Scholars Journal of Physics, Mathematics and Statistics **ISSN 2393-8056 (Print)**
Sch. J. Phys. Math. Stat. 2015; 2(3A):283-288 **ISSN 2393-8064 (Online)**

(An International Publisher for Academic and Scientific Resources)

Simulation for Performance Analysis of Some Capability Indices on Net-Volume Content of 35cl Coca-Cola Soft Drinks

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Abstract: This paper examined the statistical performance of some capability indices such as Cp and Cpk using simulation. Statistical software R version 3.1.3 was used in generating data sets on net-volume contents of 35cl coca-cola soft drink as well as the analysis. The production specification standard is 350±2.5%∗T (where T is the target value of the production). This information was collected from Quality Assurance Department of Nigerian Bottling Company. However, the natural variations are part and parcel or considered inherent to any manufacturing industries. A Nigerian Bottling Company producer of soft drinks is not free of natural variation. The values of capability indices Cp (1.48) and Cpk (1.47) were computed and reported in the capability plot histogram together with confidence interval at the indicated level of significant. The quality condition of the production based on the analysis presented and shows satisfactory performance of the production. Operating characteristics curves were established and observed that the process have better performance for not detecting shift with the associated probability of type II error.
Keywords: Simulation, Capability indices, Net-volume content, OC curves

INTRODUCTION

Capability indices are rapidly becoming a standard tool for quality assessment, especially at management level around the world of manufacturing industry and other service organizations. The term provides a numerical measure on whether a process conforms to the defined manufacturing capability prerequisite. These have been successfully applied to many companies to compete with and to lead high-profit market by evaluating the quality and productivity performance. For any manufacturing industries to succeed, therefore quality of its products, good service deliveries are some of the prime determine factors to be considered. However, quality is multifaceted. Thus, the quality of a product is judged by a number of factors such as its performance, reliability, durability, serviceability, aesthetics, features, conformance to standards and specifications, and sometimes even the reputation of the manufacturer! [8].

The science of process capability analysis was first introduced by Dr Joseph M. Juran, in early 1970 with the introduction of “Capability ratio” he began as a comparison of the process output distribution with the product tolerances [12]. This comparison is made by forming the ratio of the width between the process specification limits to the width of the of the natural tolerance limits which is measured by 6-process standard deviation units [16]. Kane [11] described six areas of application for capability indices: the prevention of the production of nonconforming products, the continuous measure of improvement, communication, prioritization, the identification of directions for improvement, and the auditing of the quality systems. Capability indices have extensively studied by several researchers considering wide range of applications over a great variety of situations in manufacturing industries and their contributions follows with various degrees of completeness. Hoskins *et.al*, [9] examined the capability indices considering manufacturing of semiconductor product. Application to the head of gimbals [18], Jet-turbine engine components parts [10]. However [3] assess process capability indices based on the lower confidence bound of Cpk for asymmetric tolerances. A Matlab computer program using a binary search method was used and observed that lower confidence bound estimates the minimum process capability, conveying critical information regarding product quality, which is essential to quality assurance. Then, [5] discussed theory and practice on process capability indices for quality assurance. The result illustrates some advantages of using the index $Cpmk$ over the indices Cpk and Cpm in measuring process capability, since $Cpmk$ always provides a better protection for the customers. Thereafter, [4] studied variables sampling inspection scheme for resubmitted lots based on the process capability index Cpk. The results show that developed resubmitted sampling plan has a better OC curve than variables single sampling plan at good quality levels and also ensures protection against consumer point of view at poor quality level.

The process capability indices including, Cp, and Cpk provide a quick indication of how a process has conformed to its specifications, which are preset by manufacturers and customers. Therefore, [17] indicated the index of capability is used for monitoring the accuracy of the manufacturing process. This paper is aim at studying the statistical performance of some capability index such as Cp and Cpk using simulated data on net-volume content of 35cl coca-cola soft drink products produced by Nigerian Bottling Company.

Manufacturing industries are not free of natural variation associated with their production processes. In particular, Nigeria Bottling Company, producers of soft drinks is not free of the natural variation. Process capability analysis would apply to this company using the said indices and examine the amount of variability present in the production, finally we recommend on the way to improve the quality of their products.

