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Fractal Universe and Atoms

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Abstract **Review Article**

Fractal universes and atoms are assigned to k-components or stable orbiting laps of simplest cycles of elliptic invariants. Cosmological redshift, expansion of the universe, origin of cosmic rays, cosmic microwave background, quantum entanglement and the cosmological constant problem are resolvable easily by fractal universes of bifurcating spacetime. Quantum entanglement is explainable by a highly correlated pseudo-congruent k-component in bifurcating spacetime. A one-dimensional complex contour around nontrivial zeros of zeta and L- functions is capable to create a zero-energy universe- action functional. Gauge coupling parameter fit into Gaussian periods of fixpoints. Many experiments in natural history support a fractal zeta universe.

Keywords: Fractal Universe, Atomic Universe, Feigenbaum Renormalization, Spacetime Curvature, Charge Definition.

1. INTRODUCTION

In natural science constants of gauge coupling G_w are predetermined from outside. Regulator indices of an information current based universe predict the order of magnitude G_w for five interactions w=1,…,5 [1]. Experimental data for the fine structure constant α_f show $\alpha_f \simeq 1/128$ GeV which questions a G_w theory [2]. The conjecture $2\pi\delta_F{}^2 \simeq \alpha_f{}^{-1}$ with Feigenbaum constant δ_F remains an open problem though first accuracy is $9.12 \cdot 10^{-4}$ [3, 4]. $2\pi\delta_F^2 \simeq 2\pi R^2 \simeq g_1^{:gn}$ on a sphere for generator $g_1 + \ldots + g_n$ renormalizes α_f [5] [6]. In extension, a fractal zeta universe (FZU) describes charge quanta as δ_F -chaotic bifurcations as nontrivial zeros of zeta functions answering the Dirac monopole problem (DM) [7, 8]. FZU relates chaotic period-doublings and periods ν_{Sh} due to the theorem of Sharkovkii to unified fields which are doubly-periodic invariants suffering Kronecker-Weber extensions. Local L-function minima yield gauge parameter as Gaussian periods in the regulator index $lnG_w = w!\,2^w \ln_3^w 2$ reflecting ratios in experiment [1]. This universe as an open thermodynamic system creates continuously matter with complex, positive and negative curvature. It contains closed thermodynamic systems with atoms and crystals as stable orbiting laps under the influence of pseudo-congruent k-components. Equations for curvature-stress-energy, one- and two-particle Green- function fit into linear relations for one-dimensional simplest cycle in Section 2. A finite Euclidean norm of iterated invariants leads to a spectral and geometric zeta function in Section 3 for an oscillating fractal string. Section 4 confirms that binary invariants envelope unified fields. In Section 5 simple nontrivial zeros of entire functions are local minima of L-functions. In Section 6 stable orbiting laps are Feynman diagrams of stable atoms and crystals. Section 7 proves a pseudo-congruence of bifurcating k-components where fractal and atomic views are interdependent. A chaotic spacetime background susceptibility offers an alternative view to cosmological redshift (CR), expansion of the universe (EU), origin of cosmic rays (GCR), origin of cosmic microwave background (CMB), the phenomenon of quantum entanglement (QE) and the cosmological constant problem (CCP) [8, 1]. Some experiments confirming a continuously created matter are summarized in Section 8.

2. Spacetime Curvature and Simplest Cycles

A Friedmann solution $c\tau = \int \frac{\sqrt{R_u}dR_u}{\sqrt{\phi_3(R_u)}}$ confirms an elliptic integral in universe radius R_u which contains already a doubly-periodic time τ [9]. Real spacetime arises from an oscillation around its square discriminant Δ_3 [8]. This oscillation is complex quadratic map γ of R, the chaotic bifurcations of which are under discussion. A period-3 cycle is a one-dimensional variable $z_k \in [0,1]$ in a quadruple $q_{sc}=\{k+3,t_{sc}\}$ where z_{k+3} is in a triple $t_{sc}=\{k, k+1, k+2\}$. Period-3 specifically means $f(f(f(z)))=z$ but $z \neq f(z)$ and $z \neq f(f(z))$. In FZU quadruples q_{sc} of simplest cycles is equated with Minkowski spacetime and speed limit c_l. Congruences $k+\mu\rightarrow k_\mu$ for steps $\mu=\{0,1,2,3\}$ of q_{sc} yield a wave vector k_μ-Fourier transform. A speed limit c_l is equivalent to the simplest cycle rate. Similarly, the Hubble relation RH=Rln'R=c_l corresponds to a self-similar diffusion process $(R^2)'=D_0$ with $c_l=D_0/R\simeq 1/\sqrt{\varepsilon_0}$ where D_0 is the diffusion coefficient. Minkowski-spacetime averages a self-similar diffusion process $R^2\simeq D_o t$ with universe radius R as a theta square. A symbolic complex quadratic map $\gamma\circ z\rightarrow z$ can be casted into a symbolic linear substitution (Hermite-Tschirnhausen) γ of cubic roots t,z

$$F(\mathrm{t},z)=\gamma(\phi_3(\mathrm{t}))\,z=\phi_3(\mathrm{t})/(\mathrm{t}-z)-⅓\phi'_3(\mathrm{t})\quad(1)$$

It should be mentioned that the form (1) of a symbolic binary substitution γ is also valid for any n^{th} order polynomial ϕ_n. A conjugate of $\gamma(\phi_3(f(\omega)))$ is the Mandelbrot map $z_{k+1}\leftarrow z_k^2+c$ which writes identically

$$\mathbf{R}^4_{\mu\nu}+2\mathcal{F}\mathbf{R}^2_{\mu\nu}-\mathcal{G}^2=0\qquad(2)$$

with skew curvature tensor $\mathbf{R}_{\mu\upsilon}=\chi_{\mu\upsilon\mu'\upsilon'}\mathbf{R}_{\mu'\upsilon'}=\mathrm{Re}z_k$, field density F=½ Re(c-z_{k+1})=½($\mathbf{H}^2$-$\mathbf{E}^2$), Poynting vector G=½ Im(c-z_{k+1})=**EH**, μ,ν=1,2,3,4 of gauge field **E**, **H**. Lorentz-invariance is a rotated cardioid normal $\mathbf{n}^2$ =2F+2iG =c-z_{k+1} as a three-dimensional complex vector as a subset of (1) with

