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Jacobi Sums and Cyclotomic Numbers of Order 14

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Abstract: In this paper we obtain unambiguous expression for cyclotomic numbers of order 14 using Jacobi sums J_7 (1 1) and J_7 (1 2).
Keywords: Cyclotomic Numbers, Jacobi sums.

1.INTRODUCTION

For a positive integer e and p be a rational prime and p≡1(mod e). Let $q = p^{\alpha}$and $F_q$be a finite field with q elements. Write q = ef + 1. Let ζ be a complex primitiveeth root of unity. If is a generator of cyclic group of $F_q$then define the multiplicative character χ: $F_q \rightarrow Q(\zeta)$ by $\chi(\gamma) = \zeta\chi(0) = 0$. (Note: χ (0) = 0 all i in Z.) For0 ≤ i j ≤e -1 define the e^2cyclotomic numbers $A_{i,j}$ by

(1.1) $A_{i,j}$= Cardinality of $X_{i,j}$,
Where$X_{i,\,j}$= $\{v \in F_q \mid \chi(v) = \zeta^i , \chi(v + 1) = \zeta^j\}$
= $\{v \in F_q - \{0, 1\} \mid ind_{\gamma} v \equiv i \pmod e, ind_{\gamma}(v + 1) \equiv j \pmod e\}$

Also, define the e^2 Jacobi sums J (i j) (or J_e (i j)) of order e by
(1.2) $J(i\,j) = \sum \chi(v)\,\chi(v + 1)$
$v \in F_q$.

In the literature of Jacobi sums and cylotomic numbers, we observe the variation in the definition of Jacobi sums. For 0 ≤ i j ≤e -1, define the Jacobi sums R (i j) (or R (i j))of order e by

(1.2) $R(i\,j) = \sum \chi(v)\,\chi(1- v)$
$v \in F_q$.

Note that Jacobi sums R(i j) defined by (1.3) and J (i, j) defined by (1.2) are relatedby J(i j) =
$\chi^i(-1)R(i\,j)$. The Jacobi sums J (i j) and the cyclotomic numbers A_{ij} arerelated by

(1.3) $\sum_i\sum_j\zeta^{-(ai+bj)}J(ij) = e^2A_{a,b,}$

(1.4) $\sum_i\sum_j\zeta^{(ai+bj)}A_{i,j} = J(a\,b).$

These relations show that if we want to determine all the $A_{i,\,j}$it is sufficient todetermine all the Jacobi sums J (i j). Also, note that if we change the generator of $F_q$then the sets $X_{i,\,j}$ get interchanged among themselves and hence the cyclotomic numbers$A_{i,\,j}$ and Jacobi sums J (ij).

In this paper we evaluate the cyclotomic numbers of order fourteen in terms of Jacobi sums $J_7(1,1)$ and $J_7(1,2)$ of order seven. We also show that $J_7(1,1)$, Jacobi sum of order seven is sufficient to determine all the Jacobi sums of order fourteen. Eventually the cyclotomic numbers of order fourteen can be expressed in terms of $J_7(1,1)$ of order seven. The cyclotomic numbers of order fourteen have been studied by Muskat [2], N.Buck and K.S.Williams [3]. Their results have the classical ambiguity discussed in [4]. We have given unambiguous evaluation of cyclotomic numbers of order fourteen. Our work is based on results of V.V.Acharya and S.A.Katre [1] and also on the work of Dickson [5]. First we state the basic properties for cyclotomic numbers of order fourteen in general set up (e = 2l, l odd).

2. PRELIMINARIES

2.1. Cyclotomic numbers of order 2l. Let e = 2l, l an odd prime. For given generator γ of F^*_q, $A_{i,j}$ denotes cyclotomic numbers of order 2l. Observe that

$$\sum_{i=0}^{2l-1}\sum_{j=0}^{2l-1} A_{i,j} = q-2.$$

Also,

$$\sum_{j=0}^{2l-1} A_{i,j} = f - n_i.$$

Where

$n_i = 1$ if ($i = 0$ and f even) or if ($i = l$ and f odd),
$= 0$ otherwise.

Further

$$\sum_{j=0}^{2l-1} A_{i,j} = f-1 \text{ if } j = 0, \quad = f \text{ otherwise.}$$

Also, if f is even we have

(2.1) $A_{i,j} = A_{j,i} = A_{i-j,-j} = A_{j-i,-i} = A_{-i,j-i} = A_{-j,i-j}$,

and if f is odd we have

(2.2) $A_{i,j} = A_{j+l,i+l} = A_{l+i-j,-j} = A_{l+j-i,l-i} = A_{-i,j-i} = A_{l-j,i-j}$

2.2. Jacobi sums of order 2l and their properties. Let ζ and ξ be complex primitive l^{th} and $2l^{th}$ roots of unities respectivelyfor which the character χ and the Jacobi sums $J_l(ij)$ and $J(ij) = J_{2l}(ij)$ of order l and 2l are respectively defined as above. Moreover, ζ and ξ are related by $\zeta = \xi^2$, equivalently $\xi = -\zeta^{(l+1)/2}$.

Theorem 2.1.(Elementary properties of Jacobi sums.)

1) If $a + b + c \equiv 0 \pmod{2l}$ then

$$J(a\ b) = J(c\ b) = \chi^c(-1)J(c\ a) = \chi^a(-1)J(b\ c) = \chi^a(-1)J(ac) = \chi^c(-1)J(b\ a).$$

In particular, $J(1\ a) = \chi(-1)J(1\ 2l-a-1)$.

2) $J(0\ j) = -1$ if $j \not\equiv 0 \pmod{2l}$,
$= q-2$ if $j \equiv 0 \pmod{2l}$.

And $J(i0) = -\chi^i(-1)$ if $i \not\equiv 0 \pmod{2l}$,
$= q-2$ if $i \equiv 0 \pmod{2l}$.

3) Let $a + b \equiv 0 \pmod{2l}$, but not both zero (mod 2l). Then $J(a\ b) = -1$.

4) For $(k, 2l) = 1$, $\tau_k J(i\ j) = J(ikjk)$, where τ_k is an automorphism $\xi \to \xi^k$ of $Q(\zeta)$ over Q. In particular, if for $(i,2l) = 1$, i^{-1} denotes the inverse of i modulo l then $\tau^{-1}_i J(ij) = J(1\ ji^{-1})$.

5) $J(2r\ 2s) = J_l(r\ s)$ where $J_l(r\ s)$ are Jacobi sums of order l.

6) $J(1\ n)\overline{J(1\ n)} = q$ if $n \not\equiv 0,-1 \pmod{2l}$,
$= 1$ if $n \equiv 0,-1 \pmod{2l}$.

7) (Product Rule for Jacobi sums): Let m, n and t be integers such that $m + n \not\equiv 0 \pmod{2l}$ and $m + t \not\equiv 0 \pmod{2l}$. Then $J(m\ n)\, J(m + n\ t) = \chi^m(-1)J(m\ t)J(n\ m + t)$.

Proof: Refer [1].

3. JACOBI SUMS AND CYCLOTOMIC NUMBERS OF ORDER FOURTEEN

Let p be a prime, $p \equiv 1 (\text{mod } 7)$ and let $q = p^{\alpha}$, $\alpha \geq 1$. Let $q = 1+14f$ and F_q denote finite field of q elements. Note that f may be even or odd. Let ζ and ξ be the complex primitive seventh and fourteenth root of unity in terms of which the character χ and Jacobi sums $J_7(ij)$ and $J(ij) = J_{14}(ij)$ are defined as in section 1. Also, observe that $\zeta = \xi^2$. From (1.4) all the cyclotomic numbers of order fourteen are known if we know following listed cyclotomic numbers of order fourteen.

Case 1: If f is even, using (2.1) it is sufficient to know the following cyclotomic numbers of order fourteen.

$A_{0,0},A_{0,1},A_{0,2},A_{0,3},A_{0,4},A_{0,5},A_{0,6},A_{0,7},A_{0,8},A_{0,9},A_{0,10},A_{0,11},A_{0,12},A_{0,13},$
$A_{1,2},A_{1,3},A_{1,4},A_{1,5},A_{1,6},A_{1,7},A_{1,8},A_{1,9},A_{1,10},A_{1,11},A_{1,12},A_{2,4},A_{2,5},$
$A_{2,6},A_{2,7},A_{2,8},A_{2,9},A_{2,10},A_{2,11},A_{3,6},A_{3,7},A_{3,8},A_{3,9},A_{3,10},A_{4,8},A_{4,9}.$

Case 2: If f is odd, using (2.2) it is sufficient to know the cyclotomic numbers of order fourteen.

$A_{0,0},A_{0,1},A_{0,2},A_{0,3},A_{0,4},A_{0,5},A_{0,6},A_{0,7},A_{0,8},A_{0,9},A_{0,10},A_{0,11},A_{0,12},A_{0,13},$
$A_{1,0},A_{1,1},A_{1,2},A_{1,3},A_{1,4},A_{1,5},A_{1,6},A_{1,10},A_{1,11},A_{1,12},A_{1,13},A_{2,0},A_{2,1},$
$A_{2,2},A_{2,3},A_{2,4},A_{2,5},A_{2,12},A_{2,13},A_{3,0},A_{3,1},A_{3,2},A_{3,3},A_{3,4},A_{4,1},A_{4,2}.$

3.1. Jacobi sums of order 14: Among 196 Jacobi sums J(a b) of order 14, $0 \leq a\ b \leq 13$, we find the minimum number of Jacobi sums of order 14 required to get the other Jacobi sums. First observe that for a =0 or b =0 or both zero, J(0 b) = -1,$(b \neq 0)$, J(a 0) = $\chi(-1)$,$(a \neq 0)$, J(0 0) = q-2. For any a, b with $1 \leq a\ b \leq 13$, such that gcd(a,14) = 1 or gcd (b,14) = 1, we observe that
J (1 a) is enough to determine the other Jacobi sums J (ab). (Refer to (4) of Theorem 2.1). Also, using (2) of Theorem 2.1, we conclude that it is enough to determine J (1 1),J (12),J (1 3),J (1 5),J (1 7) and J (1 9).
If gcd(a,14) = gcd(b,14) = 2, then J(ab) = J(2a'2b') = J_7(a'b'). (Refer to (5) of Theorem 2.1). In this case, J(a b) is either trivial Jacobi sum or J_7(1 1) and J_7(1 2) are enough to determine other Jacobi sums. From [6], the relation between J_7(1 1) and J_7(1 2) is given by
$J_7(1\ 2) = J_7(1\ 1).\sigma_2(J_7(1\ 1)/\sigma_3(J_7(1\ 1))$.
Hence J_7(1 1) is enough to determine other Jacobi sums.
If a = 7 or b = 7 then from (1) and (4) of Theorem 2.1 we see that J (1 7) is enough to determine other Jacobi sums in this case. Also, J (7 7) = -1. From above discussion we can say that 196 Jacobi sums of order 14 are either trivial or J (1 1), J (1 2),J (13), J (1 5), J (1 7) and J (1 9) together with Jacobi sum J_7(1 1) are enough to determine other Jacobi sums.
Let γ be generator of F^*_q and m = $\text{ind}_\gamma 2$ then from (1.3) and [?] we get
(3.1)$J(1\ 1) = \zeta^{-2m}\sigma_3 J_7(1\ 1)$.
(3.2) $J(1\ 2) = (-1)^f\zeta^{2m} J_7(1\ 1)$.
(3.3) $J(1\ 3) = \zeta^{-4m}\sigma_3 J_7(1\ 1)$.
(3.4) $J(15) = \zeta^{m} J_7(1\ 14)$.
(3.5)$J(1\ 7) = \zeta^{-m}\sigma_3 J_7(1\ 1)$.
(3.6)$J(1\ 9) = \zeta^{4m}\sigma_2 J_7(1,1)$.
Looking at the above relations, J_7 (1 1) is enough to determine non-trivial Jacobi sums of order 14. We conclude the above discussion as theorem below.