Statistical Performance of Capability Indices

The first process capability index appeared in the literature is Cp index sometimes called precision index [11]. This is defined as the ratio of specification width (USL-LSL) over the process spread (6σ). The specification width represents customer and / or product requirements.

Cp Indicates how well the process fit within the two specification limits and it is calculated by using (1). Cp measures the spread of the specification relative to the 6-sigma spread in the process[12].

$$Cp = \frac{USL - LSL}{6\sigma} \quad \ldots (1)$$

where σ is the standard deviation of the process, USL and LSL are upper and lower specification limits, respectively.

The six quality conditions corresponding to Cp values are summarized in Table 1 [20].

Table-1: Quality Conditions and Cp Values

Quality Conditions	Cp Values		
Super Excellent	Cp≥2.00		
Excellent	1.67≤Cp<2.00		
Satisfactory	1.33≤Cp<1.67		
Capable	1.00≤Cp<1.33		
Inadequate	0.67≤Cp<1.00		

Source: Tsai and Chen, [20]

Let $\mu = E(X), and\ S = [LSL, USL]$. The Cp process capability index is defined as $Cp = \frac{USL-LSL}{6\sigma}$

In the usual case where σ, and hence Cp, is unknown the index must be estimate based on random sample $X_1, X_2, \ldots, X_n$ from F. The most common estimate of Cp is given by $\hat{C}p = \frac{USL-LSL}{6\hat{\sigma}}$ which simply replaces the unknown parameter σ with its unbiased estimate,

$$\hat{\sigma} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\mu})^2 \quad \ldots (2)$$

$$where\ \ \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \ldots (3)$$

when F is a normal distribution, $\hat{C}p$ has a distribution that is related to the inverse of a χ^2 distribution. Hence, a 100(1-α) % confidence interval for Cp based on $\hat{C}p$ is given by Chou *et al.*[7] and Marcucci and Beazy [15] as

$$\left[\hat{C}p\left(\frac{\chi^2_{\frac{\alpha}{2};n-1}}{n-1}\right)^{1/2}, \hat{C}p\left(\frac{\chi^2_{1-\frac{\alpha}{2};n-1}}{n-1}\right)^{1/2}\right] \quad \ldots (4)$$

$where\chi^2_{\gamma,f}\ is\ the\ \gamma^{th}\ percentile\ of\ a\ \chi^2$ $distribution$ with f degrees of freedom. When σ is estimated using within subgroup variation, such as from standard control charting procedures, Bissel [1] provides an adjustment to the degrees of freedom used for the confidence interval in Eq. (4). Chan *et al.*, [7] show that $\hat{C}p$ has an asymptotic normal distribution $as\ n \to \infty$, Bissel [1] uses such a result suggests that when the sample size is large, an approximate $100(1-\alpha)\%$ confidence interval for Cp is given by

$$\left[\hat{C}p\left(1-\frac{Z_{1-\frac{\alpha}{2}}}{\sqrt{2(n-1)}}\right), \hat{C}p\left(1+\frac{Z_{1-\frac{\alpha}{2}}}{\sqrt{2(n-1)}}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where Z_γ is the γ^{th} percentile of a standard normal distribution, Li *et al* [14] provide tables of approximate lower confidence limits for Cp when the sample range is used to estimate σ. Sample size determination formulas for confidence intervals for Cp are given by Franklin [21]. The process capability ratio Cp does not take into account where the process mean is located relative to specifications [16]. Cp Focus on the dispersion of the process and this gives no indication of the actual process performance. Kane [11] introduced the index Cpk to overcome this problem. The index Cpk is used to provide an indication of the variability associated with a process or takes both magnitude of process variance and the process departure from the midpoint m into consideration.