$$det\begin{vmatrix}z+cd\tau & x+iy\\ -x+iy & z-cd\tau\end{vmatrix}=1$$

Renormalization in (2) scales $\mathbf{n}\rightarrow(e_o/e)\mathbf{n}$ with charges e_0 and e and $\mathbf{n}\simeq z_k$ [10]. Gauge parameter G_w are fixed points of invariant $z\simeq f(\omega)\simeq\mathbf{R}_{\mu\upsilon}$ $z_{k+N}\rightarrow G_w z_k\rightarrow\gamma\circ..\circ\gamma\circ z_k$ progressing into complex space $\mathbb{C}^w$, w=1,…,5 [11]. The universe radius R_u in $\phi_3(R_u)$ is a Weierstrass $\wp$-function which is parametrized on a hyperelliptic Kummer surface and Weddle surface K(X(f)=(1,-f,f^2,1)),W(Y(f)=1,-f,f^2,-f^3)) by the Weber invariant $f(\omega)=1^{\frac{-1}{48}}\frac{\eta\left(\frac{\omega+1}{2}\right)}{\eta(\omega)}$ with Dedekind eta function $\eta(\omega)$ [12]. A quadratic map $\gamma(\phi_3(f(\omega)))$ arising from a quartic polynomial ϕ_4 with one quartic root x_s shifted to a quadruple s=±∞,±i∞ is capable to explain spin indices. This generalized shift to +∞ yields a cubic invariant $\phi_3(f(\omega))$. With $\delta z\simeq(\delta x_s)^2$ the four-component tensor $\mathbf{R}_{\mu\upsilon}$ in $\mu\rightarrow\mu$,s embeds into a 4·4-component tensor $R_{\mu s\upsilon s'}$, susceptibility $\chi_{\mu\upsilon\mu'\upsilon'}$ and $\chi_{\mu s\upsilon s\ \mu's\upsilon's}$ with 4^2, 4^4, 4^4 and 4^8 components. A quadruple $q_{sc}\simeq s$ equivalent to four spin indices s is a bispinor $\psi_s=\{1,\delta_k,\delta_k\delta_k,\delta_k\delta_k\delta_k\}f(\omega)$. In a k_μ-Fourier transform triples $t_{sc}\in q_{sc}$ are spatial indices $\mathbf{n}_i\mathbf{n}_j$. Averages are components $\chi_{..\nu..\nu'..}\simeq k_\nu k_{\nu'}\simeq g_{\nu\nu'}$. Tensor $k_\nu k_{\nu'}$ tends to the metric tensor $g_{\nu\nu'}$ giving the Ricci tensor $R_{\mu\mu'}=\chi_{\mu\upsilon\mu'\upsilon'}\,g_{\nu\nu'}$ for k→∞. FZU predicts that a bispinor $\psi_s\simeq z_k\simeq\mathbf{R}_{\mu\upsilon}$ is curvature. The binary envelope is fundamental where $\frac{1}{t-z}|_{14}\simeq G_{12}\gamma^{\mu}_{23}G_{34}$ is a two-linear Green's function $G_{ss'}[\psi_s]$ mod $\phi_3(f(\omega))$. Map (1) of invariant f(ω) envelopes Einstein, Dyson and Bethe-Salpeter equations which are linear simplest cycle relations

$$\mathrm{F}(\mathrm{t},z)|_{14}\simeq A_\mu G_{12}\gamma^{\mu}_{23}G_{34}-⅓\frac{\delta A_\mu}{\delta(G_{12}\gamma^{\mu}_{23}G_{34})^{-1}}\qquad(3)$$

A cubic potential $A_\mu\simeq\phi_3(f(\omega))\simeq\phi_3(\psi_s)$ in (3) is capable to reproduce the Ginzburg-Landau-like term. In the following stable γ-orbits where detγ=1 is called laps l_ω leading to equivalent periods ω in the Weber invariant f(ω). Inequivalent, unstable, bifurcating γ-orbits detγ≠1 is called k-component. The quantum statistical Mandelstam plane s,t,u with s+t+u=0 is related to cubic roots e_i of invariants f(ω) by $\delta_k e=\Lambda^2(\omega)(\lambda,1,1-\lambda)$, e_i= ⅓$\Lambda^2(\omega)$(2-λ,2λ-1,-λ-1). The parameter s,t,u ≃λ in Mandelstam plane get proportional to λ for $\Lambda(\omega)=(2K/\omega_1)=1$. Poles of the scattering amplitude $\mathbb{A}(s)=\sum_{(n)}\frac{1}{s-m_n^2}$ are simple, nontrivial zeros z_{nt}=½+im_n of

$$\xi(z)=\left(\frac{z}{2}\right)\pi^{-\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)\zeta(z)=½\prod_n\left(1-\frac{z}{z_{nt}}\right)\simeq\boldsymbol{E}$$

[13]. A scattering amplitude M(s,t)=$\mathbb{A}$(s)+$\mathbb{A}$(u) in the S-matrix $\mathrm{S}(\mathrm{s},\mathrm{t})\simeq\mathrm{M}(\mathrm{s},\mathrm{t})=\int\frac{\mathrm{ImM}(s',t)}{s'-s-i0}ds'$ acquires a one-dimensional complex contour integral with current density ImM(s,t). Hieb's conjecture is proven by setting $\lambda=j_n/m_n$+½ with a Dirac-like current $j_{nk}=\bar{\psi}_{qk}\,\gamma_{nqq'}\psi_{q'k}\simeq ImM(s,t)$. For triples t_{sc} in $\psi_s=\{1,\delta_k,\delta_k\delta_k,\delta_k\delta_k\delta_k\}f(\omega)$ the current gets $\mathrm{j}_{\mathrm{nk+1}}\simeq\frac{1}{2\pi\delta_F^2}\mathrm{j}_{\mathrm{nk}}$ where $\oint ds\mathbb{A}(s)=2\pi$. An entire, holomorph ξ is **E**-field-like. For each interaction w=1,2,3,4,5 a conductivity plateau $j_n\simeq\mathbf{E}$ is a holomorphic map of a simple nontrivial zero z_{nt} on a complex contour $\oint_{C_g}^{z_{nt}}dz$ producing an energy-gain of a Carnot cycle of time/entropy Re(dz) and temperature Im(dz). The integral over dz traverses contour circles with mean area $2\pi\delta_F^2$ on a chaotic cloud in a $d\sigma_5$- hypersurface. Only an average defines a S-matrix which defines charge quanta e by a chaotic traversed cloud around z_{nt}

