n} \end{array}\right. \qquad (\mathrm{B})$$

$$(4)$$

$$\left.\begin{array}{ll} \sum_{r=1}^{s} u_r Y_{rj} - \sum_{f=1}^{g} w_f Z_{fj} \leq 0 & j = 1,\dots,n \\ \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \leq 0 & \mathrm{p} = 1,\dots,\mathrm{q}, \quad j = 1,\dots,n \end{array}\right. \qquad (C)$$

Be careful that the first set of the three set of A, B, and C is redundant. In addition, the first set of constraints in each group is also redundant and can be eliminated. In the best condition, the total performance in q period for system E_k^s and two-stage performance (E_k^{I} and E_k^{II}) and performances of q period ($E_k^{s(p)}$) and their two period performances ($E_k^{\mathrm{I}(p)}$ and $E_k^{\mathrm{II}(p)}$) is as follows according to basic model of (4):

$$E_k^s = \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}$$

$$E_k^{\mathrm{I}} = \sum_{f=1}^{g} w_f^* Z_{fk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}$$

$$E_k^{\mathrm{II}} = \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}$$

$$E_k^{s(p)} = \sum_{r=1}^{s} u_r^* Y_{rk}^{p} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}^{p} \quad (5)$$

$$E_k^{\mathrm{I}(p)} = \sum_{f=1}^{g} w_f^* Z_{fk}^{p} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}^{p}$$

$$E_k^{\mathrm{II}(p)} = \sum_{r=1}^{s} u_r^* Y_{rk}^{p} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}^{p}$$

Similarly, if we consider equation (3), the total performance of q period equals the multiplying of performance of its two stages $E_k^s = E_k^{\mathrm{I}} \times E_k^{\mathrm{II}}$.

In addition, the performance of each period is also obtained by multiplying the performance $E_k^{s(p)} = E_k^{\mathrm{I}(p)} \times E_k^{\mathrm{II}(p)}$ of its two stage.

If we the review figure (2) it is observed that two-phase multi-stage system for the q subsystem has parallel structure that each subsystem acts similar to a stage which consists of two stages and these stages are placed in series. According to Kao model [3] parallel structure has the property that the system efficiency is achieved from the weighted average of the efficiency of subunits. The weights are the ratio of the inputs of a subunit to the total inputs of a single unit. This feature is used to calculate the efficiency of the E_k^s system and efficiency of the E_k^{I} and E_k^{II} stages as the following equation:

$$E_k^s = \sum_{p=1}^{q} \omega^{(p)} E_k^{s(p)}, \qquad \omega^{(\mathrm{p})} = \sum_{i=1}^{m} v_i^* X_{ik}^{p} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}$$

$$E_k^{\mathrm{I}} = \sum_{p=1}^{q} \omega^{(p)} E_k^{\mathrm{I}(p)}, \qquad \omega^{(\mathrm{p})} = \sum_{i=1}^{m} v_i^* X_{ik}^{p} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \quad (6)$$

$$E_k^{\mathrm{II}} = \sum_{p=1}^{q} \omega^{(p)} E_k^{\mathrm{II}(p)}, \qquad \omega^{(\mathrm{p})} = \sum_{f=1}^{g} w_f^* Z_{fk}^{p} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}$$

This equation can easily be proved:

$$\sum_{p=1}^{q}\omega^{(p)}E_k^{s(p)}=\sum_{p=1}^{q}\left(\frac{\sum_{i=1}^{m}v_i^*X_{ik}^p}{\sum_{i=1}^{m}v_i^*X_{ik}}\right)\left(\frac{\sum_{r=1}^{s}u_r^*Y_{rk}^p}{\sum_{i=1}^{m}v_i^*X_{ik}^p}\right)=\sum_{p=1}^{q}\left(\frac{\sum_{r=1}^{s}u_r^*Y_{rk}^p}{\sum_{i=1}^{m}v_i^*X_{ik}}\right)=E_k^s$$

$$\sum_{p=1}^{q}\omega^{(p)}E_k^{\mathrm{I}(p)}=\sum_{p=1}^{q}\left(\frac{\sum_{i=1}^{m}v_i^*X_{ik}^p}{\sum_{i=1}^{m}v_i^*X_{ik}}\right)\left(\frac{\sum_{f=1}^{g}w_f^*Z_{fk}^p}{\sum_{i=1}^{m}v_i^*X_{ik}^p}\right)=\sum_{p=1}^{q}\left(\frac{\sum_{f=1}^{g}w_f^*Z_{fk}^p}{\sum_{i=1}^{m}v_i^*X_{ik}}\right)=E_k^{\mathrm{I}}$$

