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Circling an Equilateral Triangle with Straightedge and Compass in Euclidean Geometry

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Abstract **Review Article**

No scientific theory lasts forever, but specific research and discoveries continuously build upon each other. The three classic ancient Greek mathematical challenges likely referring to are “Doubling The Circle”, “Trisecting An Angle” & “Squaring The Circle”, all famously proven Impossible under strict compass-and-straightedge constraints, by Pierre Wantzel (1837) using field theory and algebraic methods, then also by Ferdinand von Lindemann (1882) after proving π is transcendental. These original Greek challenges remain impossible under classical rules since their proofs rely on deep algebraic/transcendental properties settled in the 19th century. Recent claims may involve reinterpretations or unrelated advances but do overturn these conclusions above. Among these, the "Squaring The Circle" problem and related problems involving π have captivated both professional and amateur mathematicians for millennia. The title of this paper refers to the concept of "constructing a circle that has the exact area of a given equilateral triangle " or “Circling The Triangle”. This research idea arose after the “Squaring The Circle” problem was studied, solved and published in “SJPMS” in 2024 [8]. This paper presents an exact solution to constructing a circle that is concentric with and has the same area as any given regular triangle. The solution does not rely on the number π and adheres strictly to the constraints of Euclidean geometry, using only a straightedge and compass. The technique of “GEOMETRY ANALYSIS” is employed to solve this “Circling The Triangle” problem precisely and exactly with only a straightedge and compass, without altering any premise of the problem. This independent research demonstrates the solution to the challenge using only these tools. All mathematical tools and propositions in this solution are derived from Euclidean geometry. The methodology involves geometric methods to arrange the given regular triangle and its equal-area circle into a concentric position. Building on this method of exact “Circling The triangle,” one can deduce an equivalent problem to formulate a new mathematical challenge: "Circling The Hexagon**”** (i.e., constructing a circle that has the same area as a given regular hexagon, using only a straightedge and compass).

Keywords: Circling the Triangle, Triangling the Circle, Regular Triangling the Circle, Circle Triangulated, Triangle Area of a Circle, Constructing a Cicle with the Same Area as a Triangle, Euclidean Geometry, Straightedge and Compass.

INTRODUCTION

In the past, knowledge was often considered scientific if it could be confirmed through specific evidence or experiments. However, Karl Popper, in his book *Logik der Forschung* (The Logic of Scientific Discovery), published in 1934, demonstrated that a fundamental characteristic of scientific hypotheses is their ability to be proven wrong (falsifiability). Anything that cannot be refuted by evidence is temporarily regarded as true until new evidence emerges. For instance, in astronomy, the Big Bang theory is widely accepted, but in the future, anyone who discovers a flaw in this theory will be acknowledged by the entire physics community. Furthermore, no scientific theory lasts forever; rather, it is specific research and discoveries that continually build upon one another [1].

About three thousand years ago, three well-known problems in ancient Greek mathematics emerged. Among them, Hippocrates meticulously studied the problem of squaring the circle. The problem is stated as follows: Using only a straightedge and a compass, is it possible to construct a square with an area equal to a given circle? These problems were proven unsolvable using only straightedge and compass in the 19th century.

In 1882, mathematician Ferdinand von Lindemann proved that π is an irrational number, which means that it is impossible to construct a square with the same area as a given circle using only a straightedge and a compass, as posed by Hippocrates. One of the most fascinating aspects of this problem is that it has captured the interest of mathematicians throughout the history of mathematics. From the earliest mathematical documents to today's mathematics, the problem and its relation to π have intrigued both professional and amateur mathematicians for millennia.

A significant step forward in proving the impossibility of squaring the circle occurred in 1761 when Lambert proved that π is irrational [2, 3]. This, however, was not sufficient to demonstrate the impossibility of squaring the circle using a straightedge and compass, as certain algebraic numbers can be constructed with these tools. Despite the proof of the impossibility of squaring the circle, the problem has continued to captivate mathematicians and the general public alike, remaining an important topic in the history and philosophy of mathematics.