$$S=\ \mathrm{T}\exp\{-\mathrm{ie}\int d\sigma_5\,(j_{n\mu}\,A_\mu\}.\ (4)$$

Iterates z_k around the x=y axis is like a q_{sc}-diffusion process with quadrupolar moment $Q_{xy}\rightarrow Q_{ij}$ giving $Q/R^3\simeq 1/\varepsilon R$ in $d\sigma_5$. In dependence on the average the quadrupolar q_{sc}- background permittivity $\varepsilon_{ij}=\varepsilon_0\mathbf{k}_i\mathbf{k}_j/\mathbf{k}^2$ in 1/εR appears as a Coulomb- or Kepler singularity if $\varepsilon=\varepsilon_0R^2$. Note an equivalence between elastic, non-radiative exchange scattering 1,2→1',2', a quadrupolar susceptibility $\varepsilon_{ij}=\varepsilon_0\mathbf{k}_i\mathbf{k}_j/\mathbf{k}^2$ and the moment of inertia in gravitational waves. This can be motivated for the Poynting vector G≃½Im z_k^2 for q_{sc} where $\delta_k\delta_k\delta_k\simeq 1$ and $z_k\simeq z_{k+3}$. At the beginning of any definition of spacetime q_{sc} give $Q_{\mu\nu}\simeq\dot{Q}_{\mu\nu}\simeq\ddot{Q}_{\mu\nu}$ for μ,ν=x,y a mass generation term where the stress-energy term G is a square of the third derivative $\delta_k^3 Q_{\mu\nu}=\dddot{Q}_{\mu\nu}$.

3. Binary Invariance and Geometric Zeta Function Vibrations

In elliptic theta ϑ(u,ω(λ)) the Legendre modular function λ is independent. The binary invariant cross-ratio of four points λ is capable to start an invariant process while lattice periods ω fluctuate. Iterating

$\phi_3(f(\omega))$ or $\phi_4(f(\omega))$ one gets new invariant forms $f_k(\omega)$. The cast (1) relates a shift-operator δ_k to as a discrete k-derivative which substitutes cubic roots of $f(\omega)$ and δ_k produces a new invariant. The map $f_{k+1}=\gamma\circ f_k$ is equivalent to a form generating differential process $f_{k+1} = \delta_A f_k$ with Aronhold derivative $\delta_A = c_{k+1}\frac{\partial}{\partial c_k}$ [14]. A new form f_{k+1} yields new modules $\lambda_{k+1}= \lambda[f_{k+1}]$. The algebra of invariants of any binary forms is always finitely generated where its order is limited by an iterated function k^{10k^3} [15]. In $\mathbb{C}^5$ which is a most general Riemann surface the degree of iterated optimal units increases as a tower $g_L \simeq b^{b^{\cdots\Omega w-L}}$. The degree of the optimal L(z,χ)-function regulator index is $\ln_b g_L$ as shown in Section 6. A certain base $b\to 2^k$ competes with iterated k^{10k^3}. With respect to k-components its order 2^{2^k} should be comparable with the degree of the coupling constant $G_w \simeq \gamma\circ..\circ\gamma$. An estimation yields $2^{2^k}\simeq G_w$ for k=10 and w=5 which solves the cosmological constant problem by the existence of a generator $g^{\lambda_k} = 1$ as a root of unity [1]. The $k\to\infty$ tendency $\lambda_k\to 0$ is known as macrophysics where objects can be located with high precision. An estimation for e.g. $\omega\to 2\omega$ ($\det\gamma=2$) yields $g^{\lambda_k} \simeq g^{16e^{-2^k\pi K'/K}}$ with quarter periods K, K' [16]. The classical limit $K\to 0$ corresponds to a plateau of the Heuman lambda function $\Lambda(u=a\omega,\omega)$. Physically a δ_A pseudo- congruence implies the existence of a finite time-thermal generation rate $\oint_{C_g}^{z_{nt}} dz$ on a complex closed time -thermal contour C_g for a complex one-dimensional current j. On contour C_g the Euler product of the holomorphic function $\xi(z\simeq\lambda)$ up to a quadratic $zk\to zk+1$ transformation

$$\xi(z) = e^{\int dz\mathbb{A}(z)} = \prod\frac{p^{-½z}}{sh(½z\ln p)} \tag{5}$$

acquires a doubly-periodic oscillation. A partial factor of a triple t_{sc}

$$\prod_{t_{sc}} sh(\lambda_k) = \prod_{t_{sc}}(sh\lambda_{x_k}cosy\lambda_{y_k} + ich\lambda_{x_k}sin\lambda_{y_k}) \tag{6}$$

with $\lambda_x=½\ln p\mathrm{Re}z$ and $\lambda_y=½\ln p\mathrm{Im}z$ c. The real part of doubly-periodic vibrations has long-wave oscillations $E(k)|_{k\to 0}\to 0$ and short-wave oscillations in minima of $E(k)|_{k\to c}\neq 0$ with wave vector k. For the imaginary part of doubly-periodic vibrations the temperature oscillates in reverse $T(k)|_{k\to 0}\neq 0$ and $T(k)|_{k\to c}\to 0$. Two of three strings oscillate with fixed end-points and the third string oscillates freely. Strings with time and temperature reversed. A pseudo-congruent differential shift operator $\delta_A \simeq \frac{d}{dz}$ is related to a differential shift operator where $e^{-h\frac{d}{dz}}\zeta(z) = \zeta(z-h)$ [17, 18] Proves spectral vibrations of fractal strings described by an infinite Dirichlet Laplacian Δ_D. Eigenvalues of Δ_D are strings $\mathcal{L}=\{l_s,m_s\}$ and $\nu=k_s l_{sj}^{-1}$ entering geometric zeta functions $\zeta(z,\mathcal{L}) = \sum_{j\in\mathbb{N}} m_s^j\, l_s^{-(j+1)z}$ and spectral zeta functions $\zeta(z,\nu)$) [17, 18].

4. Field Equations as Simplest Cycles

A Feigenbaum renormalization equation $-\alpha_F g(g(-z/\alpha_F))=g(z)$ is a generator shift $-\alpha_F z_{2k}=z_k$ [19]. A measured second Feigenbaum constant δ_F implies a linear relation $c_k z_k + c_{k+1}z_{k+1} + c_{k+2}z_{k+2} = 0$ between z_k, z_{k+1}, $z_{k+2}\in t_{sc}$ which is equivalent

$$\gamma^{(ren)} = \gamma + \gamma\circ\Gamma^{(ren)}\circ\gamma^{(ren)} \tag{7}$$

and charge conserving where $c_k\simeq(\delta_F-1)$ and $c_{k+1}\simeq\delta_F$. Maxwell, Dyson, Bethe-Salpeter and Einstein equation can be are arranged in (4) where z_k, z_{k+1}, z_{k+2} are field tensor $F_{\mu\nu}$, Ricci tensor $R_{\mu\nu}$, Green function G, mass operator Σ, polarization function P, vertex Ξ. Ricci curvature R, stress-energy T and cosmological constant Λ yield e.g. $4\Lambda\text{-}R=\kappa_4 T$ with $\kappa_w \simeq \frac{8\pi G_w}{c_l^4}$. However, an external charge and mass requires a linear relation between four independent functions e.g. $\partial_\mu F_{\mu\nu} = j_\nu$. Lagrange conditions for a whole set of stable orbiting laps result in at least a quadruple $c_q z_q = 0$. One-dimensional bifurcating lines are superposed by its zoom giving Huygens-Fresnel wavelets. Wavelets combine to van der Waals interacting massive clouds on a hypersurface $d\sigma_5$. This constitutes a charge quantum e in Equation (4) around one iterated nontrivial zero z_{nt} where the map contains the whole set of Feynman diagrams.