Theorem 3.1 (Relation between Jacobi sums of order 14 and Jacobi sums of order 7). All the non-trivial Jacobi sums of order 14 are determined by knowing Jacobi sum J_7 (1 1) of order 7.

3.2. Coefficients of Jacobi sums of order 14:
For the generator γ of F^*_q and m =$\text{ind}_\gamma 2$, let
l-1l-1l-1
$J_7(1,1) = \sum a_i\zeta^i$, $J_7(1,2) = \sum c_i\zeta^i$ and $J(1\ n) = \sum b_i(n)\zeta^i$. Fix a_0=0, using (3.1)-(3.6) we geti=1 i=1 i=1

J(1 n)	m≡ 0(mod 7)						m≡ 1(mod 7)					
	$b_1(n)$	$b_2(n)$	$b_3(n)$	$b_4(n)$	$b_5(n)$	$b_6(n)$	$b_1(n)$	$b_2(n)$	$b_3(n)$	$b_4(n)$	$b_5(n)$	$b_6(n)$
J(11)	a_5	a_3	a_1	a_6	a_4	a_2	$a_1 - a_3$	$a_6 - a_3$	$a_4 - a_3$	$a_2 - a_3$	$-a_3$	$a_5 - a_3$
J(13)	a_5	a_3	a_1	a_6	a_4	a_2	$a_4 - a_6$	$a_2 - a_6$	$-a_6$	$a_5 - a_6$	$a_3 - a_6$	$a_1 - a_6$
J(1 5)	a_1	a_2	a_3	a_4	a_5	a_6	$-a_6$	$a_1 -$	$a_2 - a_6$	$a_3 - a_6$	$a_4 - a_6$	$a_5 - a_6$

								a_6				
J(1 7)	a_5	a_3	a_1	a_6	a_4	a_2	$a_3 - a_5$	$a_1 - a_5$	$a_6 - a_5$	$a_4 - a_5$	$a_2 - a_5$	$-a_5$
J(1 9)	a_4	a_1	a_5	a_2	a_4	a_3	$a_2 - a_5$	$a_6 - a_5$	$a_3 - a_5$	$-a_5$	$a_4 - a_5$	$a_1 - a_5$
J(1 11)	a_1	a_2	a_3	a_4	a_5	a_6	$a_6 - a_5$	$-a_5$	$a_1 - a_5$	$a_2 - a_5$	$a_3 - a_5$	$a_4 - a_5$

J (1, n)	**m≡ 2(mod 7)**						**m≡ 3(mod 7)**					
	b_1 (n)	b_2(n)	b_3(n)	b_4 (n)	b_5 (n)	b_6 (n)	b_1 (n)	b_2(n)	b_3(n)	b_4 (n)	b_5 (n)	b_6 (n)
J (1 1)	$a_4 - a_6$	$a_2 - a_6$	$-a_6$	$a_5 - a_6$	$a_3 - a_6$	$a_1 - a_6$	$-a_2$	$a_5 - a_2$	$a_3 - a_2$	$a_1 - a_2$	$a_6 - a_2$	$a_4 - a_2$
J (13)	$a_3 - a_5$	$a_1 - a_5$	$a_6 - a_5$	$a_4 - a_5$	$a_2 - a_5$	$-a_5$	$a_2 - a_4$	$-a_4$	$a_5 - a_4$	$a_3 - a_4$	$a_1 - a_4$	$a_6 - a_4$
J (15)	$a_6 - a_5$	$-a_5$	$a_1 - a_5$	$a_2 - a_5$	$a_3 - a_5$	$a_4 - a_5$	$a_5 - a_4$	$a_6 - a_4$	$-a_4$	$a_1 - a_4$	$a_2 - a_4$	$a_3 - a_4$
J (17)	$a_1 - a_3$	$a_6 - a_3$	$a_4 - a_3$	$a_2 - a_3$	$-a_3$	$a_5 - a_3$	$a_6 - a_1$	$a_4 - a_1$	$a_2 - a_1$	$-a_1$	$a_5 - a_1$	$a_3 - a_1$
J (1 9)	$-a_3$	$a_4 - a_3$	$a_1 - a_3$	$a_5 - a_3$	$a_2 - a_3$	$a_6 - a_3$	$a_5 - a_1$	$a_2 - a_1$	$a_6 - a_1$	$a_3 - a_1$	$-a_1$	$a_4 - a_1$
J (1 11)	$a_4 - a_3$	$a_5 - a_3$	$a_6 - a_3$	$-a_3$	$a_1 - a_3$	$a_2 - a_3$	$a_2 - a_1$	$a_3 - a_1$	$a_4 - a_1$	$a_5 - a_1$	$a_6 - a_1$	$-a_1$
J (1n)	**m≡ 4(mod 7)**						**m≡ 5(mod 7)**					
	b_1 (n)	b_2(n)	b_3(n)	b_4 (n)	b_5 (n)	b_6 (n)	b_1 (n)	b_2(n)	b_3(n)	b_4 (n)	b_5 (n)	b_6 (n)
J (1 1)	$a_3 - a_5$	$a_1 - a_5$	$a_6 - a_5$	$a_4 - a_5$	$a_2 - a_5$	$-a_5$	$a_6 - a_1$	$a_4 - a_1$	$a_2 - a_1$	$-a_1$	$a_5 - a_1$	$a_3 - a_1$
J (1 3)	$a_1 - a_3$	$a_6 - a_3$	$a_4 - a_3$	$a_2 - a_3$	$-a_3$	$a_5 - a_3$	$-a_2$	$a_5 - a_2$	$a_3 - a_2$	$a_1 - a_2$	$a_6 - a_2$	$a_4 - a_2$
J (1 5)	$a_4 - a_3$	$a_5 - a_3$	$a_6 - a_3$	$-a_3$	$a_1 - a_3$	$a_2 - a_3$	$a_3 - a_2$	$a_4 - a_2$	$a_5 - a_2$	$a_6 - a_2$	$-a_2$	$a_1 - a_2$
J (1 7)	$a_4 - a_6$	$a_2 - a_6$	$-a_6$	$a_5 - a_6$	$a_3 - a_6$	$a_1 - a_6$	$a_2 - a_4$	$-a_4$	$a_5 - a_4$	$a_3 - a_4$	$a_1 - a_4$	$a_6 - a_4$
J (1 9)	$a_3 - a_6$	$-a_6$	$a_4 - a_6$	$a_1 - a_6$	$a_5 - a_6$	$a_2 - a_6$	$a_1 - a_4$	$a_5 - a_4$	$a_2 - a_4$	$a_6 - a_4$	$a_3 - a_4$	$-a_4$
J (1 11)	$-a_6$	$a_1 - a_6$	$a_2 - a_6$	$a_3 - a_6$	$a_4 - a_6$	$a_5 - a_6$	$a_5 - a_4$	$a_6 - a_4$	$-a_4$	$a_1 - a_4$	$a_2 - a_4$	$a_3 - a_4$

J (1, n)	**m≡ 6(mod 7)**					
	b_1 (n)	b_2(n)	b_3(n)	b_4 (n)	b_5 (n)	b_6 (n)
J (1 1)	$a_2 - a_4$	$-a_4$	$a_5 - a_4$	$a_3 - a_4$	$a_1 - a_4$	$a_6 - a_4$
J (1 3)	$a_6 - a_1$	$a_4 - a_1$	$a_2 - a_1$	$-a_1$	$a_5 - a_1$	$a_3 - a_1$
J (1 5)	$a_2 - a_1$	$a_3 - a_1$	$a_4 - a_1$	$a_5 - a_1$	$a_6 - a_1$	$-a_1$
J (1 7)	$-a_2$	$a_5 - a_2$	$a_3 - a_2$	$a_1 - a_2$	$a_6 - a_2$	$a_4 - a_2$
J (1 9)	$a_6 - a_2$	$a_3 - a_2$	$-a_2$	$a_4 - a_2$	$a_1 - a_2$	$a_5 - a_2$
J (111)	$a_3 - a_2$	$a_4 - a_2$	$a_5 - a_2$	$a_6 - a_2$	$-a_2$	$a_1 - a_2$

Theorem 3.2. (Main Theorem): Let p and l be odd rational primes, $p \equiv 1(\bmod l)$ (thus $p \equiv 1(\bmod 2l)$), $q = p^{\alpha}$, $\alpha \geq 1$. Let $q = 1 + 2lf$. Let $\zeta$and ξ be fixed primitive complex $l^{th}$and $2l^{th}$ roots of unity respectively. Let ζ and ξ be related by $\zeta = \xi^2$, that is, $\xi = -\zeta^{-(l+1)/2}$. Let γ be a generator of F^*_q. Let b be a rational integer such that $b = \gamma^{(q-1)/2}$in Fp. Let m = $ind_{\gamma}2$. Let $J_l(ij)$ and $J_{2l}(ij)$ denote the Jacobi sums over F_q of order l and 2l (respectively) related to $\zeta$and ξ. For (k,l) = 1, let σ_k denote the automorphism $\zeta \rightarrow \zeta^k$ of $Q(\zeta)$. For (k,2l) = 1, τ_k denote the automorphism
$\xi \rightarrow \xi^k$of $Q(\zeta)$. Thus if k is odd then $\sigma_k = \tau_k$ and if k is even then $\sigma_k = \tau_{k+l}$. Let $\lambda(r)$ and $\Lambda(r)$ denote the least non-negative integers residue modulo 2l and l respectively. Let $a_0, a_1, \ldots, a_{l-1} \in Z$
l-1
and let H =$\sum a_i \zeta^i$. Consider the arithmetic conditions (or Diophantine system)
i=0
l-1l-1
(1) q =$\sum a^2_i - \sum a_i a_{i+1}$,

i=0 i=0
l-1l-1l-1

(2) $\sum a_i a_{i+1} = \sum a_i a_{i+2} = \cdots = \sum a_i a_{i+(l-1)/2}$,
i=0 i=0i=0

(3) $1 + a_0 + a_1 + \cdots + a_{l-1} \equiv 0 \pmod l$. Let $1 \le n \le (l-2)$. If $a_0, a_1, \ldots, a_{l-1}$ satisfy (1)-(3) together with the additional conditions

(4) $a_1 + 2a_2 + \cdots + (l-1)a_{l-1} \equiv 0 \pmod l$

(5) $p \nmid \prod H^{\sigma k}$
$\lambda((n+1)k) \ge k$

(6) $\overline{P}|H\prod(b-\zeta^{\sigma k-1})$ then $H = J_l(1,n)$ for this γ and conversely. Let $1 \le n \le (2l-3)$ be an odd
$\lambda((n+1)k) \ge k$
integer. If $a_0, a_1, \ldots, a_{l-1}$ satisfy (1)-(3) together with the additional condition

(7) $a_1 + 2a_2 + \cdots + (l-1)a_{l-1} \equiv m(n+1) \pmod l$,

(8) $p \nmid \prod H^{\tau k}$,
$\lambda((n+1)k) \ge k$

(9) $\overline{p}|H\prod(b-\zeta^{\tau k-1})$ where $k-1$ is taken modulo $2l$, then $H = J_{2l}(1,n)$ for this γ and conversely.
$\lambda((n+1)k) \ge k$