$$\sum_{p=1}^{q}\omega^{(p)}E_k^{\mathrm{II}(p)}=\sum_{p=1}^{q}\left(\frac{\sum_{f=1}^{g}w_f^*Z_{fk}^p}{\sum_{f=1}^{g}w_f^*Z_{fk}}\right)\left(\frac{\sum_{r=1}^{s}u_r^*Y_{rk}^p}{\sum_{f=1}^{g}w_f^*Z_{fk}^p}\right)=\sum_{p=1}^{q}\left(\frac{\sum_{r=1}^{s}u_r^*Y_{rk}^p}{\sum_{f=1}^{g}w_f^*Z_{fk}}\right)=E_k^{\mathrm{II}}$$

We know the efficiency of the q stage is the weighted average of the efficiency of the single units and Efficiency of each DMU can be evaluated by the desired weight of the efficiency of the stages.

We have provided two methods for dissecting the efficiency of the whole system to the efficiency of the stages that are presented as follows:

$$E_k^s=E_k^{\mathrm{I}}\times E_k^{\mathrm{II}}=\left(\sum_{p=1}^{q}\omega^{(p)}E_k^{\mathrm{I}(p)}\right)\left(\sum_{p=1}^{q}\omega^{(p)}E_k^{\mathrm{II}(p)}\right)$$
$$E_k^s=\sum_{p=1}^{q}\varpi^{(p)}E_k^{\mathrm{s}(p)}=\sum_{p=1}^{q}\varpi^{(p)}\left(E_k^{\mathrm{I}(p)}\times E_k^{\mathrm{II}(p)}\right)\qquad(7)$$

The dissection shows that the two-phase multi-stage system is efficient If and only if all its distinct periods in each step are efficient. The result of the homogenization of the stages is finding the cause of the inefficiencies of the system, thus, a method is intended for improving it.
One of the causes of dissecting the efficiency of the whole system, is the inefficiencies in the system. It also makes it possible to compare the efficiency of one stage of a DMU with the efficiency of the other DMU.

On the other hand, the responses of the system may not be unique. This makes the efficiency obtained from various DMU not to be comparable. Kao and Huang [5] proposed a model that the maximum efficiency of the stages is comparable when the achieved efficiency is minimized for a system. According to this theory, the highest efficiency of the t stage (Which is the value of 1) can be obtained from the following model when the efficiency of the system in the k stage of the E_k^s is calculated from the model 4 as:

$$
\begin{aligned}
& E_k^{s(t)} = \max \sum_{r=1}^{s} u_r Y_{rk}^{(t)} \\
& s.t. \\
& \sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rk} = E_k^{s} \sum_{i=1}^{m} v_i X_{ik} \\
& \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j=1,\dots.,\mathrm{n} \\
& \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j=1,\dots,n \\
& u_r, v_i, w_f \ge \varepsilon \qquad r=1,\dots,s, \qquad i=1,\dots,m, \qquad f=1,\dots,g
\end{aligned}
\tag{8}
$$

In this model, all redundant constraints are removed. To calculate the efficiency of each period, the efficiency of the DMU has been sustained. Moreover if the goal is calculating the efficiency of the h stage, similarly the maximum efficiency for the period can be calculated. This process could be continue to the first priority (first stage).

There are two phases in each stage. Stage efficiency obtained for a single unit is comparable among all DMU that is expressed in some theories. For calculating the efficiency of the first stage not only the total efficiency of DMU (E_k^s) should be sustained but also the efficiency of the stages ($E_k^{s(t)}$) should be considered. According to the model (9), which are listed below:

$$
\begin{aligned}
& E_k^{\mathrm{I}(t)} = \max \sum_{f=1}^{g} w_f Z_{fj}^{(t)} \\
& s.t. \\
& \sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rk} = E_k^{s} \sum_{i=1}^{m} v_i X_{ik} \\
& \sum_{r=1}^{s} u_r Y_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i X_{ik}^{(t)} \\
& \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j=1,\dots.,\mathrm{n} \\
& \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j=1,\dots,n \\
& u_r, v_i, w_f \ge \varepsilon \qquad r=1,\dots,s, \qquad i=1,\dots,m, \qquad f=1,\dots,g
\end{aligned}
\tag{9}
$$

This process also for calculating the efficiency of the second phase is conducted with the same quality. Most managers are usually interested to know the performance changes between the two periods for the matters with several periods. Because the results would be able to specify the path to achieve the best performance. One method that has been widely used for this purpose is Malmquist productivity index (MPI). There are several models of MPI in the articles. Suppose the efficiency of the h and t stages is comparable where h stage is after stage t.