5. Period-Doubling Fields and Doubly-Periodic Elliptic Invariants

Regular chaotic orbits of (1) are stable Newton-Cayley solutions which emphasize a cubic invariant f(ω) and class number one fields $h_\Delta=1$ [20] [21]. For arbitrary class fields a quartic factor in f(ω) can be extracted from the elliptic invariant j(ω). Envelope (3) relates binary invariants e.g. also to a quadratic Green's function expansion quantum statistics. Period-doubling is equivalent to a split of periods $\omega_k\to\omega_{k+1},\omega_{k+2}$ and $f_{k+1}=f(\omega_{k+1})$ and $f_{k+2}=f(\omega_{k+2})$. In conjunction with enveloping periods ν_{Sh} the map (3) forces complex multiplication (CM). Laps l_ω as well as k-components exhibit simplest cycles q_{sc} in real interval [0,1]. A bispinor ψ_s is a frozen quadruple $f_{q_{sc}}$ with Euclidean norm $\sum_q f_q^{-2} = \sum_q f_q' f_q''$ and bicubic norm $Nm(f(\omega),\mathbb{K}[\partial])=f(\omega)f'(\omega)f''(\omega)$ of c.c. conjugates f',f''. Optimal k-components of degree $b^{b^{\cdots\Omega w-\mathcal{L}}}$ ($\det\gamma\neq 1$) are assigned to ultra-high-energy particles. Stable atoms are assigned to stable orbiting laps $\gamma\circ..\circ\gamma\circ f$ of degree 2^k which yield a path on Mandelstam plane $z,s,t,u\simeq\lambda[f_k]$.

6. Regulator-Index Process to Gauge Coupling

On Mandelstam plane the inverse Riemann zeta function $z=\zeta^{-1}(z)$ is viewed as a Legendre modular invariant λ which depends again on an invariant f(ω). A self-similarity is implemented from the beginning in elliptic theta with f(ω). An entire factor ξ(z) satisfies a hyperbolic Laplace equation $\Delta_h=y^2\Delta_{xy}=(\mathrm{Im}\lambda)^2\Delta_{xy}$. The Dirichlet-Hecke L-function L(z,χ) of character χ in

$$\frac{\zeta(z,\mathbb{K})}{\zeta(z)} = \frac{\Gamma(z/2)z(z-1)\zeta(z,\mathbb{K})}{2\pi^{z/2}\xi(z)} = L(z,\chi)) \tag{8}$$

is taken as a holomorphic complex action functional $L(z,\chi)\simeq h_\Delta R_\Delta/\sqrt{\Delta} \simeq \Omega_w$-L. A function that is holomorphic throughout the finite plane is generally called an entire function, and a distinction is made between entire rational and entire transcendental functions, depending on whether their power series expansions have finite or infinite terms. A Hecke L-series is an L-series for a character on a group that is a generalization of both residue class and ideal class groups and is an entire transcendental function. The transformation of Hecke L-series into a linear combination of Epstein zeta functions

$$\zeta_{[gh]}(2z) = \pi^z \Gamma^{-1}(z) z^{-1} \int_0^{\infty^z} d\omega^z \, \vartheta_{[gh]}(0,\omega)$$

shows that the quotient of the Dedekind zeta function $\zeta(z,\mathbb{K})/\zeta(z)$ can be extended holomorphically to the entire complex plane [22] [23] [24]. The Epstein zeta function $\zeta_{[gh]}(2z)$ contains theta constants $\vartheta_{[gh]} = \eta(\omega) f_i^2(\omega)$ with [00]=``,[10]=1,[01]=2 and $f^8(\omega) = f_1^8(\omega) + f_2^8(\omega)$.

Claim: Near simple, nontrivial zeros $z\simeq z_{nt}$ of $\xi(z)$ in (8) minima of $L(z,\chi)$- functions exhibit doubly-periodic oscillations of the complex variable z. While scanning z_{nt} by the map $f\rightarrow\gamma\circ f$ the oscillations correspond to feasible solutions of the regulator index of number field $\mathbb{K}$ extensions.

$\xi(z_{nt})$ in (8) requires a vanishing Epstein zeta function $\zeta_{[gh]}(2z)$. Algorithmically accessible $\zeta_{[gh]}(2z)=0$ imply cubic integrands for $\gamma=\gamma(\phi_3(f(\omega)))$ and z, ω, $\omega^z\in\mathbb{K}$ in $\zeta(z,\mathbb{K})$. Linearizing $\gamma\circ..\circ\gamma$ in $f\rightarrow\lambda$, $\xi\rightarrow\gamma\xi$, $z\rightarrow\gamma z$, $\lambda\rightarrow\gamma\lambda$, $\omega\rightarrow\gamma\omega$ the power tower $\omega^z\simeq f^f$ in $L(z,\chi)$ minima oscillates. Oscillations with wave vector k_μ due to simplest cycles q_{sc} depend adiabatically on doubly-periodic vibrations on contour $C_g[q_{sc}]$. A feasible solution of regulator index R_Δ elements $l=\ln_b f\simeq \ln_b E$ is shown to have congruences for a non-periodic sum $N(E)\zeta(z, \boldsymbol{\mathcal{L}})$ with Euclidean norm $N(E) = \sum_{q_{sc}} E_{q_{sc}}^{-2} = \sum_{q_{sc}} \psi_{q_{sc}} \bar{\psi}_{qsc}$ similar to a vibrating fractal string [17] [18] [25]. Also, variable $z\simeq\lambda[j_n[f]]$ oscillates as a current density j_n. $\zeta_{[gh]}(2z)\simeq \zeta(z,\mathbb{K})$ is capable to vanish for a pure cubic field $z,\omega\in\mathbb{K}[\partial]$ with $\partial=2^{1/3}$ and normal field $\mathbb{N}[(\sqrt{\Delta_2}]=\mathbb{K}[\partial]\ \mathbb{K}'[\partial]\ \mathbb{K}''[\partial]$ where Δ_n is the discriminant of an n^{th} order field. The finiteness condition for non-periodic iterates $N(E)\zeta(z,\boldsymbol{\mathcal{L}})$ and the existence condition of units yield a quadratic form $\sum(\mu_1 l^2 + \mu_2 l + \mu_3 b^{2l}\zeta(l,\mathcal{L}))$ with Lagrange parameter μ_1, μ_2, μ_3. This quadratic from in $l=R_{\Delta ij}$ is appropriate for local L-minima oscillations in $d\sigma_5$ in the iterated regulator index $R_\Delta = detl$ [25]. Local L-minima contain three terms, a mean density $\mu_1\simeq\Omega_w[\lambda[j_n[f]]]$, a low but finite μ_2 net rate and a low count rate μ_3 of GCR events