In the case where μ and σ are unknown Cpk must be estimated based on a sample $X_1, X_2, \ldots, X_n$ from F. A common estimator for Cpk is given by

$$\hat{C}pk = min\left\{\frac{USL-\hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu}-LSL}{3\hat{\sigma}}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where $\hat{\mu}$ and $\hat{\sigma}$ are defines in Eq. (2) and (3), respectively. The corresponding estimates of $Cpu\ and\ Cpl$ defined as follows: $\hat{C}pu = \frac{USL-\hat{\mu}}{3\hat{\sigma}}$ and $\hat{C}pl = \frac{\hat{\mu}-LSL}{3\hat{\sigma}}$ respectively. When F is a normal distribution with mean μ and variance σ^2, the expectation of $\hat{C}pk$ is given by Chou and Owen [6] as

$$E(\hat{C}pk) = C_{11}\left\{Cp - \frac{2}{3\sqrt{n}}\,\phi[3\sqrt{n}(Cp-Cpk)] - (Cp-Cpk)\left[2\phi\left(3\sqrt{n}(Cp-Cpk)\right)-1\right]\right\} \quad \ldots\ldots\ldots (7)$$

where $\phi(x)$ is the probability densities function for standard normal distribution. Zhang *et al*., [19] obtained an approximate expression for the variance of $\hat{C}pk$ and proposed the approximate 100(1-α) % lower confidence bound which is based on the normal approximation. Bissel [1] suggests that when the sample size is large the interval

$$\left\{\hat{C}pk\left[1 - Z_{1-\frac{\alpha}{2}}\left(\frac{1}{9n\hat{C}^2pk}+\frac{1}{2(n-1)}\right)^{1/2}\right], \hat{C}pk\left[1 + Z_{1-\frac{\alpha}{2}}\left(\frac{1}{9n\hat{C}^2pk}+\frac{1}{2(n-1)}\right)^{1/2}\right]\right\} \quad \ldots\ldots\ldots (8)$$

If the process has a target value, Kane [11] suggests replacing μ in Eq. (6) with the target value T to assess the capability of the process. When $\mu = m$, $Cpk = Cp$ and the indices are equivalent. However, if $\mu \neq m$, then $Cpk \leq Cp$.

METHODOLOGY

This paper aimed at studying the statistical performance of some capability indices from simulated data. Quality control or assurance becomes very necessary in keeping track of production process in manufacturing industries. For this reason, Nigerian Bottling Company, producers of soft drink was considered for this study. The specific product that would be considered in this paper is 35cl coca-cola soft drink, and the parameter under consideration is the net-volume content or simply the content of the bottle. Upon visiting the factory, we were able to met and discussed with the quality assurance manager in the Quality assurance department of the company. From the discussion, we learned the production specification requirements of their products as follows. The tolerance band permitted is ± 2.5%∗T (where T is the target value of production). In this paper, 35cl coca-cola soft drink products is of paramount important for assessing the performance of some capability indices and also help to provides the direction of improvement. The production specification target of 35cl is under consideration for this study, and this is converted to 350ml for convenience.

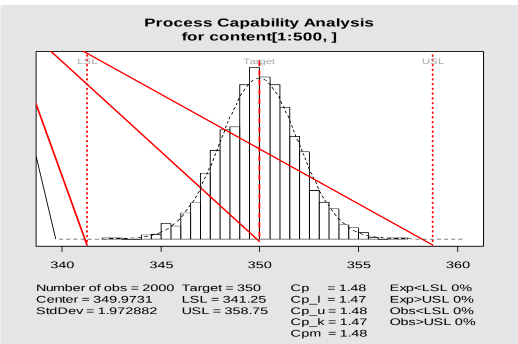

Fig. 1: Process Capability Analysis Histogram for Net-Volume Content of 35cl Coca-Cola Soft Drinks Using Simulation

The graph shown in Fig. 1, is a histogram of data values, with vertical dotted lines for the upper and lower specification limits. The target is a vertical dashed line and its value provided with the reported confidence intervals computed at the level specified.