The idea Kao and Hwang [6] is the use of t stage technology for evaluating the efficiency of the h and t stages and quotient of the last stage to the first stage in the MPI model. If this value is greater than, equal to or smaller than one, it respectively shows the progression, efficiency regression and being unchanged between the two stages of t and h.

Because the stage h can also be considered as a basis for evaluating the performance the Results may not be identical with the results of stage t. Farr et al proposed the use of the two stags together but separately, Caves et al. [7] used the geometric mean of the t stage technology for calculating the MPI. MPI can be dissected into two parts, one is the frontier technological change and technological change of efficiency.

MPI between two stages is the quotient of the efficiency of the two stages that is defined for the two stages of t and h as $MPI^{t,h}$ that is the result of the multiplication of $MPI^{t,d}$ and $MPI^{d,h}$, where d is any stage between h and t. For two-phase and multi-stage system several local technology is defined as follows:

$$\{\sum_{r=1}^{s} u_r Y_{rk} - \sum_{i=1}^{m} v_i X_{ik} \le 0,\ \forall\, j; \quad \sum_{f=1}^{g} w_f Z_{fj} - \sum_{i=1}^{m} v_i X_{ij} \le 0,\ \forall j;$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{f=1}^{g} w_f Z_{fj} \le 0,\ \forall j; \quad \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0,\ \forall p, j;$$

$$\sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0,\ \forall p, j; \quad \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0,\ \forall\, \text{p, j}\}$$

By eliminating redundant constraints we have:

$$\{\ \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0,\ \forall p, j; \quad \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0,\ \forall\, \text{p, j}\}$$

Suppose MPI is expressed between the two stages of t and h. efficiency of t stage is computed through DEA model as follows:

$$\begin{aligned}
&\tilde{E}_k^{(t)} = \max \sum_{r=1}^{s} u_r Y_{rk}^{(t)} \\
&s.t. \\
&\sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\
&\sum_{f=1}^{g} w_f Z_{fj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0 \qquad \text{p} = 1,\ldots,\text{q}, \qquad j = 1,\ldots,\text{n} \\
&\sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{f=1}^{g} w_f Z_{fj}^{(p)} \le 0 \qquad \text{p} = 1,\ldots,\text{q}, \qquad j = 1,\ldots,n \\
&u_r, v_i, w_f \ge \varepsilon \qquad r = 1,\ldots,s, \qquad i = 1,\ldots,m, \qquad f = 1,\ldots,g
\end{aligned} \tag{10}$$

The efficiency of h stage $\tilde{E}_k^{(h)}$ can be similarly computed, also local MPI can be defined as $\widetilde{MPI}^{(t,h)} = \tilde{E}_k^{(h)} / \tilde{E}_k^{(t)}$.

Comparison of model (4), with the elimination of redundant constraints, with the model (10) shows that they have the same constraints that require the same technology to be nicely defined for them. The only difference in the initial evaluation of the total efficiency E_k^s and use of equation (5) in calculating the periodical efficiencies of $E_k^{s(t)}$ and $E_k^{s(h)}$. While evaluation of the efficiencies of $E_k^{s(t)}$ and $E_k^{s(h)}$ are direct and separate. However, the computed MPI from the obtained efficiency of model (4) have normal local weights of MPI.

Many researchers believe that the computed efficiency of the different boundaries is not comparable and hence the different proposals are provided for evaluation of the weights [8, 9, 10, 11].

Normal local weights of MPI is calculated of the quotient of $E_k^{s(h)}$ to $E_k^{s(t)}$.