$$2\mu_1 l + 2\mu_2\zeta(l,\mathcal{L})e^{2l} + \mu_3 N(b^l)\zeta'(l,\mathcal{L}) = 0) \quad (9)$$

An optimal regulator index L-function processes an equilibrium holomorphic action (conductivity plateau), air ionization (finite net rate) and GCR bifurcation (scattering) on complex non-equilibrium contour C_g. In the classical limit $K'/K\rightarrow\infty$ the Heuman lambda function $\Lambda(u,\omega)$ gets constant and $\lambda\rightarrow0$ or $\lambda\rightarrow1$. The Dirac-like representation $\lambda[j_n[f]]= j_n/m_n+½$ contains as well a product of four sigma functions $U_\square\simeq\exp(i\Lambda(u,\omega)u)$ (cross ratio). For $K\rightarrow0$ a lattice gauge continuum with Lagrange condition $\mu_0(1-U_\Upsilon)$ reproduces a plaquette [26]. Thus, the equilibrium contribution $\mu_1\ln_b E \simeq \ln_b\lambda(1-\lambda)$ is capable to reproduce an action in lattice gauge theory including the spinor term. The second and third terms μ_2, μ_3 belong to an k-processed open universe. With $\lambda(1-\lambda)=2^4/f^{24}$ invariant f(ω) relates to mass.

7. Densities for Open Universes

L-functions (8) subjected to the process $f\rightarrow\lambda$, $\xi\rightarrow\gamma\xi$, $z\rightarrow\gamma z$, $\lambda\rightarrow\gamma\lambda$, $\omega\rightarrow\gamma\omega$ describe contours C_g of open thermodynamic systems. If the spacetime -density (j_n) is not an independent variable while f(ω) gets iterated experiments on CR, GCR, EU, CCP, CMB, QE can be easily explained. If the vacuum density contains the infinity of γ-processes around z_{nt} one gets the concept of quantum statistics of charge quanta with CCP. If individual k-components are resolved the vacuum density is lowered up to $2^{2^{10}}$ giving a rare count of GCR and CMB in agreement with experiment. QE is equivalent to spacetime built from correlated γ-processes $\gamma\circ..\circ\gamma$ with a quadrupolar susceptibility due to chaotic bifurcations giving an apparent EU.

8. Greens Functions, Charges, Atoms, Nuclei and Crystals

Fixing nontrivial zeros $\xi(z_{nt})$ and masses $m_n=\text{Im}\lambda$ by Lagrange parameter μ_s and μ_c up to a factor m_n^2 a γ-invariant hyperbolic Laplacian $\Delta_h\xi(z)=0$ turns out to be a one-dimensional Poisson equation where

$$\Delta_{xy}(L(z,\chi)\xi(z)) + \mu_s L(z,\chi)\xi(z) = \mu_c(\text{Im}\lambda - m_n) \quad (10)$$

Homogeneous solutions of (10) with string eigenvalues $\mu_s\simeq\boldsymbol{\mathcal{L}},\nu$ recover the relation $\frac{\zeta(z,\nu)}{\zeta(z)} = \zeta(z,\mathcal{L})$ analogous to (8). Geometric zeta functions $\zeta(z, \boldsymbol{\mathcal{L}})$ are confirmed by an infinite-dimensional Dirichlet Laplacian Δ_D [18, 17]. In L-function minima z-oscillations of units $z\simeq l$ occur in (10) for a definite set of characters χ. The road to achieve coordinates are rational solutions and fixpoints of periods n of the γ-parametrized singular determinant of a Kummer surface and Weddle surface $K(X(f)=(1,-f,f^2,1)),W(Y(f)=1,-f,f^2,-f^3))$ of points X,Y in projective space. Rational $\mathbf{R}_{\mu\upsilon}$ require square rational discriminants $\Delta_2, \Delta_3, \Delta_4$ in the time integral dτ for cubic polynomials and elliptic theta. Square discriminants Δ_2 and Δ_3 require rational Legendre modules λ. Rational λ require rational invariants f(ω). The fundamental hyperelliptic addition is a vanishing condition on Kummer surface K(X) and Weddle surface W(Y)

$$s_+(u,v)s_-(u,v)=X(f_k)jX(f_{k+1})\rightarrow0 \quad (11)$$

where $s_+(u,v)=\vartheta(u+v)/\vartheta^2(u)$, $s_-(u,v)=\vartheta(u-v)/\vartheta^2(v)$ depend on hyperelliptic characteristics [gh]. In