This "Squaring The Circle" problem has been also proven to be impossible, and the proof was provided by Ferdinand von Lindemann in 1882. He showed that π is a transcendental number, which implies that it's not a root of any polynomial equation with rational coefficients, and therefore, the circle cannot be squared using the specified constraints. The impossibility proof relies on π being a transcendental number. However, there were announcement and publication mathematics, particularly for problems like squaring the circle, the community often looks for verification from multiple reputable sources and publication of a solution to this problem in 2023 from various journals and publication platforms, that contradicts Lindemann's proof. These solutions with verification and peer-reviews, confirmed the validity and accuracy of those publications. The solution truly doesn't rely on π or any algebraic, trigonometric, or arithmetic calculations, it would likely involve a geometric construction that somehow bypasses the need for calculating π [5]. The solution also had PROOF of "how does it ensure the area of the constructed square is exactly equal to the area of the given circle". The solution comes with a proof that demonstrates the area of the constructed square is exactly equal to the area of the given circle, that would be a significant development.

In 1837, French mathematician L. Wantzel proved that the three classical problems of ancient Greece are impossible to solve using only a straightedge and compass. Although based on algebra, Pierre Laurent Wantzel (1837) declared the verdict, "Arbitrary angle trisection is impossible", I experienced that my achieved procedure was possibly not arrested by this verdict [4]. After applying my method to trisect an arbitrary angle, I hope readers can trust algebra cannot affect the successful geometrical constructions of angle trisection.

Therefore, the problems of squaring the circle, doubling the cube, and trisecting an angle have been studied for centuries and remain unsolvable with these tools (*straightedge & compass*) to this day. As of December 2022, no mathematician has found exact solutions to classical problems such as "Doubling the Cube," "Squaring the Circle," or "Trisecting an Angle" using only a compass and straightedge. However, in 2023, the research, published and titled "Exact Squaring the Circle with Straightedge and Compass Only" [5], provided a counter-proof to the impossibility stated by Wantzel in 1837 [4].

Starting from accepted premises, without proof, one uses deductive reasoning to arrive at theorems and corollaries. With different premises, we develop different mathematical systems. For example, the premise "from a point outside a line, only one parallel line can be drawn to the given line" leads to Euclidean geometry. If we assume that no parallel lines can be drawn from that point, we enter the realm of Riemannian geometry. Alternatively, Lobachevskian geometry assumes that an infinite number of parallel lines can be drawn through that point.

It is important to clarify that this does not imply a square with an equal area to a circle does not exist. If the circle has an area A, a square with a side length equal to the square root of A would have the same area. It does not imply that it is impossible to solve the problem using only a straightedge and compass. Thus, I have provided an exact solution to the "Squaring the Circle" problem without altering the premises of Euclidean geometry or the constraints of straightedge and compass. This solution was also published in SJPMS [5]. In 2024, I also solved the inverse problem, titled "Circling the Square with Straightedge & Compass in Euclidean Geometry," which was published in *SJPMS and, IJMTT* [8].

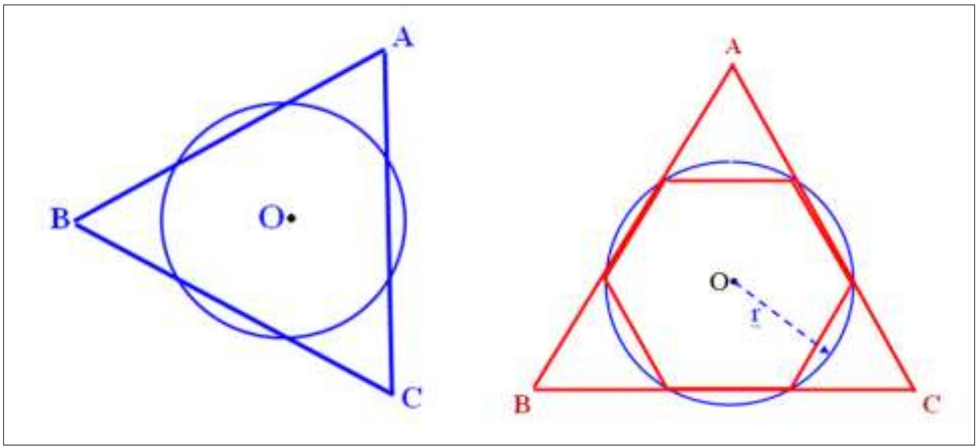
"Circling The Regular Triangle" refers to the process of "*constructing a circle that has the same area as a given equilateral triangle and is concentric with the given triangle*." This concept came to me spontaneously after solving the problem of "Circling The Square" [8]. I thought, "*If one can circle a square, then can one also circle a regular triangle using only a straightedge and compass in Euclidean Geometry?*" In other words, "Circling The Equilateral Triangle" is a new challenge problem that arose from the exact solution presented in my "Circling The Square With Straigtedge And Compass In Euclidean Geometry" paper, published in SJPMS [8]. Therefore, the "Circling The Triangle" challenge has not existed in the field of mathematics until it is solved and published today.