The Cp and Cpk are based on the short term variability of process over a given length of time. Depicted from Fig. 1, the Cp= 1.48, Cp_l= 1.47, Cp_u =1.48 and Cp_k = 1.47 respectively. Refer to the table 1 shown the Quality conditions and the corresponding Cp values. However, the results indicates that Cp and Cp_k measures are both greater than one, then the processes have the potential to meet specifications as long as the mean is probability centered. Fundamentally, the process is in control and the distribution is well within the specification limit, hence the difference between the upper specification limit (USL) and the lower specification limit (LSL) is greater than 6σ tolerance band in case of Cp. Similarly, Cpk is a process capability index that assesses how close the process mean is from the specification limit. The difference between USL and LSL from mean is greater than 3σ, show the process is in control and the distribution is well within the specification limits. More so, the Cpk is greater than 1, and then the process mean is sufficiently far from the specification limit. Finally, we re-investigate the performance by increasing the sample using simulation and observed that the more increase in the sample the better the capability indices.

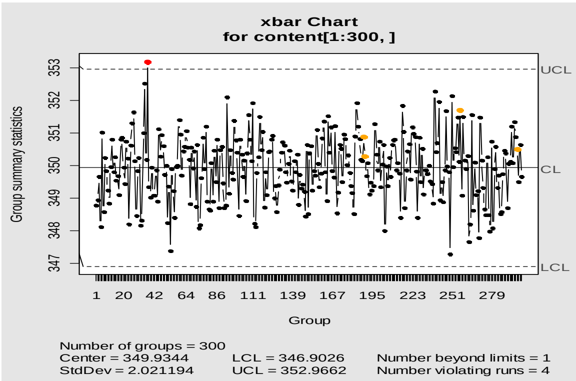

Fig. 2: Show the $\bar{X}$-Chart for Net-Volume Content of 35cl Coca-Cola Soft Drinks Using Simulation

Fig. 2 and Fig. 3, show the $\bar{X}$-Chart for net-volume content of 35cl coca-cola soft drink using simulation. The main goal of control chart is to monitor a process and see whether is in control or not. A process is declared to be in control, if all points charted lie randomly within the control limits. The $\bar{X}$-Charts are prerequisite of process capability analysis, when the process is not out-of-control. We observed that $\bar{X}$-Charts in both cases are statistically in-control even though very few points show run violation and only one observation having to be beyond the upper control limits. However, the process is still in-control with insignificant number of default to the production.

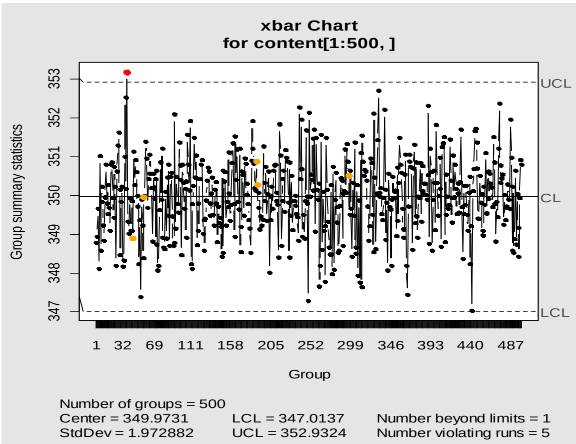

Fig. 3: Show the $\bar{X}$-Chart for Net-Volume Content of 35cl Coca-Cola Soft Drinks Using Simulation

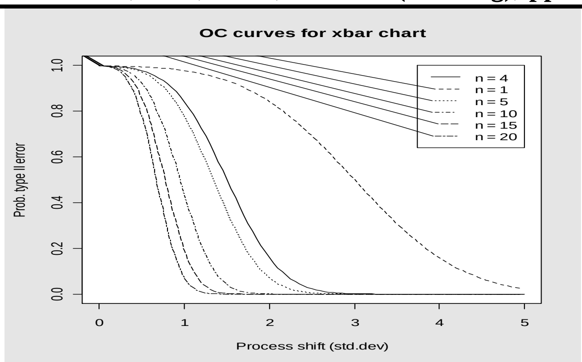

Fig. 4: Show the Operating Characteristics Curve (OC Curves) for Net-Volume Content of 35cl Coca-Cola Soft Drinks Using Simulation

Fig. 4, provide information about the probability of not detecting a shift in the process using operating characteristics curves (OC Curves). This is referring to as the type II error, the probability of erroneously accepting a process as being in control. We observed from OC curves, the process have better performance in detecting shift from $\bar{X}$-Chart.