(11) hyperelliptic theta functions, with $j^2=-1$ are parametrized by hyperelliptic ℘-functions X(f) up to an orthogonal hyperelliptic substitution. For a definite substation $f_{k+1}=\gamma\circ f_k$ expression (11) vanishes quadratically in f. Atoms and nuclei are defined by $\gamma\circ..\circ\gamma\circ f(\omega)=gf(\omega)$ for a rational period n in the generator $g^n=1$. Due to $f^{12}(\omega)\simeq m$ equation (11) is quadratic in masses m like the well-known Edington equation for the electron-to-proton-mass ratio. Statistical occurrences of characteristics [gh] and fixed points $\gamma\circ..\circ\gamma$ yield a large homogeneous factor in (11). The symbolic linear-quadratic map (1) offers simple criteria to differentiate between crystals and atoms. A complex γ is capable to describe crystals with all 32 crystallographic classes G_{32} [27]. If γ-fixpoints in the iterated q_{sc}- quadruple $\psi_s[f_q]$ depend on G_{32} the bispinor $\psi_s[f]$ belongs to a crystalline state. Subsets are called nuclei if $\gamma\circ f$ induces a rational transform of periods ω. An existing k-pseudo-congruence predicts a finite number of particles within the universe confirming the Eddington number. Further on, the n=3 transform of periods of theta constants is compatible with the ⅓ transform. This inflection point corresponds to period-3 cycles of chaotic bifurcations. In total one has 36 combinations of ⅓ transforms confirming the standard model of particle physics. There is strong indication that 120 stable combinations of hyperelliptic characteristics [gh] yield a vanishing of (11) which may explain the periodic table of elements [28, 29]. This simple theory of complex four-component curvature ψ_s extends to all interactions w=1,2,3,4,5. A coupling constant G_w is simply a scaling $z\to G_wz$ or $R_{\mu\nu}\to G_wR_{\mu\nu}$ of field z or curvature $R_{\mu\nu}$ which is $f(\omega)\to G_wf(\omega)$ or $\gamma\circ\ldots\circ\gamma\circ f(\omega)$. Parabolic γ or hyperbolic γ in a bispinor have large periods $n\to\infty$ with $j(\omega)\to\infty$ realizing the rational case $\Delta_2=\Delta_3=\Delta_4\to0$ where $\det\gamma=\phi_3$. So far fixed points of symbolic γ arose either from stable orbiting laps for $\det\gamma=1$. Unstable orbits called k-components arise for arbitrary discriminants $\Delta_\gamma=\mathrm{disc}\gamma=\frac{1}{4}\left(\frac{1}{3}\phi_3'+f\right)^2-\phi_3$. In this case the fixpoint can be any generator g_i as a root of unity. Stable regions with coupling constants are sums in the matrix R_Δ with vanishing Gaussian periods. With $\sqrt{\Delta_3}=\lambda(1-\lambda)=2^4/f^{24}\simeq\eta^{12}(\omega)$ the classical case is singular with Δ_3- a square. For $n\to\infty$ invariant f(ω) and real algebraic unit E tend to a constant. Then $E^h{}_\Delta=\frac{1}{2}f^3(\sqrt{\Delta})$ or $R_\Delta\simeq\ln E\to0$ is proportional to $G_w\to0$ if $\ln f\simeq f$. Stability of the proton and atoms is explainable by a γ-invariance of equation (11). A constant vacuum energy density throughout in the universe is understandable by a compensation of k-components by the presence of atoms in those regions where rational fixpoints are absent. It is assumed that this holds even for organic matter which must be surrounded by ionized atmospheres.

9. RC Circuit Model of a Fractal Cosmic-Ray-Charge-Cloud-Superfluid

A pseudo-congruence of cubic iterates can be demonstrated by an atmospheric active RC circuit with negative differential resistance R_g. R_g acts as an active resistance in a Gunn diode with drift-diffusion clouds. Superfluid-dynamical cloud points X(f) obeys two waves of entropy and temperature surrounding each massive shell as a Carnot heat engine. Iterated invariants f(ω) yield Kirchhoff equations of an ideal fluid in discrete form $X(f_k)$, $X(f_{k+1})$ and $X(f_{k+2})$ [1]. Chaotic f(ω)-bifurcations develop period-doubling doubly-periodic oscillations of temperature and entropy. With invariant vacuum energy density, a higher massive density region is compensated by CMB, plant growth and atmospheric clouds coupled to cosmic rays which makes a non-ergodic bifurcation. In the following a self-similarity ratio of 10^{20} for relative information density 10^{-167} proves to be essential. The stable part of the system as a whole is holomorphic and infinitely differentiable. The CM invariant $\lambda g_i^2\simeq G_wM_w^2$ contains a generator g_i which is a modular unit in an invariant dimensionless energy density [11]. Pseudo-congruence means $G_w\simeq2^{2^{\cdot k}}$ at $G_5=10^{-167}$ and k=10. With background volumetric mass $M_w\simeq g_i$ the mass ratio $\sqrt{G_1/G_5}\simeq M_5/M_1\simeq10^{83}$ is comparable to the Large Number Hypothesis [30]. The corresponding Born-Oppenheimer parameter $\kappa_{BO}=(M_5/M_1)^{1/4}\simeq10^{20}$ is viewed as a cloud moving in a cloud of w=5 bifurcations equivalent to elastic non-radiative exchange scattering 1,2→1',2' as a basic property of any spacetime as shown in Section 2. Pseudo-congruence implies a map for the Born-Oppenheimer parameter $\kappa_{BO}\to\kappa_{BO}{}^4$ which follows from $\lambda=-\frac{\vartheta^4_{[10]}}{\vartheta^4_{[01]}}$. Physically the Millikan experiment, the quantized Hall conductivity (QH), an atmospheric cloud-to-earth-mass behavior and a solar-system-to-universe mass behavior should show self-similarity each of ratio 10^{20} [8]. Then iterated fluid dynamics on K(X(f)) for a cubic invariant f(ω) allows a RC circuit model in open hypersurfaces $d\sigma_5$. A Carnot cyclic lateral current $j_{lateral}\simeq dQ/dt$ alternates in a spherical shell between low- and high-pressure areas d(pV) under influence of an altitude gradient of temperature ∇T and pressure p for variable volume. Lateral fields $\nabla T\simeq E_{altitude}$ and lateral currents $j_{lateral}\simeq E_{altitude}$ of clouds are projectable onto zeta zeros $z_{nt}\in C_g$ on Mandelstam plane giving one-dimensional relations $Q_g=CU\simeq CT$, $U=R_gI_g$ between charge Q_g, capacity C between critical stripes, negative differential resistance R_g and current I_g. Pseudo-congruence $G_w\simeq2^{2^{\cdot k}}$ means alternating fields $E_{altitude}$ and alternating low count rates of ultra-high energy GCR as the origine of capacity C. A $d\sigma_5$ current in ∂F=j is caused by variations of iterates $\gamma\circ\ldots\circ\gamma$ where the cubic invariant f(ω) causes negative differential resistances R_g. Like the Gunn effect the appearing negative differential resistance R_g is responsible for the formation of clouds, e.g. atmospheric clouds driven by zeta zeros. An altitude capacitor C subjected to k- pseudo congruent alternating currents and negative resistance is a chaotic RC circuit. This model considers a critical stripe of ζ(z) containing a seasonal growth of organic matter together with a cosmic-ray-charge-cloud-superfluid as a capacitor as a weather-climate- model.