That $\tilde{MPI}_k^{s(t,h)} = \tilde{E}_k^{s(h)} / \tilde{E}_k^{s(t)}$ is the favorable evaluation of technological change of the efficiency that is obtained from the quotient of $E_k^{s(h)}$ and $E_k^{s(t)}$. Notice that the value of the computed periodic efficiency from the model (4) never exceeds the achieved efficiency of the model (10) because the first one in choosing boundary of efficiency evaluation of performance, is mostly Inhibitor (limiting). Changes in performance for the first (and second) periods between t and h stages can be evaluated through the local normal weight of calculated MPI from the quotient ($E_k^{\mathrm{II}(h)}$) $E_k^{\mathrm{I}(h)}$ to ($E_k^{\mathrm{II}(t)}$) $E_k^{\mathrm{I}(t)}$ as $MPI_k^{\mathrm{I}(t,h)} = E_k^{\mathrm{I}(h)} / E_k^{\mathrm{I}(t)}$ and $MPI_k^{\mathrm{II}(t,h)} = E_k^{\mathrm{II}(h)} / E_k^{\mathrm{II}(t)}$.

The relationship between the periods and stages of MPI is shown in the below equation:

$$MPI_k^{s(t,h)} = \frac{E_k^{s(h)}}{E_k^{s(t)}} = \frac{E_k^{\mathrm{I}(h)} \times E_k^{\mathrm{II}(h)}}{E_k^{\mathrm{I}(t)} \times E_k^{\mathrm{II}(t)}} = MPI_k^{\mathrm{I}(t,h)} \times MPI_k^{\mathrm{II}(t,h)} \quad (11)$$

MPI between the two stages obtained from the multiplication result of MPI of each stage. Performance changes of each stage can result from performance changes of each period.

New Model

Two-phase output systems that are in reality have two processes as in the first period the main inputs of the system are used to produce primary products. Then the primary products will be the producers of the main outputs of system. According to the material presented in the previous section, for calculating the efficiency of a two-phase system in several periods of time there are two methods that is calculated from dissecting of the total efficiency according to the (7) equation. It was also stated that one of the mostly used ways to compare the performance of a system in several stages is Malmquist index. Efficiency change that is calculated according to the subsystems by the help of model proposed by Kao and Hwang [6] is an important criteria in the Malmquist index

In this section first we would develop the efficiency change of the Malmquist index according to the proposed method for calculating the efficiency. Then the assumption of non-precision data (interval) is considered and efficiency bound is calculated in the best conditions (Optimistic case) and worst (Pessimistic case) for the system.

Consider a q period of time for a set with n numbers of DMU that the units under the evaluation have two-phase structure. Also x_{ij}^p and z_{fj}^p and y_{rj}^p are the inputs, Intermediate products and the outputs of the DMU_j at the p stage. According to the (7) equation there are two methods of calculating the total efficiency for this type of systems as follows.

First method: In this way, the efficiency of each step is calculated at a specified time period and of the multiplication result of the weighted resultant of each stage, efficiency of the system is calculated. Model proposed by Kao and Hwang [6] to this matter gives multiple favorable answers that to overcome this problem they proposed (8) and (9) models. So the efficiency of the first stage and second stage in t period will be calculated as follows:

$$
\begin{aligned}
& E_k^{\mathrm{I}(t)} = \max \sum_{f=1}^{g} w_f Z_{fj}^{(t)} \\
& s.t. \\
& \sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rk} = E_k^{s} \sum_{i=1}^{m} v_i X_{ik} \\
& \sum_{r=1}^{s} u_r Y_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i X_{ik}^{(t)} \\
& \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\ldots,\mathrm{q} \qquad j=1,\ldots.,\mathrm{n} \\
& \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\ldots,\mathrm{q} \qquad j=1,\ldots,n \\
& u_r, v_i, w_f \ge \varepsilon \qquad r=1,\ldots,s, \qquad i=1,\ldots,m, \qquad f=1,\ldots,g
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
& E_k^{\mathrm{II}(t)} = \max \sum_{r=1}^{s} u_r Y_{rj}^{(t)} \\
& s.t. \\
& \sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rk} = E_k^{s} \sum_{i=1}^{m} v_i X_{ik} \\
& \sum_{r=1}^{s} u_r Y_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i X_{ik}^{(t)} \\
& \sum_{f=1}^{g} w_f Z_{fj}^{p} - \sum_{i=1}^{m} v_i X_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\ldots,\mathrm{q} \qquad j=1,\ldots.,\mathrm{n} \\
& \sum_{r=1}^{s} u_r Y_{rj}^{p} - \sum_{f=1}^{g} w_f Z_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\ldots,\mathrm{q} \qquad j=1,\ldots,n \\
& u_r, v_i, w_f \ge \varepsilon \qquad r=1,\ldots,s, \qquad i=1,\ldots,m, \qquad f=1,\ldots,g
\end{aligned} \tag{13}
$$

The proposed weights at each efficiency stage will be as $\omega_{\mathrm{I}}^{(p)} = \sum_{i=1}^{m} v_i^{*} X_{ik}^{p} \Big/ \sum_{i=1}^{m} v_i^{*} X_{ik}$ and $\omega_{\mathrm{II}}^{(p)} = \sum_{f=1}^{g} w_f^{*} Z_{fk}^{p} \Big/ \sum_{f=1}^{g} w_f^{*} Z_{fk}$.