(In (8) and (9), k varies over only those values which satisfy $1 \le k \le (2l-1)$ and $(k,2l) = 1$). Moreover, for $1 \le n \le (l-2)$ if $a_0, a_1, \ldots, a_{l-1}$ satisfy the conditions (1)-(6) and if we fix $a_0 = 0$ at the outset then we have $J_l(1,n) = \sum_{i=0}^{l-1} a_i \zeta^i$ and the cyclotomic numbers of order l are given by

l-2l-2 l-1
(3.7) $l^2 A(i,j)_l = q-3l + 1 + \varepsilon(i) + \varepsilon(j) + \varepsilon(i-j) + l\sum a_{in+j}(n) + \sum \sum a_k(n)$
n=0 n=1 k=1

where $\varepsilon(i) = 0$ if $l|i$, or 1 otherwise. The suffixes in $a_{in+j}(n)$ are considered modulo l. For n odd, $1 \le n \le (2l-3)$, if $a_0, a1, \ldots, a_{l-1}$ satisfy (1)-(3) and (7)-(9) and if we fix $a_0 = 0$ at the outset and write the a_i corresponding to given n as $b_i(n)$ then we have

$J_{2l}(1,n) = \sum_{i=1}^{l-1} b_i \zeta^i$ and the $4l^2$ cyclotomic numbers $A(i,j)_{2l}$ are given by

l-2l-2 l-1
$4l^2 A(i,j)_{2l} = q-3l + 1 + \varepsilon(i) + \varepsilon(j) + \varepsilon(i-j) + l\sum a_{in+j}(n) + \sum \sum a_k(n)$
n=0 n=1 k=1
l-1 l-2 l-1
$- \{(-1)^i + (-1)^{i+f} + (-1)^{i+j}\}\{l + \sum bk(l) + \sum \sum b_k(2u+1)\}$
k=0 u=0k=0
l-1l-1
$+ (-1)^j (l(b_{\upsilon(-i)}(l) + \sum b_{\upsilon(j-2iu-2i)}(2u+1)) + (-1)^{i+j}(l(b_{\upsilon(j)}(l) + \sum b_{\upsilon(i+2ju+j)}(2u+1))$
u=0 u=0
l-1
$+ (-1)^{i+f}(l(b_{\upsilon(-j)}(l) + \sum b_{\upsilon(i-2ju-2j)}(2u+1))$
u=0

where,
$\upsilon(i) = \Lambda(j)/2$ if j is even,
$\Lambda(j + l)/2$ if j is odd.

Proof. Refer [1].

3.3. Cyclotomic numbers of order 14. Using above theorem and section 3 the cyclotomic numbers are given in the appendix. Here we give some examples of cyclotomic numbers of order 14.

Case: f even. Let q = p = 29, m = $\mathrm{ind}_\gamma 2 \equiv 2 \pmod 7$. The Jacobi sums $J_7(1\ 1)$, $J_7(1\ 2)$ of order 7 are given by $J_7(1\ 1) = 2\zeta-\zeta^3-4\zeta^4+2\zeta^5$, $J_7(1\ 2) = 3\zeta+3\zeta^2-\zeta^3+3\zeta^4-\zeta^5-\zeta^6$ and some cyclotomic numbers of order 14 are $A_{0,0} = 0, A_{1,7} = A_{1,11} = A_{3,9} = 1$.

Case: f odd. Let q = p = 43, m = $\mathrm{ind}_\gamma 2 \equiv 1 \pmod 7$. The Jacobi sums $J_7(1\ 1)$, $J_7(1\ 2)$ of order 7 are given by $J_7(1,1) = 4\zeta - 2\zeta^2 + 2\zeta^3 - 3\zeta^5 - 2\zeta^6$,
$J_7(1,2) = -7\zeta-7\zeta^2-5\zeta^3-7\zeta^4-5\zeta^5-5\zeta^6$ and some cyclotomic numbers of order 14 are
$A_{0,0}= 0$, $A_{1,13} = 1$.

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REFERENCES

1. Acharyaand VV, Katre SA; Cyclotomic numbers of orders 2l. lanoddprime. Acta Arith. 1995; 69(1):51-74.
2. Muskat JB; The cyclotomic numbers of order fourteen. Acta Arith, 1996;11:263-279.
3. Buck N, Williams KS; Sequel to Muskat's evaluation of the cyclotomic numbers of order four-teen, Carleton Mathematical Series 216, November 1985, Carleton University Ottawa ,22pp.
4. Katreand SA, Rajwade RA; Complete solution of the cyclotomic problemin Fq for any prime modulusl, $q=p^{\alpha}$, $p\equiv 1(\mathrm{mod}\, l)$, Acta Arith, 1985; 45:183-199.
5. Dickson LE; Cyclotomy and trinomial congruences, Trans. Amer. Math. Soc, 1935;37:363-380.
6. Parnami JC, Agarwal MK, Rajwade AR; Jacobi Sums and cyclotomic Numbers for a finite field, Acta. Arith, 1982; 41:1-13.

Appendix

Table-1

`Ai,j	$m \equiv 0 (\text{mod } 7)$,f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	-2	-24	-2	-24	-2	-24	-2	-24	-2	-24	-2	-24
$196A_{0,1}$	q-13	-2	-38	-2	11	-2	-10	-2	4	-2	-3	12	46
$196A_{0,2}$	q-13	-2	-24	-2	18	-2	-3	-2	-17	12	18	-2	18
$196A_{0,3}$	q-13	-2	-3	-2	-10	-2	-38	12	46	-2	4	-2	11
$196A_{0,4}$	q-13	-2	-17	-2	-24	12	18	-2	18	-2	18	-2	-3
$196A_{0,5}$	q-13	-2	-10	12	46	-2	11	-2	-3	-2	-38	-2	4
$196A_{0,6}$	q-13	12	18	-2	-3	-2	-24	-2	18	-2	-17	-2	18
$196A_{0,7}$	q-13	-2	4	-2	4	-2	4	-2	4	-2	4	-2	4
$196A_{0,8}$	q-13	-2	18	-2	-17	-2	18	-2	-24	-2	-3	12	18
$196A_{0,9}$	q-13	-2	4	-2	-38	-2	-3	-2	11	12	46	-2	-10
$196A_{0,10}$	q-13	-2	-3	-2	18	-2	18	12	18	-2	-24	-2	-17
$196A_{0,11}$	q-13	-2	11	-2	4	12	46	-2	-38	-2	-10	-2	-3
$196A_{0,12}$	q-13	-2	18	12	18	-2	-17	-2	-3	-2	18	-2	-24
$196A_{0,13}$	q-13	12	46	-2	-3	-2	4	-2	-10	-2	11	-2	-38
$196A_{1,2}$	q+1	5	-3	-2	4	5	-3	-2	-3	-2	4	-2	-3
$196A_{1,3}$	q+1	-2	-3	5	-17	-2	18	-2	11	-2	-10	-2	4
$196A_{1,4}$	q+1	5	-10	-2	11	-2	-3	-2	-3	-2	11	5	-10
$196A_{1,5}$	q+1	-2	18	-2	4	-2	-17	-2	-10	5	-3	-2	11
$196A_{1,6}$	q+1	-2	-3	-2	-10	-2	11	5	11	-2	-10	5	-3
$196A_{1,7}$	q+1	-2	4	-2	11	5	-10	-2	4	5	-3	-2	-10
$196A_{1,8}$	q+1	-2	-10	5	-3	-2	4	5	-10	-2	11	-2	4
$196A_{1,9}$	q+1	5	-3	-2	-10	5	11	-2	11	-2	-10	-2	-3
$196A_{1,10}$	q+1	-2	11	5	-3	-2	-10	-2	-17	-2	4	-2	18
$196A_{1,11}$	q+1	5	-10	-2	11	-2	-3	-2	-3	-2	11	5	-10
$196A_{1,12}$	q+1	-2	4	-2	-10	-2	11	-2	18	5	-17	-2	-3
$196A_{2,4}$	q+1	-2	-3	5	-3	-2	4	-2	4	-2	-3	5	-3
$196A_{2,5}$	q+1	5	11	-2	-3	-2	-10	-2	-10	5	-3	-2	11
$196A_{2,6}$	q+1	-2	-3	-2	-3	-2	4	5	-3	-2	4	-2	4
$196A_{2,7}$	q+1	-2	4	-2	4	5	-3	-2	11	-2	-10	5	-10
$196A_{2,8}$	q+1	-2	-3	5	4	-2	-3	-2	-3	5	4	-2	-3
$196A_{2,9}$	q+1	5	-10	-2	-10	-2	11	5	-3	-2	4	-2	4
$196A_{2,10}$	q+1	-2	4	-2	4	5	-3	-2	4	-2	-3	-2	-3
$196A_{2,11}$	q+1	-2	11	5	-3	-2	-10	-2	-10	-2	-3	5	11
$196A_{3,6}$	q+1	-2	4	5	-3	5	-3	-2	-3	-2	-3	-2	4
$196A_{3,7}$	q+1	5	-3	5	-10	-2	4	-2	-10	-2	4	-2	11
$196A_{3,8}$	q+1	5	-17	-2	11	-2	4	-2	-3	-2	18	-2	-10
$196A_{3,9}$	q+1	-2	-10	-2	18	-2	-3	-2	4	-2	11	5	-17
$196A_{3,10}$	q+1	-2	11	-2	4	-2	-10	-2	4	5	-10	5	-3
$196A_{4,8}$	q+1	-2	4	-2	-3	-2	-3	5	-3	5	-3	-2	4
$196A_{4,9}$	q+1	-2	-3	-2	-3	5	4	5	4	-2	-3	-2	-3