10. Experiments in Natural History

The FZU model suggests a net persistent creation of organic and non-organic matter from a zeta-zero-energy-universe. Seasonal sun radiation is rather a catalyst to set the loop $\gamma\circ\ldots\circ\gamma\circ f(\omega)$ in motion. K-components with very low GCR count rate induce air ionization. The seasonal variation of GCR counts has been proven over a large period of time [31]. A predicted influence of cosmic rays on earth's climate as well atmospheric cloud-cosmic ray- global temperature correlations has been proven and is under discussion [32-36]. The plateau-like $Q_g \simeq CT$, $U=R_g I_g$ transition between the next fixpoints predicts oscillations of temperature in agreement with statistical data over 10^8 years [37]. Microwave emission at QH stands for a continuous creation of CMB on earth [38]. Diurnal variations of air ionization in vegetation areas seem to indicate the presence of continuously created organic matter and GCR [39, 40]. A superfluid contains the number of segmental displacements up to reaching a congruence level. The bifurcating potential from the lowest up to the highest GCR energies is replaced by a continuous potential with a gauge parameter e which originates from z_{nt}. Pseudo-congruence of k-components $\gamma\circ..\circ\gamma$ implies a highly correlated spacetime on earth up to the exosphere. This is simply the criterion of atmospheric stability. Correlated signals of large array GCR-detectors are equivalent to a surface stability. A superfluid is a perfect fluid having a first and a second sound. A second sound in cosmic-ray-charge-cloud-superfluid is trivially proven by flash bang and thunder in atmospheric excitations. Moreover, a seasonal and a diurnal variation is a doubly-periodic oscillation present in the whole universe. FZU is realized by a doubly-periodic oscillation of time τ in the Friedmann solution. Accordingly, the notion of a superfluid is equivalent to iterating Kirchhoff equations in terms of doubly-periodic elliptic invariants. The validity of the binary substitution (1) in creating new binary invariants is compared to continuous creation of organic matter where a one-to-one correspondence exists between binary forms and organic molecules [41]. The superfluid as a chaotic bifurcating perfect liquid is non-turbulent, non-dissipative, non-radiative where emission is secondary process. The chaotic RC circuit model is felt by seasonal superimposed by diurnal changes of plant growth. Also, for organic matter and atmospheric clouds a continuous creation of matter is suspected. Existing matter is a catalyst for a van-der Waals-like minimum for a cloud of $n\rightarrow\infty$ particles with a nonlinear quadrupolar, non-radiative, non-dissipative but highly-correlated interaction potential. This ultra-high energy limit is realized as an ultra-large mass near a simple nontrivial zero z_{nt} in (6) which is invariant for simultaneous changes of $\gamma\circ..\circ\gamma\circ\xi$ and $\gamma\circ..\circ\gamma\circ z$. [42-44].

11. CONCLUSIONS

The prerequisite for unified fields is a solution of CCP, QE, DM which is easily achieved by a bifurcated correlated spacetime. The connection between the algebra- mathematical definition of spinors and unified fields as four-component simplest cycles of spacetime curvature offers a new framework to connect uniformly zeta and L- functions with field-theoretic action functionals for all interactions. Dark matter is the all-pervading bifurcation of the simplest cycles felt as tidal forces, a quadrupole-like doubly-periodic wave of Carnot cycles which gains energy. In this respect chaotic bifurcation is the prerequisite for stability. Biopower plants use the massive content of matter. A radiational part of matter migrates into the higher atmosphere. The measured diurnal variations of air ionization in vegetation areas confirm a possible a new technological use of radiational biopower. The question is whether emitting a low count rate of ultra-high particles can compete with the use of the molecular part of biopower. The measured ionization rate of 10^4-10^5 ions$\cdot$cm^{-3} in vegetation areas is much higher than the GCR vacuum energy density of a few protons per cm^3 in the higher atmosphere layers. From CMB and GCR with energy density of $\simeq$1eV$\cdot$cm^{-3} energy hardly can be extracted. However, a charge quantum requires an environment cloud mass as large as a Planck energy. This indicates that a GCR-like energy density generated by bifurcated spacetime can be much higher than that in higher atmosphere layers. Moreover, using nanolayers the predicted GCR and CMB emission can be controlled by QH-like experiments. FZU as an open universe is a stationary non-equilibrium open thermodynamic state. Spacetime as a texture of simplest cycles alone yields an apparent expansion via cosmological redshift caused by the increasing background permittivity in the Rydberg constant. This is equivalent to drift-diffusion around a point with a compensating electric field. A point radius (universe radius) as theta square $\vartheta(u,\omega)$ is surrounded by constant diffusion D_o if the Gaussian kernel of $\vartheta(u,\omega)$ has standard deviation δ_F. This chaotic $k\rightarrow\infty$ limit explains a finite speed limit.

REFERENCES

1. Ziep, O. (2025). Matter and Quantum Entanglement. Journal of Applied Mathematics and Physics, 13(4),1125-1137,https://doi.org/10.4236/jamp.2025.134059.
2. Fritzsch, H. (2002). Fundamental Constants at High Energy. 100 Years Werner Heisenberg: Works and Impact, 89–95.
3. Hieb, M. (1995). Feigenbaum's Constant and the Sommerfeld Fine-Structure Constant, https://www.rxiv.org/abs/1704.0365.
4. Manasson, V. A. (2017). An Emergence of a Quantum World in a Self-Organized Vacuum-A Possible Scenario. Journal of Modern Physics, 08, 1330-1381. https://doi.org/10.4236/jmp.2017.88086.
5. Ganter, B. (2020). Deciphering the Fine Structure Constant, www.epubli.de, http://inis.iaea.org/search/search.aspx?orig_q=RN: 52007997.