The second method proposed for calculating the total efficiency of the system is as follows.

Second method: In this method, the efficiency of each step is calculated then the efficiency of the supposed unit is achieved of the multiplication result of the efficiency of the two phases by the time separation, Then to calculate the efficiency of the whole system, the weighted resultants of the efficiency of $E_k^{\mathrm{I}(p)}$ and $E_k^{\mathrm{II}(p)}$ are calculated as the (10) and (11) equations but the recommended weight for the achieved efficiency from the multiplication result of the efficiencies of the two stages is as $\varpi_{\mathrm{I}}^{(p)} = \sum_{i=1}^{m} v_i X_{ik}^{p} \Big/ \sum_{i=1}^{m} v_i X_{ik}$

It is noteworthy that $\sum_{p=1}^{q} \varpi^{(p)} = 1$ that $\varpi^{(p)} \geq 0$.

In the original DEA model to assess the efficiency, data are used accurately but in reality we are not dealing with the exact data. Also note that when the data are inaccurate it is expected that the efficiency value that is achieved of the data to be inaccurate. In this section we will express the presented models for calculating the efficiency of the two-phase systems for the interval data and obtain efficiency bound for the supposed systems.

One of the inaccurate data, is interval input and output that are placed within the upper and lower bounds given by the intervals. Suppose that all data (input - intermediate products - output) are interval. That:

$$\tilde{y}_{rj}^{p} \in \left[\underline{y}_{rj}^{p}, \bar{y}_{rj}^{p}\right], \qquad \tilde{x}_{ij}^{p} \in \left[\underline{x}_{ij}^{p}, \bar{x}_{ij}^{p}\right], \qquad \tilde{z}_{fj}^{p} \in \left[\underline{z}_{fj}^{p}, \bar{z}_{fj}^{p}\right]$$

So the efficiency of the whole system according to the above data will be as the (14) equation.

$$\begin{aligned}
& E_k^{s} = \max \sum_{r=1}^{s} u_r \tilde{Y}_{rk} \\
& s.t. \\
& \sum_{i=1}^{m} v_i \tilde{X}_{ik} = 1 \\
& \sum_{f=1}^{g} w_f \tilde{Z}_{fj}^{(p)} - \sum_{i=1}^{m} v_i \tilde{X}_{ij}^{(p)} \leq 0 \qquad \mathrm{p} = 1,\ldots,\mathrm{q}, \ \ j = 1,\ldots.,\mathrm{n} \\
& \sum_{r=1}^{s} u_r \tilde{Y}_{rj}^{(p)} - \sum_{f=1}^{g} w_f \tilde{Z}_{fj}^{(p)} \leq 0 \qquad \mathrm{p} = 1,\ldots,\mathrm{q}, \ \ j = 1,\ldots,n \\
& u_r, v_i, w_f \geq \varepsilon \qquad r - = 1,\ldots,s, \ \ i = 1,\ldots,m, \ \ f = 1,\ldots,g
\end{aligned} \tag{14}$$

Note that in equation (14) the redundant constraints have been removed. The most optimistic state for a system is when desired DMU has the minimum input and the maximum output in comparison to other DMUs. In these circumstances, we can achieve the upper bound of efficiency from model (15).

$$
\begin{aligned}
&\bar{E}_k^s = \max \sum_{r=1}^{s} u_r \bar{Y}_{rk} \\
&s.t. \\
&\sum_{i=1}^{m} v_i \underline{X}_{ik} = 1 \\
&\sum_{f=1}^{g} w_f \bar{Z}_{fj}^{(p)} - \sum_{i=1}^{m} v_i \underline{X}_{ij}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad \text{k} \ne j = 1,\dots,\text{n} \\
&\sum_{f=1}^{g} w_f \underline{Z}_{fj}^{(p)} - \sum_{i=1}^{m} v_i \bar{X}_{ij}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad j = 1,\dots,\text{n} \\
&\sum_{r=1}^{s} u_r \bar{Y}_{rj}^{(p)} - \sum_{f=1}^{g} w_f \underline{Z}_{fj}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad \text{k} \ne j = 1,\dots,n \\
&\sum_{r=1}^{s} u_r \underline{Y}_{rj}^{(p)} - \sum_{f=1}^{g} w_f \bar{Z}_{fj}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad j = 1,\dots,n \\
&u_r, v_i, w_f \ge \varepsilon \qquad r = 1,\dots,s, \quad i = 1,\dots,m, \quad f = 1,\dots,g
\end{aligned}
\tag{15}
$$