Table-2

Ai,j	m ≡ 1(mod 7),f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	-2	18	-2	60	-2	-24	-2	-3	-2	-24	-2	-24
$196A_{0,1}$	q-13	-2	32	-2	-10	-2	-10	-2	4	-2	-24	12	18
$196A_{0,2}$	q-13	-2	-10	-2	-10	-2	-10	-2	-3	12	4	-2	-10
$196A_{0,3}$	q-13	-2	4	-2	-10	-2	-10	12	-10	-2	-3	-2	39
$196A_{0,4}$	q-13	-2	11	-2	-10	12	-3	-2	-24	-2	18	-2	18
$196A_{0,5}$	q-13	-2	11	12	-10	-2	-3	-2	32	-2	4	-2	-24
$196A_{0,6}$	q-13	12	4	-2	-10	-2	18	-2	-10	-2	11	-2	-3
$196A_{0,7}$	q-13	-2	-10	-2	-24	-2	4	-2	-3	-2	4	-2	4
$196A_{0,8}$	q-13	-2	4	-2	18	-2	-10	-2	4	-2	4	12	-10
$196A_{0,9}$	q-13	-2	-10	-2	-10	-2	-10	-2	-3	12	4	-2	-10
$196A_{0,10}$	q-13	-2	-24	-2	-10	-2	18	12	18	-2	-3	-2	11
$196A_{0,11}$	q-13	-2	-17	-2	-10	12	25	-2	4	-2	18	-2	-10
$196A_{0,12}$	q-13	-2	-17	12	46	-2	-3	-2	4	-2	-24	-2	4
$196A_{0,13}$	q-13	12	4	-2	-10	-2	18	-2	-10	-2	11	-2	-3
$196A_{1,2}$	q+1	5	-24	-2	4	5	4	-2	18	-2	-3	-2	-3
$196A_{1,3}$	q+1	-2	4	5	-3	-2	11	-2	-10	-2	11	-2	-10
$196A_{1,4}$	q+1	5	4	-2	4	-2	-10	-2	-10	-2	4	5	4
$196A_{1,5}$	q+1	-2	11	-2	-3	-2	18	-2	4	5	-24	-2	-3
$196A_{1,6}$	q+1	-2	4	-2	4	-2	4	5	-10	-2	11	5	-17
$196A_{1,7}$	q+1	-2	-24	-2	4	5	4	-2	-10	5	4	-2	18
$196A_{1,8}$	q+1	-2	18	5	4	-2	-24	5	4	-2	-3	-2	-3
$196A_{1,9}$	q+1	5	-10	-2	4	5	-10	-2	4	-2	-3	-2	11
$196A_{1,10}$	q+1	-2	4	5	-3	-2	11	-2	-10	-2	11	-2	-10
$196A_{1,11}$	q+1	5	-10	-2	4	-2	4	-2	4	-2	4	5	-10
$196A_{1,12}$	q+1	-2	11	-2	-3	-2	-24	-2	4	5	4	-2	11
$196A_{2,4}$	q+1	-2	4	5	-24	-2	-10	-2	18	-2	11	5	-3
$196A_{2,5}$	q+1	5	4	-2	4	-2	18	-2	-10	5	-31	-2	11
$196A_{2,6}$	q+1	-2	-10	-2	-3	-2	-3	5	4	-2	-3	-2	18
$196A_{2,7}$	q+1	-2	4	-2	4	5	4	-2	11	-2	18	5	4
$196A_{2,8}$	q+1	-2	18	5	-24	-2	18	-2	4	5	-17	-2	-3
$196A_{2,9}$	q+1	5	-3	-2	4	-2	11	5	-24	-2	4	-2	4
$196A_{2,10}$	q+1	-2	-3	-2	25	5	4	-2	-10	-2	4	-2	-17
$196A_{2,11}$	q+1	-2	18	5	4	-2	-10	-2	-24	-2	11	5	-3
$196A_{3,6}$	q+1	-2	-10	5	4	5	4	-2	4	-2	4	-2	-10
$196A_{3,7}$	q+1	5	4	5	4	-2	-10	-2	18	-2	-3	-2	-17
$196A_{3,8}$	q+1	5	-10	-2	-3	-2	-3	-2	4	-2	-3	-2	18
$196A_{3,9}$	q+1	-2	-3	-2	-3	-2	-10	-2	18	-2	4	5	-3
$196A_{3,10}$	q+1	-2	11	-2	4	-2	11	-2	4	5	-24	5	-10
$196A_{4,8}$	q+1	-2	4	-2	4	-2	-10	5	-10	5	4	-2	4
$196A_{4,9}$	q+1	-2	-10	-2	4	5	-10	5	4	-2	-3	-2	11

Table-3

Ai,j	$m \equiv 2 \pmod 7$, f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	2	-3	-2	18	-2	-24	-2	60	-2	-24	-2	-24
$196A_{0,1}$	q-13	-2	4	-2	-17	-2	18	-2	-10	-2	-10	12	25
$196A_{0,2}$	q-13	-2	4	-2	4	-2	4	-2	18	12	-10	-2	-10
$196A_{0,3}$	q-13	-2	32	-2	11	-2	4	12	-10	-2	-24	-2	-3
$196A_{0,4}$	q-13	-2	-3	-2	-10	12	4	-2	-10	-2	-10	-2	-10
$196A_{0,5}$	q-13	-2	-10	12	4	-2	11	-2	-10	-2	-3	-2	18
$196A_{0,6}$	q-13	12	18	-2	-24	-2	-3	-2	-10	-2	11	-2	18
$196A_{0,7}$	q-13	-2	-3	-2	-10	-2	4	-2	-24	-2	4	-2	4
$196A_{0,8}$	q-13	-2	-24	-2	11	-2	18	-2	-10	-2	18	12	-3
$196A_{0,9}$	q-13	-2	4	-2	32	-2	-24	-2	-10	12	18	-2	-10
$196A_{0,10}$	q-13	-2	4	-2	-17	-2	-24	12	46	-2	4	-2	-3
$196A_{0,11}$	q-13	-2	-3	-2	-10	12	4	-2	-10	-2	-10	-2	-10
$196A_{0,12}$	q-13	-2	-10	12	4	-2	11	-2	-10	-2	-3	-2	18
$196A_{0,13}$	q-13	12	-10	-2	4	-2	-3	-2	-10	-2	39	-2	-10
$196A_{1,2}$	q+1	5	4	-2	-10	5	4	-2	4	-2	-10	-2	4
$196A_{1,3}$	q+1	-2	4	5	-10	-2	-3	-2	-3	-2	18	-2	-3
$196A_{1,4}$	q+1	5	-24	-2	18	-2	11	-2	4	-2	-3	5	-10
$196A_{1,5}$	q+1	-2	4	-2	11	-2	4	-2	-3	5	11	-2	-24
$196A_{1,6}$	q+1	-2	-10	-2	4	-2	4	5	4	-2	4	5	-10
$196A_{1,7}$	q+1	-2	4	-2	11	5	-24	-2	4	5	-10	-2	11
$196A_{1,8}$	q+1	-2	18	5	4	-2	-3	5	4	-2	-17	-2	-10
$196A_{1,9}$	q+1	5	4	-2	-10	5	4	-2	4	-2	-10	-2	4
$196A_{1,10}$	q+1	-2	-10	5	4	-2	11	-2	-3	-2	-10	-2	11
$196A_{1,11}$	q+1	5	-10	-2	4	-2	-31	-2	4	-2	11	5	18
$196A_{1,12}$	q+1	-2	18	-2	-3	-2	4	-2	-3	5	-3	-2	-10
$196A_{2,4}$	q+1	-2	4	5	18	-2	-17	-2	-24	-2	-3	5	18
$196A_{2,5}$	q+1	5	4	-2	-10	-2	-3	-2	4	5	11	-2	-10
$196A_{2,6}$	q+1	-2	4	-2	-10	-2	-3	5	-3	-2	18	-2	-3
$196A_{2,7}$	q+1	-2	-10	-2	-24	5	4	-2	4	-2	18	5	4
$196A_{2,8}$	q+1	-2	-10	5	4	-2	4	-2	4	5	4	-2	-10
$196A_{2,9}$	q+1	5	4	-2	18	-2	-3	5	4	-2	-3	-2	-24
$196A_{2,10}$	q+1	-2	-10	-2	-3	5	4	-2	25	-2	-17	-2	4
$196A_{2,11}$	q+1	-2	-10	5	4	-2	11	-2	4	-2	-17	5	4
$196A_{3,6}$	q+1	-2	4	5	-10	5	-3	-2	4	-2	11	-2	-10
$196A_{3,7}$	q+1	5	-24	5	-3	-2	4	-2	4	-2	4	-2	11
$196A_{3,8}$	q+1	5	-10	-2	4	-2	11	-2	-3	-2	-10	-2	11
$196A_{3,9}$	q+1	-2	4	-2	11	-2	-24	-2	-3	-2	-3	5	18
$196A_{3,10}$	q+1	-2	11	-2	4	-2	18	-2	4	5	4	5	4
$196A_{4,8}$	q+1	-2	18	-2	4	-2	11	5	-24	5	-3	-2	-10
$196A_{4,9}$	q+1	-2	18	-2	-24	5	-3	5	4	-2	-3	-2	4

Table-4

Ai,j	m ≡ 3(mod 7),f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	-2	-24	-2	-24	-2	18	-2	-24	-2	-3	-2	60
$196A_{0,1}$	q-13	-2	4	-2	-3	-2	11	-2	-24	-2	32	12	-10
$196A_{0,2}$	q-13	-2	-3	-2	18	-2	-24	-2	11	12	18	-2	-10
$196A_{0,3}$	q-13	-2	-24	-2	-10	-2	32	12	18	-2	4	-2	-10
$196A_{0,4}$	q-13	-2	11	-2	18	12	4	-2	-3	-2	-10	-2	-10
$196A_{0,5}$	q-13	-2	18	12	25	-2	-17	-2	-10	-2	4	-2	-10
$196A_{0,6}$	q-13	12	4	-2	-10	-2	-10	-2	-10	-2	-3	-2	-10
$196A_{0,7}$	q-13	-2	4	-2	4	-2	-10	-2	4	-2	-3	-2	-24
$196A_{0,8}$	q-13	-2	-24	-2	-3	-2	-17	-2	4	-2	4	12	46
$196A_{0,9}$	q-13	-2	-3	-2	-10	-2	4	-2	39	12	-10	-2	-10
$196A_{0,10}$	q-13	-2	4	-2	-10	-2	4	12	-10	-2	4	-2	18
$196A_{0,11}$	q-13	-2	11	-2	18	12	4	-2	-3	-2	-10	-2	-10
$196A_{0,12}$	q-13	-2	18	12	-3	-2	11	-2	18	-2	-24	-2	-10
$196A_{0,13}$	q-13	12	4	-2	-10	-2	-10	-2	-10	-2	-3	-2	-10
$196A_{1,2}$	q+1	5	-3	-2	-10	5	-10	-2	11	-2	4	-2	4
$196A_{1,3}$	q+1	-2	-24	5	18	-2	11	-2	-3	-2	4	-2	-3
$196A_{1,4}$	q+1	5	-3	-2	-10	-2	-10	-2	11	-2	4	5	4
$196A_{1,5}$	q+1	-2	-3	-2	-3	-2	-10	-2	18	5	4	-2	-3
$196A_{1,6}$	q+1	-2	-31	-2	18	-2	4	5	11	-2	-10	5	4
$196A_{1,7}$	q+1	-2	4	-2	11	5	-3	-2	4	5	-24	-2	4
$196A_{1,8}$	q+1	-2	18	5	4	-2	4	5	4	-2	11	-2	4
$196A_{1,9}$	q+1	5	11	-2	-10	5	18	-2	-3	-2	-24	-2	4
$196A_{1,10}$	q+1	-2	4	5	-10	-2	-3	-2	-3	-2	18	-2	-3
$196A_{1,11}$	q+1	5	11	-2	4	-2	4	-2	-17	-2	-10	5	4
$196A_{1,12}$	q+1	-2	11	-2	11	-2	4	-2	-10	5	-10	-2	-3
$196A_{2,4}$	q+1	-2	4	5	-10	-2	4	-2	4	-2	-10	5	4
$196A_{2,5}$	q+1	5	4	-2	-10	-2	4	-2	4	5	-10	-2	4
$196A_{2,6}$	q+1	-2	4	-2	4	-2	-3	5	-17	-2	-10	-2	25
$196A_{2,7}$	q+1	-2	-3	-2	-10	5	4	-2	-17	-2	18	5	4
$196A_{2,8}$	q+1	-2	11	5	-10	-2	4	-2	-3	5	18	-2	-24
$196A_{2,9}$	q+1	5	-24	-2	11	-2	11	5	-10	-2	4	-2	4
$196A_{2,10}$	q+1	-2	-3	-2	-3	5	-10	-2	18	-2	4	-2	-3
$196A_{2,11}$	q+1	-2	4	5	4	-2	-10	-2	-10	-2	4	5	4
$196A_{3,6}$	q+1	-2	-3	5	4	5	-24	-2	-3	-2	18	-2	4
$196A_{3,7}$	q+1	5	4	5	4	-2	-24	-2	18	-2	-10	-2	4
$196A_{3,8}$	q+1	5	4	-2	-24	-2	11	-2	11	-2	4	-2	-3
$196A_{3,9}$	q+1	-2	11	-2	11	-2	4	-2	-10	-2	-10	5	-3
$196A_{3,10}$	q+1	-2	-3	-2	-24	-2	18	-2	-3	5	4	5	4
$196A_{4,8}$	q+1	-2	-17	-2	18	-2	18	5	-3	5	4	-2	-24
$196A_{4,9}$	q+1	-2	4	-2	4	5	-10	5	-10	-2	4	-2	4

