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Classification of All Single Traveling Wave Solutions to the BBMP Equation with dual Power-law

LI Wenhe
College of mathematics and statistics, Northeast Petroleum University, Daqing 163318, China

***Corresponding Author:**
LI Wenhe
Email: xiongdi163@163.com

Abstract: Under the travelling wave transformation, BBMP equation with power-law can be changed into ordinary differential equation. In this article, by using polynomial completely discriminant classification system ,we can give classification of all single traveling wave solutions to the equation. The solutions can't be obtained by any indirect method.
Keywords: BBMP equation with dual power-law, traveling wave transform, complete discrimination system for polynomial, classification

INTRODUCTION

Many nonlinear partial differential equation, such as coupled Klein- Gordon- Schrodinger equation, higher-order dispersive cubic-qintic nonlinear Schrodinger equation, NNV equation, Camassa-Holm equation with dispersion, Compound KdV equation with any order nonlinear terms, Sine-Gordan equation, KdV equation, Getmanou equation and so on,can be turned into ordinary differential equation and further into elementary integral form. Then we discuss the situation of the polynomial roots and find out the solutions to the equation by using polynomial completely discriminant classification system. In this article, we use BBMP equation with power-law as an example to explain the application of the method for the sake of simplicity. These solutions can't be received by any indirect method [1-9].

Transforming BBMP equation with dual Power-law into the form of elementary integrals

BBMP equation with power-law read as

$$u_t + au_x + (bu^n + cu^{2n})u_x + mu_{xxt} = 0 \tag{1}$$

Taking the traveling wave transformation $u = u(\xi)$, $\xi = kx + \omega t$, we obtain $u_t = \omega u'$ $u_x = ku'$ $u_{xxt} = k^2\omega u'''$ .We substitute u_t , u_x and u_{xxt} into Eq.(1),we have

$$(\omega + ak)u' + (bu^n + cu^{2n})ku' + mk^2\omega u''' = 0 \tag{2}$$

Integrating Eq.(2) with respect to ξ once, we obtain

$$(\omega + ak)u + mk^2\omega u'' + \frac{bk}{n+1}u^{n+1} + \frac{kc}{2n+1}u^{2n+1} + c_1 = 0 \tag{3}$$

Multipling u' at both sides of the equation, we get

$$(\omega + ak)uu' + mk^2\omega u''u' + \frac{bk}{n+1}u^{n+1}u' + \frac{kc}{2n+1}u^{2n+1}u' + c_1u' = 0 \tag{4}$$

Integrating Eq.(4) with respect to ξ once, we yield

$$\frac{\omega + ak}{2}u^2 + \frac{mk^2\omega}{2}(u')^2 + \frac{bk}{(n+1)(n+2)}u^{n+2} + \frac{kc}{(2n+1)(2n+2)}u^{2n+2} + c_1u + c_2 = 0 \tag{5}$$

When $n = 1$, Simplifying Eq. (5), we get

$$\frac{\omega + ak}{2}u^2 + \frac{mk^2\omega}{2}(u')^2 + \frac{bk}{6}u^3 + \frac{kc}{12}u^4 + c_1u + c_2 = 0 \tag{6}$$

$$(u')^2 = A(u^4 + tu^3 + p_1u^2 + q_1u + r_1) \tag{7}$$

where $A=-\dfrac{c}{6mk\omega}$, $t=\dfrac{2b}{c}$, $p_1=\dfrac{6(\omega+ak)}{kc}$, $q_1=\dfrac{12c_1}{kc}$, $r_1=\dfrac{12c_2}{kc}$, let $u_1=u+\theta$, we have

$$(u_1')^2=A(u_1^4+pu_1^2+qu_1+r) \tag{8}$$

where $\theta=\dfrac{t}{4}$, $p=p_1-\dfrac{3}{8}t^2$, $q=q_1-\dfrac{p_1t}{2}+\dfrac{t^3}{8}$, $r=r_1+\dfrac{p_1t^2}{16}-\dfrac{q_1t}{4}-\dfrac{3t^4}{256}$

Turning into the form of elementary integrals as follows

$$\pm\sqrt{\varepsilon A}(\xi-\xi_0)=\int\frac{du_1}{\sqrt{\varepsilon(u_1^4+pu_1^2+qu_1+r)}} \tag{9}$$

when $A>0$ $\varepsilon=1$ $A<0$ $\varepsilon=-1$.

Obtaining exact solutions by using polynomial completely discriminant classification system

Let $F(u_1)=u_1^4+pu_1^2+qu_1+r$, its complete discrimination system is

$$\begin{cases} D_1=4 \\ D_2=-p \\ D_3=-2p^3+8pr-9q^2 \\ D_4=-p^3q^2+4p^4r+36pq^2r-32p^2r^2-\dfrac{27}{4}q^4+64r^3 \\ E_2=9p^2-32pr \end{cases} \tag{8}$$

Case1 $D_4=0$, $D_3=0$, $D_2<0$, $F(u_1)=[(u_1-l_1)^2+s_1^2]^2$, where l_1, s_1 are real numbers and $s_1>0$.