6. Ganter, B. (2013). The emergence of the dimensions and fundamental forces in the universe, an information-theoretical approach for the explaining of the quantity ratios of the fundamental interactions, 4. rev. and enl. Ed.
7. Dirac, P. A. (1931). Quantised singularities in the electromagnetic field. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 133, 60–72.
8. Ziep, O. (2025). Charge Quanta as Zeros of the Zeta Function in Bifurcated Spacetime. Journal of Modern Physics, 16, 249–262, https://doi.org/10.4236/jmp.2025.162011.
9. Friedmann, A. (1922) On the Curvature of Space. Zeitschrift fur Physik, 10, 377
10. Schwinger, J. (1948). Quantum electrodynamics. I. A covariant formulation. Physical Review, 74, 1439.
11. Ziep, O. (2024). Fractal zeta universe and cosmic-ray-charge-cloud superfluid. www.epubli.de, https://doi.org/10.5281/zenodo.14193126
12. Baker, H. F.(1907) An introduction to the theory of multiply periodic functions, University Press
13. Remmen, G. N. (2021). Amplitudes and the Riemann Zeta Function. Physical Review letters, 127, 241602. https://doi.org/10.1103/physrevlett.127.241602
14. Gordan, P.(1885) Dr. Paul Gordan's Vorlesungen über Invariantentheorie, Bd. 1, BG Teubner
15. Weitzenböck, R. &Turnbull H. W., (1933)„Ueber die Endlichkeit der Invarianten binärer Formen," Proceedings of the Edinburgh Mathematical Society, 3, 223–230.
16. Weber, H. (1908)„Lehrbuch der Algebra, Band III," *Elliptische Funktionen und Algebraische Zahlen, F.* Vieweg und Sohn. Braunschweig.
17. Lapidus, M. L. (2015). Towards quantized number theory: spectral operators and an asymmetric criterion for the Riemann hypothesis. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 373, 20140240.https://doi.org/10.1098/rsta.2014.0240.
18. Lapidus, M. L. (1993). Vibrations of fractal drums, the Riemann hypothesis, waves in fractal media and the Weyl-Berry conjecture. Pitman Research Notes in Mathematics Series, 126–126.
19. Coppersmith, S. N. (1999). A Simpler Derivation of Feigenbaum's Renormalization Group Equation for the Period-Doubling Bifurcation Sequence. Am. J. Phys., 67, 52–54, https://doi.org/10.1119/1.19190.
20. Billings, L., Curry, J. H., & Robins, V. (1999). Chaos in relaxed Newton's method: The quadratic case. Contemporary Mathematics, 252, 63–70.https://doi.org/10.1090/conm/252/1747278.
21. Babajee, D. K., Cordero, A., & Torregrosa, J. R. (2016). Study of iterative methods through the Cayley Quadratic Test. J. Comput. Appl. Math., 291, 358–369. https://doi.org/10.1016/j.cam.2014.09.020.
22. Epstein, P. (1906). Zur Theorie allgemeiner Zetafunktionen. II. Mathematische Annalen, 63, 205–216.
23. Hecke, E. (1917). Ueber die L- Funktionen und den Dirichletschen Primzahlsatz für einen beliebigen Zahlkörper. Nachr. Ges. Wiss.Göttingen, Math.-Phys. Kl., 1917, 299-2. http://eudml.org/doc/58995.
24. Lemmermeyer, F. (2016). Dedekinds Arbeit über Rein Kubische Zahlkörper, Schriften zur Geschichte der Mathematik und ihrer Didaktik, WTM-Verlag, https://nbn-resolving.de/urn:nbn:de:hbz:6-49189652025.
25. Pohst, M., & Zassenhaus, H. (1997). Algorithmic Algebraic Number Theory (Bd. 30). Cambridge University Press.
26. Ziep, O. (2025). Topological entropy of the optimal chaotic quadratic ma. Scholars Journal of Physics, Mathematics and Statistics_, 12, 43-58. https://doi.org/10.36347/sjpms.2025.v12i03.002.
27. Weber, H. (1896). Lehrbuch der Algebra: Vol. 2. (F. Vieweg und Sohn. Braunschweig).
28. Ziep, O. (2024). Linear map and spin II. Kummer surface and focal error. J Mod Appl Phy, 7(1),1-5. https://doi.org/10.5281/zenodo.10444161.
29. Krazer, A. (1903). Lehrbuch der Thetafunktionen (Bd. 12). BG Teubner.
30. Dirac, P. A. (7 1974). Cosmological Models and the Large Numbers Hypothesis. Proc. Roy. Soc. A: Mathematical, Physical and Engineering Sciences, 338, 439-446. https://doi.org/10.1098/rspa.1974.0095
31. Duperier, A. (1941). The Seasonal Variations of Cosmic-Ray Intensity and Temperature of the Atmosphere. Proc. Roy. Soc., A. Mathematical and Physical Sciences, 177, 204–216.
32. Edward P., N. E. (1959). Cosmic Radiation and the Weather. Nature, 183, 451–452. https://doi.org/10.1038/183451a0.
33. Dickinson, R. E. (1975). Solar variability and the Lower Atmosphere. Bull. Amer. Meteor.Soc., 56, 1240–1248.
34. Svensmark, H. (1998). Influence of Cosmic Rays on Earth's Climate. Phys. Rev. Lett., 81(22), 5027–5030. https://doi.org/10.1103/PhysRevLett.81.5027.
35. Svensmark, H., Bondo, T., & Svensmark, J. (2009). Cosmic ray decreases affect atmospheric aerosols and clouds. Geophysical Research Letters, 36(15), https://doi.org/10.1029/2009GL038429.
36. Svensmark, H., Svensmark, J., Enghoff, M. B., & Shaviv, N. J. (2021). Atmospheric ionization and cloud radiative forcing. Scientific Reports, 11, 19668, https://doi.org/10.1038/s41598-021-99033-1.
37. Shaviv, N.J. and Veizer, J. (2003) Celestial Driver of Phanerozoic Climate? GSA Today, 13, 4-10, https://doi.org/10.1130/1052-5173(2003)013<0004:cdopc>2.0.co;2.
38. Bisognin, R. , Bartolomei, H. Kumar, M. , Safi, I. , Berroir, J.-M. , Bocquillon, E. Plaçais, B. Cavanna,

A. Gennser, U. Jin, Y. (2019). Microwave photons emitted by fractionally charged quasiparticles, Nat. Commun.10, 1708, https://doi.org/10.1038/s41467-019-09758-x.

39. Patil, G. B., Pawar, S. D., Bhosale, J. L., & Patil, P. G. (2021). Pollution Index and Air Ion Variation in Different Vegetation area at the Rural Station Bhilawadi (16059'N, 740 28'E). Journal of Physics: Conference Series, 1964, S. 032006, https://doi.org/10.1088/1742-6596/1964/3/032006.
40. Pawara, S. D., Patilb, G. B., Patilc, P. G., Patild, P. G., & Bhosalee, J. L. (kein Datum). Air Quality and Air Ion Assessment Ratio for Papaya, Chickpea, and Guava Vegetation Area at Rural Station Khatav (16o57'N, 74o 31'E). https://doi.org/10.21275/sr23424144257.
41. Gordan, P., & Alexejeff, W. (1900). Uebereinstimmung der Formeln der Chemie und der Invariantentheorie. Zeitschrift für Physikalische Chemie, 35, 610–633.
42. Ziep, O. (2024). Nucleosynthesis in Thin Layers. www.epubli.de, https://doi.org/10.5281/zenodo.10848579.
43. Ziep, O. (2025), Quantized Conductivity as a Neutral Quadrupolar Generator. Scholars Journal of Physics, Mathematics and Statistics, 12, 1–5. https://doi.org/ 10.36347/sjpms.2025.v12i01.001.
44. Ziep, O. (2025), A quantum entangled fractal superfluid universe, Journal of High Energy Physics, Gravitation and Cosmology, submitted.