CONCLUSION

Data handling software version R.3.1.3 was used for simulation of net-volume content of 35cl coca-cola soft drinks. However, the same software was used throughout the analysis in this paper and visualized the performance of Cp and Cpk indices respectively. These indices gives effective information about the process summary in a succinctly manner. At the same time, Cp, Cp_l, Cp_u and Cpk form a group complementary measures that comprises a unit less systems. Perhaps greater value of these indices indicates how well the process is capable to prevent production of non-conforming products, and they provide a strategy for monitoring and continuous improvement scheme.

REFERENCES

1. Bissel AF; How reliable is your Capability Index? Applied Statistics, 1990; 39:331-340
2. Chan LK, Xiong Z, Zhamg D; On the Asymptotic Distributions of some Process Capability Indices, Comm. Statist Theory Methods, 1990; 19:11-18
3. Chang YC, Chien-Wei Wu; Assessing Process Capability Based on the lower confidence bound of Cpk for asymmetric tolerance. European Journal of Operational Research, 2008; 190:205-227.
4. Chien W, Aslam A, Chi-Hyuck Jun; Variables Sampling Inspection Scheme for resubmitted lots based on the process capability index Cpk, European Journal of Operational Research, 2012; 217:560-566.
5. Chien-Wei, Pearn WL, Samuel Kotz; An overview of theory and practice on process capability indices for quality assurance. International Journal of production Economics, 2009; 11:338-359
6. Chou YM, Owen DB; On the Distributions of the Estimated Process Capability Indices, Comm Statis Theory Methods, 1989; 18:4549-4560
7. Chou YM, Owen DB, Borrego SA; Lower Confidence Limits on Process Capability Indices, Journal of Quality Technology, 1990;22:223-229.
8. Garvin DA; Competing in the Eight Dimensions of Quality, Harvard Business Review, 1987.
9. Hoskins J, Stuart B, Taylor J; A Motorola commitment: a six sigma mandate. The Motorola Guide to Statistical Process for Continuous Improvement, Towards Six-Sigma Quality, Dearborn, MI, 1988.
10. Hubele NF, Montgomery DC, Chin WH; An application of statistical process control in jet-turbine engine component manufacturing, Journal of Quality Engineering, 1991;4(2):197-210
11. Kane VE; Process Capability Indices. Journal of Quality Technology, 1986;18:41-52
12. Koltz S, Johnson NL; Process Capability Indices-A Review. 1992-2000. Journal of Quality Technology, 2002; 34(1):2-39.
13. Kotz S, Lovelace C; Process Capability Indices in Theory and Practice, Arnold, London, 1998.
14. Li Hi, Owen DB, Borrego SA; Lower Confidence Limits on Process Capability Indices Based on the Range. Comm. Statist. Simulation Computing, 1990;19:1-24
15. Marcucci MO, Beazey CC; Capability Indices: Process Performance Measures. In ASQC Quality Congress Transaction- Dallas, 1988; 516-523
16. Montgomery DC; Introduction to Statistical Quality Control, New York, John Wiley and Sons, 2005.
17. Pearn WL, Shu MH, Hsu BM; Monitoring Manufacturing Quality for multiple Li-BPIC processes based on capability index Cpmk, International Journal of Production Research, 2005;43(12):2493-2512
18. Rado IG; Enhanced product development by using capability indexes, Quality Progress, 1989;22(4):38-41.

19. Zhang NF, Stenback GA, Wardrop DM; Interval Estimation of Process Capability Index Cpk. Comm. Statist Theory Methods, 1990; 19:4455-4470.
20. TsaiYI, Chen CL; Atmospheric aerosol composition and source apportionments to aerosol in southern Taiwan. Atmospheric Environment, 2006; 40:4751–4763.
21. Franklin LA, Wasserman G; Bootstrap lower confidence limits for capability indices. Journal of Quality Technology, 1992;22(30):223-229.