Table-5

Ai,j	m ≡ 4(mod 7),f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	-2	60	-2	-3	-2	-24	-2	18	-2	-24	-2	-24
$196A_{0,1}$	q-13	-2	-10	-2	-3	-2	-10	-2	-10	-2	-10	12	4
$196A_{0,2}$	q-13	-2	-10	-2	-24	-2	18	-2	11	12	-3	-2	18
$196A_{0,3}$	q-13	-2	-10	-2	-10	-2	-3	12	4	-2	18	-2	11
$196A_{0,4}$	q-13	-2	18	-2	4	12	-10	-2	4	-2	-10	-2	4
$196A_{0,5}$	q-13	-2	-10	12	-10	-2	39	-2	4	-2	-10	-2	-3
$196A_{0,6}$	q-13	12	46	-2	4	-2	4	-2	-17	-2	-3	-2	-24
$196A_{0,7}$	q-13	-2	-24	-2	-3	-2	4	-2	-10	-2	4	-2	4
$196A_{0,8}$	q-13	-2	-10	-2	-3	-2	-10	-2	-10	-2	-10	12	4
$196A_{0,9}$	q-13	-2	-10	-2	4	-2	-10	-2	-17	12	25	-2	18
$196A_{0,10}$	q-13	-2	-10	-2	-10	-2	-3	12	4	-2	18	-2	11
$196A_{0,11}$	q-13	-2	-10	-2	4	12	18	-2	32	-2	-10	-2	-24
$196A_{0,12}$	q-13	-2	-10	12	18	-2	11	-2	-24	-2	18	-2	-3
$196A_{0,13}$	q-13	12	-10	-2	32	-2	-24	-2	11	-2	-3	-2	4
$196A_{1,2}$	q+1	5	4	-2	4	5	11	-2	-10	-2	-10	-2	-3
$196A_{1,3}$	q+1	-2	-3	5	-10	-2	-10	-2	4	-2	11	-2	11
$196A_{1,4}$	q+1	5	4	-2	-10	-2	-17	-2	4	-2	4	5	11
$196A_{1,5}$	q+1	-2	-3	-2	18	-2	-3	-2	-3	5	-10	-2	4
$196A_{1,6}$	q+1	-2	4	-2	-24	-2	-3	5	18	-2	-10	5	11
$196A_{1,7}$	q+1	-2	4	-2	11	5	4	-2	4	5	4	-2	18
$196A_{1,8}$	q+1	-2	4	5	-24	-2	4	5	-3	-2	11	-2	4
$196A_{1,9}$	q+1	5	4	-2	-10	5	11	-2	4	-2	18	-2	-31
$196A_{1,10}$	q+1	-2	-3	5	4	-2	18	-2	-10	-2	-3	-2	-3
$196A_{1,11}$	q+1	5	4	-2	4	-2	11	-2	-10	-2	-10	5	-3
$196A_{1,12}$	q+1	-2	-3	-2	4	-2	-3	-2	11	5	18	-2	-24
$196A_{2,4}$	q+1	-2	4	5	-10	-2	4	-2	4	-2	-10	5	4
$196A_{2,5}$	q+1	5	4	-2	4	-2	-10	-2	-10	5	4	-2	4
$196A_{2,6}$	q+1	-2	-3	-2	4	-2	18	5	-10	-2	-3	-2	-3
$196A_{2,7}$	q+1	-2	4	-2	4	5	-10	-2	11	-2	11	5	-24
$196A_{2,8}$	q+1	-2	-24	5	18	-2	-3	-2	4	5	-10	-2	11
$196A_{2,9}$	q+1	5	4	-2	18	-2	-17	5	4	-2	-10	-2	-3
$196A_{2,10}$	q+1	-2	25	-2	-10	5	-17	-2	-3	-2	4	-2	4
$196A_{2,11}$	q+1	-2	4	5	-10	-2	4	-2	4	-2	-10	5	4
$196A_{3,6}$	q+1	-2	4	5	18	5	-3	-2	-24	-2	4	-2	-3
$196A_{3,7}$	q+1	5	4	5	4	-2	-3	-2	18	-2	-24	-2	-3
$196A_{3,8}$	q+1	5	-3	-2	-10	-2	-10	-2	4	-2	11	-2	11
$196A_{3,9}$	q+1	-2	-3	-2	4	-2	11	-2	11	-2	-24	5	4
$196A_{3,10}$	q+1	-2	4	-2	-10	-2	18	-2	-24	5	4	5	4
$196A_{4,8}$	q+1	-2	-24	-2	4	-2	-3	5	18	5	18	-2	-17
$196A_{4,9}$	q+1	-2	4	-2	4	5	-10	5	-10	-2	4	-2	4

Table-6

Ai,j	m ≡ 5(mod 7),f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	-2	-24	-2	-24	-2	60	-2	-24	-2	18	-2	-3
$196A_{0,1}$	q-13	-2	-10	-2	39	-2	-10	-2	-3	-2	4	12	-10
$196A_{0,2}$	q-13	-2	18	-2	-3	-2	-10	-2	11	12	4	-2	-10
$196A_{0,3}$	q-13	-2	-10	-2	-10	-2	-10	12	4	-2	-10	-2	-3
$196A_{0,4}$	q-13	-2	-3	-2	4	12	46	-2	-24	-2	-17	-2	4
$196A_{0,5}$	q-13	-2	-10	12	18	-2	-10	-2	-24	-2	32	-2	4
$196A_{0,6}$	q-13	12	-3	-2	18	-2	-10	-2	18	-2	11	-2	-24
$196A_{0,7}$	q-13	-2	4	-2	4	-2	-24	-2	4	-2	-10	-2	-3
$196A_{0,8}$	q-13	-2	18	-2	11	-2	-10	-2	-3	-2	-24	12	18
$196A_{0,9}$	q-13	-2	18	-2	-3	-2	-10	-2	11	12	4	-2	-10
$196A_{0,10}$	q-13	-2	-10	-2	-10	-2	-10	12	4	-2	-10	-2	-3
$196A_{0,11}$	q-13	-2	-3	-2	-24	12	-10	-2	4	-2	11	-2	32
$196A_{0,12}$	q-13	-2	-10	12	-10	-2	18	-2	4	-2	4	-2	4
$196A_{0,13}$	q-13	12	25	-2	-10	-2	-10	-2	18	-2	-17	-2	4
$196A_{1,2}$	q+1	5	4	-2	-10	5	4	-2	4	-2	-10	-2	4
$196A_{1,3}$	q+1	-2	-10	5	-3	-2	-3	-2	4	-2	-3	-2	18
$196A_{1,4}$	q+1	5	18	-2	11	-2	4	-2	-31	-2	4	5	-10
$196A_{1,5}$	q+1	-2	11	-2	-10	-2	-3	-2	11	5	4	-2	-10
$196A_{1,6}$	q+1	-2	4	-2	-10	-2	4	5	4	-2	-10	5	4
$196A_{1,7}$	q+1	-2	-10	-2	-17	5	4	-2	-3	5	4	-2	18
$196A_{1,8}$	q+1	-2	11	5	-10	-2	4	5	-24	-2	11	-2	4
$196A_{1,9}$	q+1	5	-10	-2	4	5	4	-2	4	-2	4	-2	-10
$196A_{1,10}$	q+1	-2	-24	5	11	-2	-3	-2	4	-2	11	-2	4
$196A_{1,11}$	q+1	5	-10	-2	-3	-2	4	-2	11	-2	18	5	-24
$196A_{1,12}$	q+1	-2	-3	-2	18	-2	-3	-2	-3	5	-10	-2	4
$196A_{2,4}$	q+1	-2	18	5	-3	-2	-24	-2	-17	-2	18	5	4
$196A_{2,5}$	q+1	5	4	-2	-17	-2	4	-2	11	5	4	-2	-10
$196A_{2,6}$	q+1	-2	4	-2	-17	-2	25	5	4	-2	-3	-2	-10
$196A_{2,7}$	q+1	-2	-24	-2	-3	5	4	-2	-3	-2	18	5	4
$196A_{2,8}$	q+1	-2	-10	5	4	-2	4	-2	4	5	4	-2	-10
$196A_{2,9}$	q+1	5	4	-2	18	-2	4	5	4	-2	-24	-2	-10
$196A_{2,10}$	q+1	-2	-3	-2	18	5	-3	-2	-3	-2	-10	-2	4
$196A_{2,11}$	q+1	-2	-10	5	11	-2	4	-2	-3	-2	-10	5	4
$196A_{3,6}$	q+1	-2	-10	5	11	5	4	-2	-3	-2	-10	-2	4
$196A_{3,7}$	q+1	5	4	5	4	-2	4	-2	18	-2	4	-2	11
$196A_{3,8}$	q+1	5	18	-2	-3	-2	-3	-2	-24	-2	11	-2	4
$196A_{3,9}$	q+1	-2	11	-2	-10	-2	-3	-2	11	-2	4	5	-10
$196A_{3,10}$	q+1	-2	11	-2	4	-2	4	-2	4	5	-3	5	-24
$196A_{4,8}$	q+1	-2	-10	-2	-3	-2	-24	5	11	5	4	-2	18
$196A_{4,9}$	q+1	-2	4	-2	-3	5	4	5	-3	-2	-24	-2	18