When $\varepsilon=1$

$$u_1=s_1\tan[s_1\sqrt{A}(\xi-\xi_0)]+l_1 \tag{9}$$

Case2 $D_4=0$, $D_3=0$, $D_2=0$, $F(u_1)=u_1^4$,

When $\varepsilon=1$,

$$u_1=-\frac{1}{\sqrt{A}(\xi-\xi_0)} \tag{10}$$

Case3 $D_4=0$, $D_3=0$, $D_2>0$, $E_2=0$, $F(u_1)=(u_1-\alpha)^2(u_1-\beta)^2$, where α, β are real numbers and $\alpha>\beta$.

When $\varepsilon=1$, $u>\alpha$ or $u<\beta$, we have

$$u_1=\frac{\beta-\alpha}{2}[\coth\frac{\beta-\alpha}{2}\sqrt{A}(\xi-\xi_0)-1]+\beta \tag{11}$$

When $\varepsilon=1$, $\alpha<u<\beta$, we have

$$u_1=\frac{\beta-\alpha}{2}[\tanh\frac{\beta-\alpha}{2}\sqrt{A}(\xi-\xi_0)-1]+\beta \tag{12}$$

Case4 $D_4=0$, $D_3>0$, $D_2>0$, $F(u_1)=(u_1-\alpha)^2(u_1-\beta)(u_1-\gamma)$, where α, β, γ are real numbers and $\beta>\gamma$,

When $\varepsilon=1$, $\alpha>\beta$ and $u>\beta$ or $\alpha<\gamma$ or $u<\gamma$时,

$$\pm\sqrt{A}(\xi-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(u_1-\beta)(\alpha-\gamma)}-\sqrt{(\alpha-\beta)(u_1-\gamma)}]^2}{|u_1-\alpha|} \tag{13}$$

当$\varepsilon=1$, $\alpha>\beta$且$u_1<\gamma$或$\alpha<\gamma$或$u_1<\beta$,

$$\pm\sqrt{A}(\xi-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(u_1-\beta)(\gamma-\alpha)}-\sqrt{(\beta-\alpha)(u_1-\gamma)}]^2}{|u_1-\alpha|} \quad (14)$$

When $\varepsilon=1$, $\alpha>\beta$ and $u>\beta$ or $\alpha<\gamma$ or $u<\gamma$,

$$\pm\sqrt{A}(\xi-\xi_0)=\frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}\arcsin\frac{(u_1-\beta)(\alpha-\gamma)+(\alpha-\beta)(u_1-\gamma)}{|(u_1-\alpha)(\beta-\gamma)|} \quad (15)$$

When $\varepsilon=1$, $\alpha>\beta$ and $u<\gamma$ or $\alpha<\gamma$ or $u<\beta$,

$$\pm\sqrt{-A}(\xi-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(u_1-\beta)(\alpha-\gamma)}-\sqrt{(\alpha-\beta)(u_1-\gamma)}]^2}{|u_1-\alpha|} \quad (16)$$

When $\varepsilon=1$, $\beta>\alpha>\gamma$,

$$\pm\sqrt{-A}(\xi-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(u_1-\beta)(\gamma-\alpha)}-\sqrt{(\beta-\alpha)(u_1-\gamma)}]^2}{|u_1-\alpha|} \quad (17)$$

When $\varepsilon=-1$, $\alpha>\beta$ and $u>\beta$ or $\alpha<\gamma$ or $u<\gamma$,

$$\pm\sqrt{-A}(\xi-\xi_0)=\frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}\arcsin\frac{(u_1-\beta)(\alpha-\gamma)+(\alpha-\beta)(u_1-\gamma)}{|(u_1-\alpha)(\beta-\gamma)|} \quad (18)$$

Case5 $D_4=0$, $D_3=0$, $D_2>0$, $E_2=0$, $F(u_1)=(u_1-\alpha)^3(u_1-\beta)$, where α, β are real numbers.
When $\varepsilon=1$, $u>\alpha$ and $u>\beta$ or $u<\alpha$ and $u<\beta$,

$$u_1=\frac{4(\alpha-\beta)}{A(\alpha-\beta)^2(\xi-\xi_0)^2-4}+\alpha \quad (19)$$

When $\varepsilon=-1$, $u>\alpha$ and $u<\beta$ or $u<\alpha$ and $u>\beta$,

$$u_1=\frac{4(\beta-\alpha)}{4-A(\alpha-\beta)^2(\xi-\xi_0)^2}+\alpha \quad (20)$$

Case6 $D_4=0$, $D_2D_3<0$, $F(u_1)=(u_1-\alpha)^2[(u_1-l_1)^2+s_1^2]$, where α, l_1, s_1 are real numbers.
When $\varepsilon=1$,

$$u_1=\frac{\exp[\pm\sqrt{(\alpha-l_1)^2+s_1^2}\sqrt{A}(\xi-\xi_0)]-\gamma+\sqrt{(\alpha-l_1)^2+s_1^2}}{\{\exp[\pm\sqrt{(\alpha-l_1)^2+s_1^2}\sqrt{A}(\xi-\xi_0)]-\gamma\}^2-1} \quad (21)$$