Similarly, the lower bound of efficiency or the pessimistic mode of the system occurs when the desired DMU has most inputs to produce the less output than other DMUs. The lower bound of the whole system efficiency is calculated as follows through equation (16).

$$
\begin{aligned}
&\underline{E}_k^s = \max \sum_{r=1}^{s} u_r \underline{Y}_{rk} \\
&s.t. \\
&\sum_{i=1}^{m} v_i \bar{X}_{ik} = 1 \\
&\sum_{f=1}^{g} w_f \underline{Z}_{fj}^{(p)} - \sum_{i=1}^{m} v_i \bar{X}_{ij}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad \text{k} \ne j = 1,\dots,\text{n} \\
&\sum_{f=1}^{g} w_f \bar{Z}_{fj}^{(p)} - \sum_{i=1}^{m} v_i \underline{X}_{ij}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad j = 1,\dots,\text{n} \\
&\sum_{r=1}^{s} u_r \underline{Y}_{rj}^{(p)} - \sum_{f=1}^{g} w_f \bar{Z}_{fj}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad \text{k} \ne j = 1,\dots,n \\
&\sum_{r=1}^{s} u_r \bar{Y}_{rj}^{(p)} - \sum_{f=1}^{g} w_f \underline{Z}_{fj}^{(p)} \le 0 \qquad \text{p} = 1,\dots,\text{q}, \quad j = 1,\dots,n \\
&u_r, v_i, w_f \ge \varepsilon \qquad r = 1,\dots,s, \quad i = 1,\dots,m, \quad f = 1,\dots,g
\end{aligned}
\tag{16}
$$

According to what noted in models (15) and (16), the efficiency of the unit under evaluation is evaluated to be smaller equal in the best conditions that is $E_k^s \le \bar{E}_k^s$ and greater equal to the calculated efficiency in worst conditions $\underline{E}_k^s \le E_k^s$. So we can say that $E_k^s \in [\underline{E}_k^s, \bar{E}_k^s]$. According to what stated above, the proof of the theorem above is clear.

Theorem 1: If $\bar{E}_k^s$ is the upper bound and $\underline{E}_k^s$ is the lower bound of the total efficiency range for the unit under evaluation, then $\underline{E}_k^s \le E_k^s \le \bar{E}_k^s$.

Result 2: If $\underline{E}_k^s = 1$, then $E_k^s = \bar{E}_k^s$ and DMU is the whole efficiency.

Also the efficiency bound of the first second stage will be as follows by considering the interval data. The upper bound of the first stage is obtained through relationship (17).

$$
\begin{aligned}
&\bar{E}_k^{1(t)} = \max \sum_{f=1}^{g} w_f \bar{Z}_{fj}^{(t)} \\
&s.t. \sum_{i=1}^{m} v_i \underline{X}_{ik}^{(t)} = 1 \\
&\sum_{r=1}^{s} u_r \bar{Y}_{rk} = E_k^s \sum_{i=1}^{m} v_i \underline{X}_{ik} \\
&\sum_{r=1}^{s} u_r \bar{Y}_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i \underline{X}_{ik}^{(t)} \\
&\sum_{f=1}^{g} w_f \bar{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \underline{X}_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots,\mathrm{n} \\
&\sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \bar{X}_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j = 1,\dots,\mathrm{n} \\
&\sum_{r=1}^{s} u_r \bar{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots,n \\
&\sum_{r=1}^{s} u_r \underline{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \bar{Z}_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j = 1,\dots,n \\
&u_r, v_i, w_f \ge \varepsilon \qquad r=1,\dots,s, \qquad i=1,\dots,m, \qquad f=1,\dots,g
\end{aligned} \tag{17}
$$

Lower bound of the first stage efficiency where DMU is at the worst conditions (the maximum input to produce the minimum intermediate products) is calculated from the following model.