Table-7

Ai,j	m ≡ 6(mod 7),f even												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-41	-2	-24	-2	-24	-2	-3	-2	-24	-2	60	-2	18
$196A_{0,1}$	q-13	-2	-3	-2	11	-2	-10	-2	18	-2	-10	12	4
$196A_{0,2}$	q-13	-2	4	-2	-24	-2	4	-2	-3	12	46	-2	-17
$196A_{0,3}$	q-13	-2	-10	-2	18	-2	4	12	25	-2	-10	-2	-17
$196A_{0,4}$	q-13	-2	11	-2	-3	12	18	-2	18	-2	-10	-2	-24
$196A_{0,5}$	q-13	-2	-10	12	4	-2	-3	-2	-10	-2	-10	-2	-10
$196A_{0,6}$	q-13	12	-10	-2	4	-2	4	-2	-10	-2	18	-2	4
$196A_{0,7}$	q-13	-2	4	-2	4	-2	-3	-2	4	-2	-24	-2	-10
$196A_{0,8}$	q-13	-2	-3	-2	11	-2	-10	-2	18	-2	-10	12	4
$196A_{0,9}$	q-13	-2	-24	-2	4	-2	32	-2	-3	12	-10	-2	11
$196A_{0,10}$	q-13	-2	18	-2	18	-2	-24	12	-3	-2	-10	-2	11
$196A_{0,11}$	q-13	-2	39	-2	-3	12	-10	-2	-10	-2	-10	-2	4
$196A_{0,12}$	q-13	-2	-10	12	4	-2	-3	-2	-10	-2	-10	-2	-10
$196A_{0,13}$	q-13	12	18	-2	-24	-2	4	-2	-10	-2	-10	-2	32
$196A_{1,2}$	q+1	5	-3	-2	-3	5	18	-2	4	-2	4	-2	-24
$196A_{1,3}$	q+1	-2	11	5	4	-2	4	-2	-24	-2	-3	-2	11
$196A_{1,4}$	q+1	5	-10	-2	4	-2	4	-2	4	-2	4	5	-10
$196A_{1,5}$	q+1	-2	-10	-2	11	-2	-10	-2	11	5	-3	-2	4
$196A_{1,6}$	q+1	-2	11	-2	-3	-2	4	5	-10	-2	4	5	-10
$196A_{1,7}$	q+1	-2	-3	-2	-3	5	4	-2	-24	5	4	-2	18
$196A_{1,8}$	q+1	-2	18	5	4	-2	-10	5	4	-2	4	-2	-24
$196A_{1,9}$	q+1	5	-17	-2	11	5	-10	-2	4	-2	4	-2	4
$196A_{1,10}$	q+1	-2	-3	5	-24	-2	4	-2	18	-2	-3	-2	11
$196A_{1,11}$	q+1	5	4	-2	4	-2	-10	-2	-10	-2	4	5	4
$196A_{1,12}$	q+1	-2	-10	-2	11	-2	-10	-2	11	5	-3	-2	4
$196A_{2,4}$	q+1	-2	-3	5	11	-2	18	-2	-10	-2	-24	5	4
$196A_{2,5}$	q+1	5	-3	-2	11	-2	-24	-2	-10	5	4	-2	18
$196A_{2,6}$	q+1	-2	-17	-2	4	-2	-10	5	4	-2	25	-2	-3
$196A_{2,7}$	q+1	-2	4	-2	4	5	-24	-2	11	-2	4	5	-3
$196A_{2,8}$	q+1	-2	-3	5	-17	-2	4	-2	18	5	-24	-2	18
$196A_{2,9}$	q+1	5	4	-2	18	-2	11	5	4	-2	4	-2	4
$196A_{2,10}$	q+1	-2	18	-2	-3	5	4	-2	-3	-2	-3	-2	-10
$196A_{2,11}$	q+1	-2	11	5	-31	-2	-10	-2	18	-2	4	5	4
$196A_{3,6}$	q+1	-2	-10	5	4	5	4	-2	4	-2	4	-2	-10
$196A_{3,7}$	q+1	5	-10	5	-24	-2	4	-2	11	-2	4	-2	11
$196A_{3,8}$	q+1	5	-3	-2	4	-2	18	-2	-10	-2	-3	-2	-3
$196A_{3,9}$	q+1	-2	18	-2	-3	-2	4	-2	-3	-2	-3	5	-10
$196A_{3,10}$	q+1	-2	-17	-2	-3	-2	18	-2	-10	5	4	5	4
$196A_{4,8}$	q+1	-2	4	-2	4	-2	-10	5	-10	5	4	-2	4
$196A_{4,9}$	q+1	-2	11	-2	-3	5	4	5	-10	-2	4	-2	-10

Table-8

Ai,j	m ≡0(mod 7),f odd											
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	-10	-2	-10	-2	-10	-2	-10	-2	-10	-2	-10
$196A_{0,1}$	q+1	-2	-24	-2	25	-2	-24	-2	18	-2	-3	12	4
$196A_{0,2}$	q+1	-2	-10	-2	32	-2	-3	-2	-3	12	-24	-2	4
$196A_{0,3}$	q+1	-2	-3	-2	-24	-2	-24	12	4	-2	18	-2	25
$196A_{0,4}$	q+1	-2	-3	-2	-10	12	-24	-2	32	-2	4	-2	-3
$196A_{0,5}$	q+1	-2	-24	12	4	-2	25	-2	-3	-2	-24	-2	18
$196A_{0,6}$	q+1	12	-24	-2	-3	-2	-10	-2	4	-2	-3	-2	32
$196A_{0,7}$	q+1	-2	18	-2	18	-2	18	-2	18	-2	18	-2	18
$196A_{0,8}$	q+1	-2	32	-2	-3	-2	4	-2	-10	-2	-3	12	-24
$196A_{0,9}$	q+1	-2	18	-2	-24	-2	-3	-2	25	12	4	-2	-24
$196A_{0,10}$	q+1	-2	-3	-2	4	-2	32	12	-24	-2	-10	-2	-3
$196A_{0,11}$	q+1	-2	25	-2	18	12	4	-2	-24	-2	-24	-2	-3
$196A_{0,12}$	q+1	-2	4	12	-24	-2	-3	-2	-3	-2	32	-2	-10
$196A_{0,13}$	q+1	12	4	-2	-3	-2	18	-2	-24	-2	25	-2	-24
$196A_{1,0}$	q-13	-2	-10	-2	-3	5	4	-2	-10	5	-3	-2	32
$196A_{1,1}$	q-13	-2	32	5	-3	-2	-10	5	4	-2	-3	-2	-10
$196A_{1,2}$	q+1	5	-3	-2	4	5	-3	-2	-3	-2	4	-2	-3
$196A_{1,3}$	q+1	-2	-17	5	-3	-2	18	-2	11	-2	4	-2	-10
$196A_{1,4}$	q+1	5	4	-2	-3	-2	-3	-2	-3	-2	-3	5	4
$196A_{1,5}$	q+1	-2	4	-2	18	-2	-17	-2	-10	5	11	-2	-3
$196A_{1,6}$	q+1	-2	-3	-2	-10	-2	11	5	11	-2	-10	5	-3
$196A_{1,10}$	q+1	-2	-3	5	11	-2	-10	-2	-17	-2	18	-2	4
$196A_{1,11}$	q+1	5	4	-2	-3	-2	-3	-2	-3	-2	-3	5	4
$196A_{1,12}$	q+1	-2	-10	-2	4	-2	11	-2	18	5	-3	-2	-17
$196A_{1,13}$	q+1	-2	-3	-2	4	-2	-3	5	-3	-2	4	5	-3
$196A_{2,0}$	q-13	-2	-10	-2	-10	5	-3	-2	-3	-2	32	5	4
$196A_{2,1}$	q+1	-2	-3	5	-10	-2	11	-2	11	5	-10	-2	-3
$196A_{2,2}$	q-13	5	4	-2	32	-2	-3	5	-3	-2	-10	-2	-10
$196A_{2,3}$	q+1	-2	4	-2	4	5	-3	-2	4	-2	-3	-2	-3
$196A_{2,4}$	q+1	-2	-3	5	-3	-2	4	-2	4	-2	-3	5	-3
$196A_{2,5}$	q+1	5	11	-2	-3	-2	-10	-2	-10	5	-3	-2	11
$196A_{2,12}$	q+1	5	-3	-2	-3	-2	4	-2	4	5	-3	-2	-3
$196A_{2,13}$	q+1	-2	-3	-2	-3	-2	4	5	-3	-2	4	-2	4
$196A_{3,0}$	q-13	5	-3	5	4	-2	-10	-2	32	-2	-10	-2	-3
$196A_{3,1}$	q+1	5	11	-2	-17	-2	4	-2	-3	-2	-10	-2	18
$196A_{3,2}$	q+1	-2	18	-2	-10	-2	-3	-2	4	-2	-17	5	11
$196A_{3,3}$	q-13	-2	-3	-2	-10	-2	32	-2	-10	5	4	5	-3
$196A_{3,4}$	q+1	-2	-10	-2	11	-2	-3	5	-3	5	11	-2	-10
$196A_{4,1}$	q+1	-2	-10	-2	11	-2	-3	5	-3	5	11	-2	-10
$196A_{4,2}$	q+1	-2	11	-2	-3	5	-10	5	-10	-2	-3	-2	11

Table-9

Ai,j	m ≡1(mod 7),f odd												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	4	-2	18	-2	-10	-2	-3	-2	-10	-2	-10
$196A_{0,1}$	q+1	-2	-10	-2	-10	-2	4	-2	-10	-2	-10	12	32
$196A_{0,2}$	q+1	-2	4	-2	4	-2	4	-2	11	12	18	-2	4
$196A_{0,3}$	q+1	-2	18	-2	4	-2	-24	12	4	-2	-3	-2	-3
$196A_{0,4}$	q+1	-2	25	-2	4	12	-3	-2	-10	-2	-24	-2	4
$196A_{0,5}$	q+1	-2	11	12	-24	-2	11	-2	-10	-2	18	-2	-10
$196A_{0,6}$	q+1	12	18	-2	4	-2	-24	-2	4	-2	-3	-2	-3
$196A_{0,7}$	q+1	-2	-24	-2	-66	-2	18	-2	-3	-2	18	-2	18
$196A_{0,8}$	q+1	-2	-38	-2	18	-2	4	-2	-10	-2	18	12	4
$196A_{0,9}$	q+1	-2	4	-2	4	-2	4	-2	11	12	18	-2	4
$196A_{0,10}$	q+1	-2	-10	-2	4	-2	4	12	32	-2	-3	-2	-31
$196A_{0,11}$	q+1	-2	-3	-2	4	12	25	-2	18	-2	-24	-2	-24
$196A_{0,12}$	q+1	-2	-17	12	32	-2	11	-2	-38	-2	-10	-2	18
$196A_{0,13}$	q+1	12	18	-2	4	-2	-24	-2	4	-2	-3	-2	-3
$196A_{1,0}$	q-13	-2	18	-2	4	5	-10	-2	4	5	-10	-2	4
$196A_{1,1}$	q-13	-2	4	5	-10	-2	18	5	-10	-2	11	-2	-3
$196A_{1,2}$	q+1	-2	18	-2	4	5	-10	-2	4	5	-10	-2	4
$196A_{1,3}$	q+1	-2	-10	5	-3	-2	-3	-2	4	-2	-3	-2	18
$196A_{1,4}$	q+1	5	4	-2	4	-2	-10	-2	-10	-2	4	5	4
$196A_{1,5}$	q+1	-2	-3	-2	11	-2	18	-2	4	5	-10	-2	-17
$196A_{1,6}$	q+1	-2	18	-2	-10	-2	4	5	-10	-2	-3	5	-3
$196A_{1,10}$	q+1	-2	-10	5	-3	-2	-3	-2	4	-2	-3	-2	18
$196A_{1,11}$	q+1	5	-10	-2	4	-2	4	-2	4	-2	4	5	-10
$196A_{1,12}$	q+1	-2	-3	-2	11	-2	-24	-2	4	5	18	-2	-3
$196A_{1,13}$	q+1	-2	-10	-2	-10	-2	4	5	18	-2	-17	5	11
$196A_{2,0}$	q-13	-2	-10	-2	-10	5	-10	-2	-3	-2	4	5	-10
$196A_{2,1}$	q+1	-2	-24	5	18	-2	-10	-2	4	5	11	-2	-3
$196A_{2,2}$	q-13	5	-3	-2	18	-2	-3	5	18	-2	-10	-2	-10
$196A_{2,3}$	q+1	-2	11	-2	-17	5	-10	-2	18	-2	-10	-2	11
$196A_{2,4}$	q+1	-2	-10	5	-10	-2	4	-2	18	-2	-17	5	11
$196A_{2,5}$	q+1	5	4	-2	18	-2	4	-2	-24	5	-17	-2	11
$196A_{2,12}$	q+1	5	4	-2	-10	-2	-24	-2	4	5	25	-2	-3
$196A_{2,13}$	q+1	-2	4	-2	-3	-2	11	5	-10	-2	11	-2	-10
$196A_{3,0}$	q-13	5	-10	5	-10	-2	4	-2	4	-2	-3	-2	25
$196A_{3,1}$	q+1	5	4	-2	-3	-2	11	-2	-10	-2	11	-2	-10
$196A_{3,2}$	q+1	-2	11	-2	11	-2	4	-2	-10	-2	-10	5	-3
$196A_{3,3}$	q-13	-2	-3	-2	-10	-2	11	-2	-10	5	18	5	4
$196A_{3,4}$	q+1	-2	4	-2	4	-2	-10	5	-10	5	4	-2	4
$196A_{4,1}$	q+1	-2	-10	-2	4	-2	4	5	4	5	4	-2	-10
$196A_{4,2}$	q+1	-2	18	-2	-10	5	4	5	-10	-2	-3	-2	-3