Where $\gamma=\dfrac{\alpha-2l_1}{\sqrt{(\alpha-l_1)^2+s_1^2}}$

Case7 $D_4>0$, $D_3>0$, $D_1>0$, $F(u_1)=(u_1-\alpha_1)(u_1-\alpha_2)(u_1-\alpha_3)(u_1-\alpha_4)$, where α_1, α_2, α_3, α_4 are real numbers and $\alpha_1>\alpha_2>\alpha_3>\alpha_4$.
When $\varepsilon=1$, $u>\alpha_1$ or $u<\alpha_4$,

$$u_1=\frac{\alpha_2(\alpha_1-\alpha_4)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-\alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-(\alpha_2-\alpha_4)} \quad (22)$$

When $\varepsilon=1$, $\alpha_2<u<\alpha_3$,

$$u_1 = \frac{\alpha_4(\alpha_1-\alpha_4)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-\alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-(\alpha_2-\alpha_4)} \quad (23)$$

When $\varepsilon=-1$, $\alpha_2<u<\alpha_1$,

$$u_1 = \frac{\alpha_3(\alpha_1-\alpha_2)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-\alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-(\alpha_1-\alpha_3)} \quad (24)$$

When $\varepsilon=-1$, $\alpha_3<u<\alpha_4$,

$$u_1 = \frac{\alpha_1(\alpha_3-\alpha_4)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-\alpha_4(\alpha_1-\alpha_3)}{(\alpha_3-\alpha_4)\mathrm{sn}^2[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}\sqrt{A}(\xi-\xi_0),m]-(\alpha_3-\alpha_1)} \quad (25)$$

Where $m^2=\frac{(\alpha_1-\alpha_2)(\alpha_3-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$

Case8 $D_4<0$, $D_2D_3\geq 0$, $F(u_1)=(u_1-\alpha)(u_1-\beta)[(u_1-l_1)^2+s_1^2]$, Where α, β, l_1, s_1 are real numbers and $\alpha>\beta$, $s_1>0$,

$$u_1 = \frac{a_1\mathrm{cn}[\frac{\sqrt{\mp 2s_1m_1(\alpha-\beta)}}{2mm_1}\sqrt{-A}(\xi-\xi_0),m]+b_1}{c_1\mathrm{cn}[\frac{\sqrt{\mp 2s_1m_1(\alpha-\beta)}}{2mm_1}\sqrt{-A}(\xi-\xi_0),m]+d_1} \quad (26)$$

Where $a_1=\frac{1}{2}[(\alpha+\beta)c_1-(\alpha-\beta)d_1]$, $b_1=\frac{1}{2}[(\alpha+\beta)d_1-(\alpha-\beta)c_1]$, $c_1=a_1-l_1-\frac{s_1}{m_1}$,

$d_1=a_1-l_1-s_1m_1$, $E=\frac{s_1^2+(\alpha-l_1)(\beta-l_1)}{s_1(\alpha-\beta)}$, $m_1=E\pm\sqrt{E^2+1}$, $m^2=\frac{1}{1+m_1^2}$, and m_1 satisfy the condition $\varepsilon m_1<0$.

Case9 $D_4>0$, $D_2D_3\leq 0$, $F(u_1)=[(u_1-l_1)^2+s_1^2][(u_1-l_2)^2+s_2^2]$, where l_1, l_2, s_1, s_2 are real numbers and $s_1>s_2>0$.

When $\varepsilon=1$,

$$u_1 = \frac{a_1\mathrm{sn}[\eta\sqrt{A}(\xi-\xi_0),m]+b_1\mathrm{cn}[\eta\sqrt{A}(\xi-\xi_0),m]}{c_1\mathrm{sn}[\eta\sqrt{A}(\xi-\xi_0),m]+d_1\mathrm{cn}[\eta\sqrt{A}(\xi-\xi_0),m]} \quad (27)$$

Where $c_1=-s_1-\frac{s_2}{m_1}$, $d_1=l_1-l_2$, $a_1=l_1c_1+s_1d_1$, $b_1=l_1d_1-s_1c_1$, $E=\frac{s_1^2+s_2^2+(l_1-l_2)^2}{2s_1s_2}$,

$m_1=E+\sqrt{E^2-1}$, $m^2=1-\frac{1}{m_1^2}$, $\eta=s_2\sqrt{\frac{m_1^2c^2+d^2}{c^2+d^2}}$.

CONCLUSIONS

In this article, we take the traveling wave transformation $u=u(\xi)$, $\xi=kx+\omega t$ to BBMP equation with power-law and turn it into ordinary differential equations. Then we given its precise solutions of single traveling wave method by complete discrimination system for polynomial(8)-(27).

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