In seeking solutions to such problems, geometers developed a special technique called

"GEOMETRY ANALYSIS." They would assume the problem had been solved, and by investigating the properties of the solution, they would work backward to identify an equivalent problem that could be solved based on the given conditions. To obtain the formally correct solution to the original problem, geometers would then reverse the process: starting with the data to solve the equivalent problem derived through analysis, and then using that solution to solve the original problem. This reversed procedure is known as "SYNTHESIS." I have adopted both the "GEOMETRY ANALYSIS" and "SYNTHESIS" techniques to solve the "Circling An Equilateral Triangle" problem exactly, using only a straightedge, compass, in Euclidean Geometry, without involving the irrational number π.

The inspiration for this research arose after the exact solution to the "Circling the Square" problem was published in SJPMS in 2024 [8]. If it is possible to circle a square using Euclidean geometry, the question arises: why is it difficult to circle a regular triangle? In this context, the given equilateral triangle contains an inscribed hexagon with three non-consecutive sides overlap the triangle's sides, leading to an the resulting circle. This circle serves as the solution to the new problem of "Circling The Regular Triangle." The remaining task is to prove that the area of this circle is exactly equal to the area of the given equilateral triangle. This article paper also introduces a new mathematical tool, the "Hexagon Ruler," and provides a proof for the following: **(1)** a concentric circle intersects the given regular triangle at the six vertices of a special hexagon, with two non-consecutive sides symmetrically positioned; **(2)** the construction process forms a regular hexagon inscribed in the given triangle.

This paper's solution to the "Circling The Regular Triangle" problem naturally leads to the concept of "Circling The Regular Hexagon," "Circling The Regular Heptagon," "Circling The Regular Octagon," …, and so on.



I. PROOFS OF NEW PROPOSITIONS

I.1 Theorem 1:

Given an equilateral triangle ABC with side length a. If there exists a concentric circle (O, r) that has the same area as the given equilateral triangle ABC then the circle intersects the three sides of the triangle at 6 points, thereby forming an inscribed hexagon within the circle.

PROOF:

Assume there exists a concentric circle (O, r) such that its area is equal to the area of the given equilateral triangle ABC. It follows that:

- The area of the circle (O, r) is less than the area of the circumscribed circle of triangle ABC, (as indicated by the red dashed circle in Figure 1 below).
- The area of the circle (O, r) is larger than the area of the inscribed circle (O, r') of triangle ABC, (as indicated by the green dashed circle in Figure 1 below).

Consequently, the circle (O, r) lies between the two circles: the circumscribed circle (O, R) and the inscribed circle (O, r') as depicted in Figure 1 below. The circle (O, r) intersects three sides of triangle ABC at 6 distinct points, denotes as a, b, c, d, e and f. These six points form the vertices of an inscribed hexagon within the triangle ABC, where segment bc overlaps with side AC, segment ed overtlaps with side AB, and segment af overlaps with side AB.

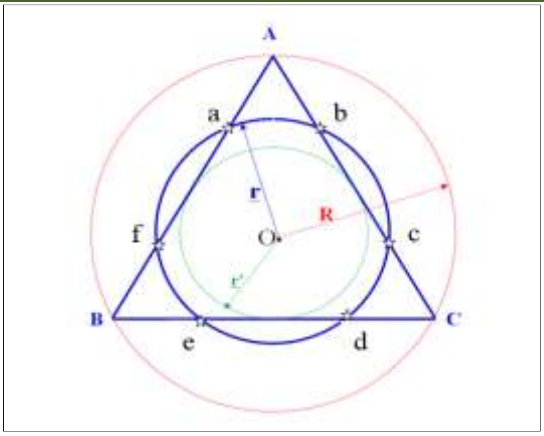


Figure 1: The concentric Circle (O, r) is located between the two concentric dashed circles (O, R) & (O, r'), which represent the circumscribed & inscribed circles of the given equilateral triangle ABC, respectively, and illustrated the inscribed hexagon ABCDEF by the given equilateral triangle ABC

I.2 Theorem 2

Let ABC be an equilateral triangle with side length a. Suppose there exists a concentric circle (O, r) - i.e., with the same centre as the triangle's circumscribed circle- such that the area of this circle is equal to the area of triangle ABC. Then:

- The circle (O, r) intersects the three sides of triangle ABC at six distinct points a,b,c,d,e,f, forming a regular hexagon inscribed in the circle.
- This regular hexagon is also inscribed within the equilateral triangle ABC.