Table-10

Ai,j	m ≡2(mod 7),f odd												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	-3	-2	4	-2	-10	-2	18	-2	-10	-2	-10
$196A_{0,1}$	q+1	-2	18	-2	-3	-2	-24	-2	4	-2	-24	12	25
$196A_{0,2}$	q+1	-2	-10	-2	-38	-2	18	-2	18	12	4	-2	4
$196A_{0,3}$	q+1	-2	-10	-2	11	-2	18	12	-24	-2	-10	-2	11
$196A_{0,4}$	q+1	-2	11	-2	4	12	18	-2	4	-2	4	-2	4
$196A_{0,5}$	q+1	-2	4	12	18	-2	-3	-2	4	-2	-3	-2	-24
$196A_{0,6}$	q+1	12	32	-2	-10	-2	-3	-2	4	-2	-31	-2	4
$196A_{0,7}$	q+1	-2	-3	-2	-24	-2	18	-2	-66	-2	18	-2	18
$196A_{0,8}$	q+1	-2	-10	-2	25	-2	-24	-2	4	-2	4	12	-3
$196A_{0,9}$	q+1	-2	-10	-2	-10	-2	-10	-2	-10	12	32	-2	4
$196A_{0,10}$	q+1	-2	-38	-2	-17	-2	-10	12	32	-2	18	-2	11
$196A_{0,11}$	q+1	-2	11	-2	4	12	18	-2	4	-2	4	-2	4
$196A_{0,12}$	q+1	-2	4	12	18	-2	-3	-2	4	-2	-3	-2	-24
$196A_{0,13}$	q+1	12	4	-2	18	-2	-3	-2	4	-2	-3	-2	-24
$196A_{1,0}$	q-13	-2	-10	-2	-3	5	18	-2	-10	5	4	-2	11
$196A_{1,1}$	q-13	-2	4	5	-10	-2	-3	5	-10	-2	25	-2	4
$196A_{1,2}$	q+1	5	-10	-2	4	5	4	-2	4	-2	4	-2	-10
$196A_{1,3}$	q+1	-2	4	5	-10	-2	-3	-2	-3	-2	18	-2	-3
$196A_{1,4}$	q+1	5	4	-2	4	-2	25	-2	-10	-2	-3	5	-24
$196A_{1,5}$	q+1	-2	-10	-2	11	-2	-10	-2	11	5	-3	-2	4
$196A_{1,6}$	q+1	-2	-10	-2	4	-2	4	5	4	-2	4	5	-10
$196A_{1,10}$	q+1	-2	-10	5	4	-2	11	-2	-3	-2	-10	-2	11
$196A_{1,11}$	q+1	5	18	-2	-10	-2	-17	-2	-10	-2	11	5	4
$196A_{1,12}$	q+1	-2	4	-2	-3	-2	-10	-2	11	5	-17	-2	18
$196A_{1,13}$	q+1	-2	4	-2	-10	-2	4	5	4	-2	-10	5	4
$196A_{2,0}$	q-13	-2	4	-2	18	5	-10	-2	4	-2	4	5	-10
$196A_{2,1}$	q+1	-2	4	5	-10	-2	4	-2	4	5	-10	-2	4
$196A_{2,2}$	q-13	5	-10	-2	4	-2	11	5	-10	-2	-3	-2	18
$196A_{2,3}$	q+1	-2	18	-2	11	5	-10	-2	-17	-2	11	-2	-10
$196A_{2,4}$	q+1	-2	18	5	-10	-2	-17	-2	-10	-2	11	5	4
$196A_{2,5}$	q+1	5	4	-2	-24	-2	11	-2	18	5	-3	-2	-10
$196A_{2,12}$	q+1	5	4	-2	4	-2	-3	-2	-10	5	-17	-2	18
$196A_{2,13}$	q+1	-2	-10	-2	4	-2	11	5	-3	-2	-10	-2	11
$196A_{3,0}$	q-13	5	18	5	-3	-2	-10	-2	18	-2	-10	-2	-3
$196A_{3,1}$	q+1	5	4	-2	-10	-2	-3	-2	-3	-2	18	-2	-3
$196A_{3,2}$	q+1	-2	4	-2	-3	-2	18	-2	11	-2	-3	5	-24
$196A_{3,3}$	q-13	-2	-3	-2	-10	-2	4	-2	-10	5	-10	5	-10
$196A_{3,4}$	q+1	-2	-10	-2	18	-2	-3	5	-10	5	-3	-2	4
$196A_{4,1}$	q+1	-2	-24	-2	4	-2	-17	5	18	5	11	-2	4
$196A_{4,2}$	q+1	-2	-10	-2	18	5	-3	5	-10	-2	-3	-2	4

Table-11

Ai,j	m ≡3(mod 7),f odd												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	-10	-2	-10	-2	4	-2	-10	-2	-3	-2	18
$196A_{0,1}$	q+1	-2	18	-2	11	-2	11	-2	-10	-2	-10	12	-24
$196A_{0,2}$	q+1	-2	-3	-2	4	-2	-10	-2	-31	12	32	-2	4
$196A_{0,3}$	q+1	-2	-10	-2	4	-2	-10	12	32	-2	-10	-2	-10
$196A_{0,4}$	q+1	-2	-3	-2	-24	12	18	-2	-3	-2	4	-2	4
$196A_{0,5}$	q+1	-2	-24	12	25	-2	-3	-2	-24	-2	18	-2	4
$196A_{0,6}$	q+1	12	18	-2	4	-2	4	-2	4	-2	11	-2	4
$196A_{0,7}$	q+1	-2	18	-2	18	-2	-24	-2	18	-2	-3	-2	-66
$196A_{0,8}$	q+1	-2	-10	-2	11	-2	-17	-2	18	-2	-38	12	32
$196A_{0,9}$	q+1	-2	-3	-2	-24	-2	18	-2	-3	12	4	-2	4
$196A_{0,10}$	q+1	-2	18	-2	4	-2	-38	12	4	-2	-10	-2	18
$196A_{0,11}$	q+1	-2	-3	-2	-24	12	18	-2	-3	-2	4	-2	4
$196A_{0,12}$	q+1	-2	-24	12	-3	-2	25	-2	4	-2	-10	-2	4
$196A_{0,13}$	q+1	12	18	-2	4	-2	4	-2	4	-2	11	-2	4
$196A_{1,0}$	q-13	-2	-10	-2	-3	5	-3	-2	-10	5	18	-2	18
$196A_{1,1}$	q-13	-2	4	5	-10	-2	-10	5	-10	-2	-3	-2	-10
$196A_{1,2}$	q+1	5	-17	-2	4	5	-10	-2	11	-2	18	-2	-10
$196A_{1,3}$	q+1	-2	-10	5	4	-2	11	-2	-3	-2	-10	-2	11
$196A_{1,4}$	q+1	5	-17	-2	4	-2	-10	-2	11	-2	18	5	-10
$196A_{1,5}$	q+1	-2	-3	-2	-3	-2	-10	-2	18	5	4	-2	-3
$196A_{1,6}$	q+1	-2	-3	-2	4	-2	18	5	-3	-2	-10	5	-10
$196A_{1,10}$	q+1	-2	18	5	-24	-2	-3	-2	-3	-2	4	-2	11
$196A_{1,11}$	q+1	5	-3	-2	18	-2	4	-2	-17	-2	4	5	-10
$196A_{1,12}$	q+1	-2	11	-2	11	-2	4	-2	-10	5	-10	-2	-3
$196A_{1,13}$	q+1	-2	25	-2	-24	-2	4	5	-3	-2	4	5	-10
$196A_{2,0}$	q+1	-2	-3	-2	4	5	-10	-2	25	-2	4	5	-10
$196A_{2,1}$	q+1	-2	-17	5	4	-2	4	-2	11	5	-24	-2	18
$196A_{2,2}$	q+1	5	18	-2	11	-2	-3	5	4	-2	-10	-2	-10
$196A_{2,3}$	q+1	-2	11	-2	11	5	4	-2	-10	-2	-10	-2	-3
$196A_{2,4}$	q+1	-2	4	5	-10	-2	4	-2	4	-2	-10	5	4
$196A_{2,5}$	q+1	5	4	-2	4	-2	-10	-2	-10	5	4	-2	4
$196A_{2,12}$	q+1	5	4	-2	4	-2	-10	-2	-10	5	4	-2	4
$196A_{2,13}$	q+1	-2	-10	-2	-10	-2	11	5	11	-2	18	-2	-17
$196A_{3,0}$	q-13	5	-10	5	-10	-2	18	-2	4	-2	4	-2	4
$196A_{3,1}$	q+1	5	-10	-2	18	-2	-3	-2	-17	-2	4	-2	11
$196A_{3,2}$	q+1	-2	-3	-2	-3	-2	-10	-2	18	-2	4	5	-3
$196A_{3,3}$	q-13	-2	11	-2	18	-2	4	-2	-3	5	-10	5	-10
$196A_{3,4}$	q+1	-2	-3	-2	4	-2	18	5	-3	5	-10	-2	-10
$196A_{4,1}$	q+1	-2	11	-2	-10	-2	-24	5	-3	5	4	-2	18
$196A_{4,2}$	q+1	-2	4	-2	-10	5	4	5	4	-2	-10	-2	4

Table-12

Ai,j	m ≡4(mod 7),f odd												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	18	-2	-3	-2	-10	-2	4	-2	-10	-2	-10
$196A_{0,1}$	q+1	-2	4	-2	11	-2	4	-2	4	-2	4	12	18
$196A_{0,2}$	q+1	-2	4	-2	-10	-2	4	-2	25	12	-3	-2	-24
$196A_{0,3}$	q+1	-2	4	-2	4	-2	-3	12	18	-2	-24	-2	-3
$196A_{0,4}$	q+1	-2	18	-2	-10	12	4	-2	-38	-2	4	-2	18
$196A_{0,5}$	q+1	-2	4	12	4	-2	-3	-2	18	-2	-24	-2	-3
$196A_{0,6}$	q+1	12	32	-2	-38	-2	18	-2	-17	-2	11	-2	-10
$196A_{0,7}$	q+1	-2	-66	-2	-3	-2	18	-2	-24	-2	18	-2	18
$196A_{0,8}$	q+1	-2	4	-2	11	-2	4	-2	4	-2	4	12	18
$196A_{0,9}$	q+1	-2	4	-2	18	-2	-24	-2	-3	12	25	-2	-24
$196A_{0,10}$	q+1	-2	4	-2	4	-2	-3	12	18	-2	-24	-2	-3
$196A_{0,11}$	q+1	-2	-10	-2	-10	12	32	-2	-10	-2	4	-2	-10
$196A_{0,12}$	q+1	-2	4	12	32	-2	-31	-2	-10	-2	4	-2	-3
$196A_{0,13}$	q+1	12	-24	-2	-10	-2	-10	-2	11	-2	11	-2	18
$196A_{1,0}$	q-13	-2	-10	-2	-3	5	10	-2	-10	5	-10	-2	4
$196A_{1,1}$	q-13	-2	18	5	18	-2	-10	5	-3	-2	-3	-2	-10
$196A_{1,2}$	q+1	5	-10	-2	4	5	-3	-2	4	-2	-24	-2	25
$196A_{1,3}$	q+1	-2	-3	5	-10	-2	-10	-2	4	-2	11	-2	11
$196A_{1,4}$	q+1	5	-10	-2	4	-2	-17	-2	4	-2	18	5	-3
$196A_{1,5}$	q+1	-2	11	-2	4	-2	-3	-2	-3	5	-24	-2	18
$196A_{1,6}$	q+1	-2	-10	-2	-10	-2	-3	5	18	-2	4	5	-3
$196A_{1,10}$	q+1	-2	-3	5	4	-2	18	-2	-10	-2	-3	-2	-3
$196A_{1,11}$	q+1	5	-10	-2	18	-2	11	-2	-10	-2	4	5	-17
$196A_{1,12}$	q+1	-2	11	-2	-10	-2	-3	-2	11	5	4	-2	-10
$196A_{1,13}$	q+1	-2	-10	-2	18	-2	11	5	-10	-2	4	5	-17
$196A_{2,0}$	q+1	-2	-10	-2	-10	5	4	-2	-3	-2	11	5	18
$196A_{2,1}$	q+1	-2	18	5	-24	-2	11	-2	4	5	4	-2	-17
$196A_{2,2}$	q+1	5	-10	-2	4	-2	25	5	-10	-2	4	-2	-3
$196A_{2,3}$	q+1	-2	-17	-2	18	5	11	-2	11	-2	-10	-2	-10
$196A_{2,4}$	q+1	-2	4	5	4	-2	-10	-2	-10	-2	4	5	4
$196A_{2,5}$	q+1	5	4	-2	4	-2	-10	-2	-10	5	4	-2	4
$196A_{2,12}$	q+1	5	4	-2	-10	-2	4	-2	4	5	-10	-2	4
$196A_{2,13}$	q+1	-2	-3	-2	-10	-2	-10	5	4	-2	11	-2	11
$196A_{3,0}$	q-13	5	-10	5	-10	-2	-3	-2	4	-2	18	-2	11
$196A_{3,1}$	q+1	5	-3	-2	4	-2	18	-2	-10	-2	-3	-2	-3
$196A_{3,2}$	q+1	-2	11	-2	4	-2	-17	-2	-3	-2	18	5	-10
$196A_{3,3}$	q-13	-2	4	-2	4	-2	4	-2	18	5	-10	5	-10
$196A_{3,4}$	q+1	-2	-10	-2	-10	-2	-3	5	18	5	4	-2	-3
$196A_{4,1}$	q+1	-2	18	-2	4	-2	-3	5	-24	5	-10	-2	11
$196A_{4,2}$	q+1	-2	4	-2	-10	5	4	5	4	-2	-10	-2	4