$$
\begin{aligned}
&\underline{E}_k^{1(t)} = \max \sum_{f=1}^{g} w_f \underline{Z}_{fk}^{(t)} \\
&s.t. \sum_{i=1}^{m} v_i \bar{X}_{ik}^{(t)} = 1 \\
&\sum_{r=1}^{s} u_r \underline{Y}_{rk} = E_k^s \sum_{i=1}^{m} v_i \bar{X}_{ik} \\
&\sum_{r=1}^{s} u_r \underline{Y}_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i \bar{X}_{ik}^{(t)} \\
&\sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \bar{X}_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots,\mathrm{n} \\
&\sum_{f=1}^{g} w_f \bar{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \underline{X}_{ij}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j = 1,\dots,\mathrm{n} \\
&\sum_{r=1}^{s} u_r \underline{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \bar{Z}_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots,n \\
&\sum_{r=1}^{s} u_r \bar{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} \le 0 \qquad \mathrm{p}=1,\dots,\mathrm{q} \qquad j = 1,\dots,n \\
&u_r, v_i, w_f \ge \varepsilon \qquad r=1,\dots,s, \qquad i=1,\dots,m, \qquad f=1,\dots,g
\end{aligned} \tag{18}
$$

Similarly, for the second stage we can obtain the upper bound efficiency through equation (19) and achieve the lower bound efficiency through equation (20).

$$\overline{E}_k^{\mathrm{II}(t)} = \max \sum_{r=1}^{s} u_r \overline{Y}_{rj}^{(t)}$$

$$s.t. \sum_{i=1}^{m} v_i \underline{X}_{ik}^{(t)} = 1$$

$$\sum_{r=1}^{s} u_r \overline{Y}_{rk} = E_k^s \sum_{i=1}^{m} v_i \underline{X}_{ik}$$

$$\sum_{r=1}^{s} u_r \overline{Y}_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i \underline{X}_{ik}^{(t)}$$

$$\sum_{f=1}^{g} w_f \overline{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \underline{X}_{ij}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots.,\mathrm{n}$$

$$\sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \overline{X}_{ij}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad j = 1,\dots.,\mathrm{n}$$

$$\sum_{r=1}^{s} u_r \overline{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots,n$$

$$\sum_{r=1}^{s} u_r \underline{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \overline{Z}_{fj}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad j = 1,\dots,n$$

$$u_r, v_i, w_f \ge \varepsilon \qquad r = 1,\dots,s, \qquad i = 1,\dots,m, \qquad f = 1,\dots,g \quad (19)$$

$$\underline{E}_k^{\mathrm{II}(t)} = \max \sum_{r=1}^{s} u_r \underline{Y}_{rj}^{(t)}$$

$$s.t. \sum_{i=1}^{m} v_i \overline{X}_{ik}^{(t)} = 1$$

$$\sum_{r=1}^{s} u_r \underline{Y}_{rk} = E_k^s \sum_{i=1}^{m} v_i \overline{X}_{ik}$$

$$\sum_{r=1}^{s} u_r \underline{Y}_{rk}^{(t)} = E_k^{s(t)} \sum_{i=1}^{m} v_i \overline{X}_{ik}^{(t)}$$

$$\sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \overline{X}_{ij}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots.,\mathrm{n}$$

$$\sum_{f=1}^{g} w_f \overline{Z}_{fj}^{p} - \sum_{i=1}^{m} v_i \underline{X}_{ij}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad j = 1,\dots.,\mathrm{n}$$

$$\sum_{r=1}^{s} u_r \underline{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \overline{Z}_{fj}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad \mathrm{k} \ne j = 1,\dots,n$$

$$\sum_{r=1}^{s} u_r \overline{Y}_{rj}^{p} - \sum_{f=1}^{g} w_f \underline{Z}_{fj}^{p} \le 0 \qquad \mathrm{p} = 1,\dots,\mathrm{q} \qquad j = 1,\dots,n$$

$$u_r, v_i, w_f \ge \varepsilon \qquad r - = 1,\dots,s, \qquad i = 1,\dots,m, \qquad f = 1,\dots,g \quad (20)$$

One of the methods that is widely used to compare the performance changes between the two periods, is MPI. This index consists of two parts.

Part one is TEC which is the technological change of efficiency and part 2 is Fs, which indicates the measure of boundary technology between the two periods.