PROOF:

Let (O, R) be the circumscribed circle of the given equilateral triangle ABC, which shares the same centre O as the given circle (O, r). From Theorem 1, the circle (O, r) and the triangle ABC together define an hexagon abcdef, inscribed in both the circle and the triangle.

Now, consider the side ab of the hexagon. Extend ab until it intersects the circumscribed circle (O, R) at points D and E. Connect point E to the centre O.

Construct an angle symetric to angle $b\widehat{E}O$, denoted $O\widehat{E}c$, such that the side cd of the new angle overlaps with the corresponding side of the hexagon. Due to the symmetry of the hexagon, sides ab and cd are congruent and symmetric about the center O implying DE=EF. Similarly, applying the same symmetry to sides cd and ef, we observe the following:

- Line segment FO bisects angle $d\widehat{F}e$, and side fe overlaps with the corresponding side of the hexagon.
- Extend side fe to intersect the circumscribed circle (O, R) at points D and F.

It follows that triangle DEF is equilateral and inscribed in circle (O, R). Since triangle DEF and triangle ABC are both equilateral and share the same circumscribed circle, they are congruent and equual.

Therefore,

- the hexagon abcdef is a regular hexagon, and
- this regular hexagon inscribes in both triangle ABC and triangle DEF as well as in circle (O, r).

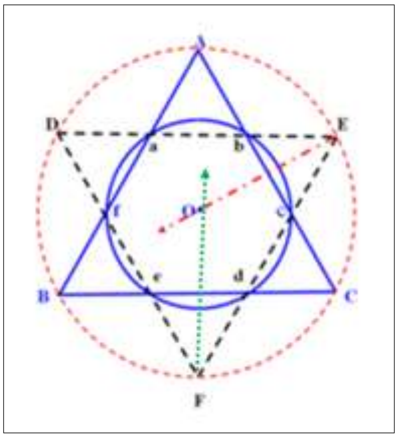


Figure 2: The concentric Circle (O, r) along with the given equilateral triangle ABC, centered O, creates the regular hexagon ABCDEF which serves as an inscribed regular hexagon for both the circle and the triangle

I.3 Theorem 3: **METHOD 1 FOR CIRCLING AN EQULATERAL TRIANGLE**

Given an equilateral triangle with side length a, it is possible, using only a straightedge and compass (within the rules of classical construction geometry), to construct a circle (O, r) whose area is exactly equal to the area of the triangle.

PROOF:

Let triangle ABC be the given equilateral triangle with side length a. Using a straightedge and compass, construct the perpendicular bisectors of all three sides of triangle ABC. These bisectors intersect at the center O, which is also the center of the circumcircle (O, R) of the triangle (Figure 3 below).

Now, extend the construction so that the perpendicular bisectors intersect the circumcircle at three new points D, E, and F. Connecting these points forms triangle DEF, which is also equilateral and has the same area as triangle ABC.

The intersection of the two equilateral triangles, triangle ABC and triangle DEF, defines a regular hexagon ABCDEF, which is inscribed in both triangles.

Next, construct the circumcircle (O, r) of the regular hexagon ABCDEF.

By Theorem 2, this circle (O, r) has an area exactly equal to that of the original triangle ABC, thus completing the construction.

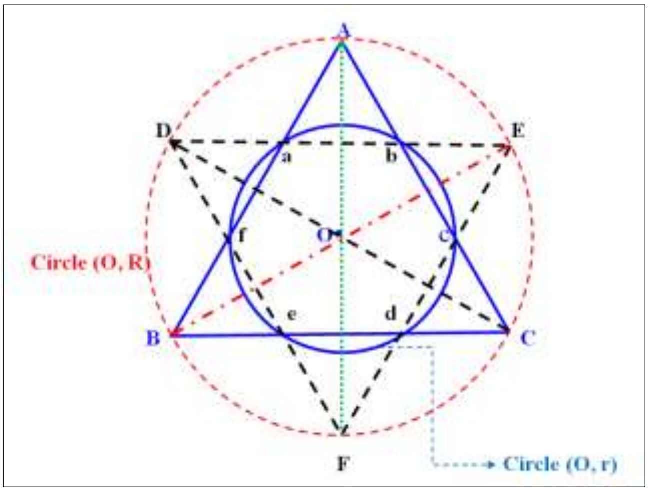


Figure 3: The figure illustrates the concentric circle (O, r) and the equilateral triangle ABC, both centered at O. This figure represents the exact geometric solution to the problem of "CIRCLING AN EQUILATERAL TRIANGLE."