Table-13

Ai,j	m ≡5(mod 7),f odd												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	-10	-2	-10	-2	18	-2	-10	-2	4	-2	-3
$196A_{0,1}$	q+1	-2	-24	-2	-3	-2	4	-2	-3	-2	18	12	4
$196A_{0,2}$	q+1	-2	-24	-2	-3	-2	4	-2	-3	12	18	-2	4
$196A_{0,3}$	q+1	-2	4	-2	4	-2	4	12	18	-2	4	-2	11
$196A_{0,4}$	q+1	-2	11	-2	18	12	32	-2	-10	-2	-17	-2	-38
$196A_{0,5}$	q+1	-2	4	12	32	-2	-10	-2	-10	-2	-10	-2	-10
$196A_{0,6}$	q+1	12	-3	-2	4	-2	4	-2	-24	-2	25	-2	-10
$196A_{0,7}$	q+1	-2	18	-2	18	-2	-66	-2	18	-2	-24	-2	-3
$196A_{0,8}$	q+1	-2	4	-2	-31	-2	4	-2	-3	-2	-10	12	32
$196A_{0,9}$	q+1	-2	-24	-2	-3	-2	4	-2	-3	12	18	-2	4
$196A_{0,10}$	q+1	-2	4	-2	4	-2	4	12	18	-2	4	-2	11
$196A_{0,11}$	q+1	-2	11	-2	-10	12	-24	-2	18	-2	11	-2	-10
$196A_{0,12}$	q+1	-2	4	12	4	-2	18	-2	18	-2	-38	-2	-10
$196A_{0,13}$	q+1	12	25	-2	-24	-2	4	-2	-24	-2	-3	-2	18
$196A_{1,0}$	q-13	-2	4	-2	25	5	-10	-2	-3	5	-10	-2	4
$196A_{1,1}$	q-13	-2	11	5	4	-2	-10	5	18	-2	-3	-2	-10
$196A_{1,2}$	q+1	5	4	-2	-10	5	4	-2	4	-2	-10	-2	4
$196A_{1,3}$	q+1	-2	18	5	-17	-2	11	-2	-10	-2	-3	-2	4
$196A_{1,4}$	q+1	5	4	-2	11	-2	-10	-2	-17	-2	-10	5	18
$196A_{1,5}$	q+1	-2	11	-2	-10	-2	-3	-2	11	5	4	-2	-10
$196A_{1,6}$	q+1	-2	-10	-2	4	-2	4	5	4	-2	4	5	-10
$196A_{1,10}$	q+1	-2	4	5	-3	-2	11	-2	-10	-2	11	-2	-10
$196A_{1,11}$	q+1	5	-24	-2	-3	-2	-10	-2	25	-2	4	5	4
$196A_{1,12}$	q+1	-2	-3	-2	18	-2	-3	-2	-3	5	-10	-2	4
$196A_{1,13}$	q+1	-2	-10	-2	4	-2	4	5	4	-2	4	5	-10
$196A_{2,0}$	q-13	-2	18	-2	-3	5	-10	-2	11	-2	4	5	-10
$196A_{2,1}$	q+1	-2	4	5	-10	-2	4	-2	4	5	-10	-2	4
$196A_{2,2}$	q-13	5	-10	-2	4	-2	4	5	-10	-2	18	-2	4
$196A_{2,3}$	q+1	-2	11	-2	-10	5	-3	-2	11	-2	4	-2	-10
$196A_{2,4}$	q+1	-2	18	5	-17	-2	-10	-2	-3	-2	4	5	4
$196A_{2,5}$	q+1	5	-10	-2	-3	-2	18	-2	11	5	-24	-2	4
$196A_{2,12}$	q+1	5	4	-2	11	-2	-10	-2	-17	5	-10	-2	18
$196A_{2,13}$	q+1	-2	-10	-2	11	-2	-17	5	-10	-2	11	-2	18
$196A_{3,0}$	q-13	5	-10	5	-10	-2	-10	-2	4	-2	-10	-2	-3
$196A_{3,1}$	q+1	5	-24	-2	-3	-2	11	-2	18	-2	-3	-2	4
$196A_{3,2}$	q+1	-2	-3	-2	18	-2	-3	-2	-3	-2	-10	5	4
$196A_{3,3}$	q-13	-2	-3	-2	-10	-2	18	-2	-10	5	-3	5	18
$196A_{3,4}$	q+1	-2	4	-2	-3	-2	-10	5	-3	5	18	-2	-10
$196A_{4,1}$	q+1	-2	4	-2	11	-2	18	5	-17	5	4	-2	-24
$196A_{4,2}$	q+1	-2	4	-2	-3	5	-10	5	-3	-2	18	-2	-10

Table-14

Ai,j	m ≡6(mod 7),f odd												
	constant	c_6	a_6	c_5	a_5	c_4	a_4	c_3	a_3	c_2	a_2	c_1	a_1
$196A_{0,0}$	q-27	-2	-10	-2	-10	-2	-3	-2	-10	-2	18	-2	4
$196A_{0,1}$	q+1	-2	-3	-2	-3	-2	4	-2	-24	-2	4	12	18
$196A_{0,2}$	q+1	-2	18	-2	-10	-2	-38	-2	11	12	32	-2	-17
$196A_{0,3}$	q+1	-2	-24	-2	-24	-2	18	12	25	-2	4	-2	-3
$196A_{0,4}$	q+1	-2	-31	-2	-3	12	32	-2	4	-2	4	-2	-10
$196A_{0,5}$	q+1	-2	4	12	18	-2	11	-2	4	-2	4	-2	4
$196A_{0,6}$	q+1	12	4	-2	18	-2	-10	-2	4	-2	18	-2	-38
$196A_{0,7}$	q+1	-2	18	-2	18	-2	-3	-2	18	-2	-66	-2	-24
$196A_{0,8}$	q+1	-2	-3	-2	-3	-2	4	-2	-24	-2	4	12	18
$196A_{0,9}$	q+1	-2	-10	-2	18	-2	-10	-2	11	12	-24	-2	11
$196A_{0,10}$	q+1	-2	4	-2	-24	-2	-10	12	-3	-2	4	-2	25
$196A_{0,11}$	q+1	-2	-3	-2	-3	12	4	-2	-24	-2	4	-2	18
$196A_{0,12}$	q+1	-2	4	12	18	-2	11	-2	4	-2	4	-2	4
$196A_{0,13}$	q+1	12	32	-2	-10	-2	-10	-2	4	-2	-10	-2	-10
$196A_{1,0}$	q-13	-2	-3	-2	11	5	-10	-2	18	5	-10	-2	4
$196A_{1,1}$	q-13	-2	4	5	-10	-2	4	5	-10	-2	4	-2	18
$196A_{1,2}$	q+1	5	11	-2	-17	5	18	-2	4	-2	-10	-2	-10
$196A_{1,3}$	q+1	-2	-3	5	18	-2	4	-2	-24	-2	11	-2	-3
$196A_{1,4}$	q+1	5	-10	-2	4	-2	4	-2	4	-2	4	5	-10
$196A_{1,5}$	q+1	-2	18	-2	-3	-2	4	-2	-3	5	-3	-2	-10
$196A_{1,6}$	q+1	-2	-3	-2	-3	-2	-10	5	4	-2	-10	5	18
$196A_{1,10}$	q+1	-2	-17	5	-10	-2	4	-2	18	-2	11	-2	-3
$196A_{1,11}$	q+1	5	4	-2	4	-2	-10	-2	-10	-2	4	5	4
$196A_{1,12}$	q+1	-2	18	-2	-3	-2	4	-2	-3	5	-3	-2	-10
$196A_{1,13}$	q+1	-2	-17	-2	-3	-2	4	5	18	-2	-10	5	4
$196A_{2,0}$	q-13	-2	-10	-2	-10	5	18	-2	-3	-2	18	5	-3
$196A_{2,1}$	q+1	-2	-3	5	11	-2	4	-2	-10	5	18	-2	-24
$196A_{2,2}$	q-13	5	-10	-2	4	-2	-3	5	-10	-2	-10	-2	-10
$196A_{2,3}$	q+1	-2	-10	-2	11	5	-10	-2	11	-2	-3	-2	4
$196A_{2,4}$	q+1	-2	-3	5	25	-2	4	-2	-24	-2	-10	5	4
$196A_{2,5}$	q+1	5	11	-2	-17	-2	-24	-2	4	5	18	-2	4
$196A_{2,12}$	q+1	5	11	-2	-17	-2	18	-2	4	5	-10	-2	-10
$196A_{2,13}$	q+1	-2	11	-2	-10	-2	18	5	-10	-2	-17	-2	11
$196A_{3,0}$	q+1	5	4	5	18	-2	-10	-2	11	-2	-10	-2	-3
$196A_{3,1}$	q+1	5	-3	-2	-10	-2	-10	-2	4	-2	11	-2	11
$196A_{3,2}$	q+1	-2	-10	-2	11	-2	-10	-2	11	-2	-3	5	4
$196A_{3,3}$	q-13	-2	25	-2	-3	-2	4	-2	4	5	-10	5	-10
$196A_{3,4}$	q+1	-2	4	-2	4	-2	-10	5	-10	5	4	-2	4
$196A_{4,1}$	q+1	-2	-10	-2	4	-2	4	5	4	5	4	-2	-10
$196A_{4,2}$	q+1	-2	-3	-2	-3	5	-10	5	4	-2	-10	-2	18