Now suppose that efficiencies of period's t and h are comparable. Also assume that period h is after period t. In this way, for calculating the technological changes of efficiency of MPI, we use the ratio of the whole system efficiency in period h to the whole system efficiency in period t.

$$TEC_k^{s(t,h)}=\frac{E_k^{s(h)}}{E_k^{s(t)}}=\frac{E_k^{\mathrm{I}(h)}\times E_k^{\mathrm{II}(h)}}{E_k^{\mathrm{I}(t)}\times E_k^{\mathrm{II}(t)}}=TEC_k^{\mathrm{I}(t,h)}\times TEC_k^{\mathrm{II}(t,h)}$$

Technological changes of the boundary are assumed as fixed in this study that is we have assumed $Fs=1$. In this case, the technological changes of the boundary in period t will be the same in period h. Thus we can calculate the MPI for two phase systems at several time intervals, only in terms of technological efficiency changes.

$$MPI_k^{s(t,h)}=TEC_k^{s(t,h)}\times Fs_k^{s(t,h)}=TEC_k^{s(t,h)}\times 1=TEC_k^{s(t,h)}$$

$$TEC_k^{s(t,h)}=TEC_k^{\mathrm{I}(t,h)}\times TEC_k^{\mathrm{II}(t,h)}=MPI_k^{\mathrm{I}(t,h)}\times MPI_k^{\mathrm{II}(t,h)}$$

$$\Rightarrow MPI_k^{s(t,h)}=MPI_k^{\mathrm{I}(t,h)}\times MPI_k^{\mathrm{II}(t,h)}$$

As was mentioned, the information is not always provided accurately to the manager in order to evaluate the organizational performance under his own management and evaluate efficiency changes according to the detailed data. Accordingly, in this section we will calculate the upper band for the MPI. If the system is in the best conditions, that is could produce the maximum output from the minimal input, the technological changes will be calculated according to models (17) and (18) as below:

$$T\bar{E}C_k^{s(t,h)}=\frac{\bar{E}_k^{s(h)}}{\bar{E}_k^{s(t)}}=\frac{\bar{E}_k^{\mathrm{I}(h)}\times \bar{E}_k^{\mathrm{II}(h)}}{\bar{E}_k^{\mathrm{I}(t)}\times \bar{E}_k^{\mathrm{II}(t)}}=T\bar{E}C_k^{\mathrm{I}(t,h)}\times T\bar{E}C_k^{\mathrm{II}(t,h)}$$

Therefore, the upper bound of Malmquist index is calculated according to the following equation.

$$M\bar{P}I_k^{s(t,h)}=T\bar{E}C_k^{s(t,h)}=T\bar{E}C_k^{\mathrm{I}(t,h)}\times T\bar{E}C_k^{\mathrm{II}(t,h)}=M\bar{P}I_k^{\mathrm{I}(t,h)}\times M\bar{P}I_k^{\mathrm{II}(t,h)}$$

Similarly, we can calculate the lower bound of the efficiency in the worst conditions for efficiency changes and the productivity index.

CONCLUSION

The technique of data envelopment analysis is widely used to evaluate the efficiency of a set of decision making units in a period of time. Traditionally, data collected from combined periods, either overall or average are used to measure efficiency. This study has examined the effects of the efficiency of distinct periods on the overall performance of a DMU in a two-step process in a certain time period. The overall model developed by Kao and Hwang [5] was extended to incorporate changes in individual courses. This model not only can evaluate a variety of efficiencies, but also calculates MPI according to local weights.

When the performances of distinct periods were considered, it was stated that a DMU is efficient if and only if it is efficient in each period and the measured efficiency number from a multi-period model is smaller than the total.

Similarly, changes in the efficiency of a DMU in separate courses are extended for inaccurate data and proposed models to calculate efficiency in the best and worst conditions which are considered as the upper and lower bounds of efficiency. Both DEA and MPI raised in this research have been developed under the assumption of constant returns to scale (CRS). Future research can be more useful if the models be developed under the assumption of variable returns to scale (VRS). Kao and Hwang [6] developed their CRS model in 2008, and this study is conducted based on the VRS model. However, they are limited to a two-step structure and cannot be used for multi-step mode. In this regard, many details must be clear and therefore there is a topic for future research. The multi-periodic system studied in this paper has two parts. The results are also applicable to systems with more than two steps. There are two basic structures of series and parallel for the main network systems. Another controversial issue, is to review efficiencies and MPI for major network systems in several periods of time which is recommended for the researchers to conduct a research in this regard.

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