I.4 Corollary 1:

Given an equilateral triangle triangle ABC with side length a, the radius r of a circle (O, r), whose area is exactly equal to the area of the triangle, is equal to one-third of a; that is $\frac{1}{3}$a.

PROOF:

Let triangle ABC be an equilateral triangle with side length a. According to Theorem 3, there exists a concentric circle (O, r) whose area is equal to the area of triangle ABC. This circle intersects the triangle at six vertices of a regular hexagon abcdef inscribed within triangle ABC (see Figure 4 below).

Extensions of three non-consecutive sides of this hexagon intersect the circumscribed (O, R) of triangle ABC at points D, E, and F, forming triangle DEF, which is congruent and equal to triangle ABC, (see Figure 4 below).

Now, consider line segment BC // DE, which intersects the perpendicular bisector of circle (O, r) at points e and d. Let M be the midpoint of BC, so that BM = MC, (see Figure 4 below). Since segment AF is the perpendicular bisector of BC, quadrilateral eOdF is a rhombus with angles $e\widehat{O}d = e\widehat{F}d = 60°$. Hence, triangles eOd and eFd are equilateral with.

ed = r (1)

Similarly, consider quadrilateral cOdC. By an argument analogous to the one above, cOdC is also a rhombus with angles $d\widehat{O}c = d\widehat{C}c = 60°$, so

Dc = dC= r (2)

Applying the same reasoning to quadrilateral BeOf, we obtain

ef = Be = r (3)

From equations (1), (2), and (3), we conclude that

Be = ed = dC = r = $\boldsymbol{\frac{1}{3}a}$

Therefore, the radius r of the circle (O, r) is exactly one-third of the side length a of the given equilateral triangle ABC, as required.

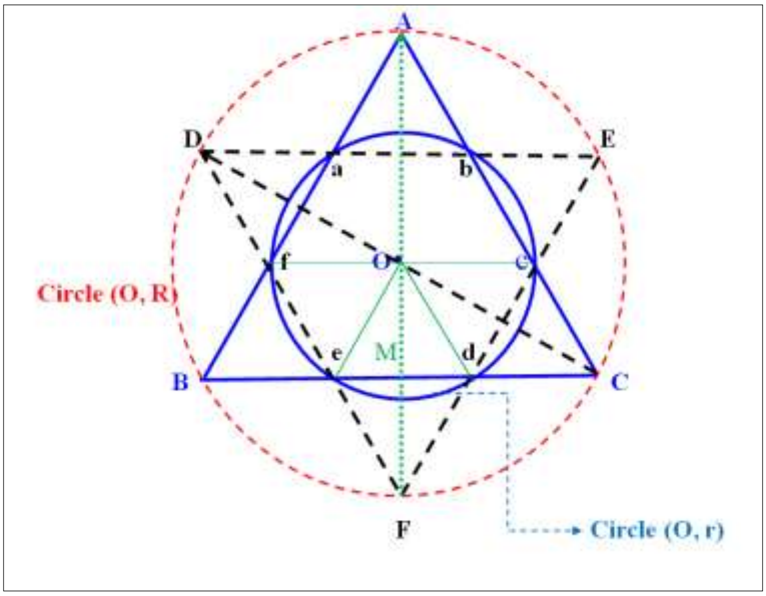


Figure 4: The figure illustrates that the radius r of the concentric circle (O, r) is equal to one-third the side length a of triangle ABC (or triangle DEF)

I.5. Corollary 2: **METHOD 2 FOR CIRCLING AN EQULATERAL TRIANGLE**

It is certainly possible to construct a circle (O, r) with the same area as a given equilateral triangle ABC with side length a, using only a straightedge and compass in Euclidean Geometry.

PROOF:

Let ABC be the given equilateral triangle with side length a. By Corollary 1 above, the radius r of a circle (O, r) with the same area as triangle ABC is exactly $\frac{1}{3}\boldsymbol{a}$.

Now, consider an arbitrary length Bf'. Extend this length into f′a′ and then a′B′ using a straightedge and compass, so that Bf′= f′a′=a′B′. Connect B′A. Then, from points a′ and f′, use a straightedge and compass to draw parallel line segments, each parallel to B′A that these parallel segments intersect AB at points a and f. As a result, AB is divided into three equal parts, each of length is $\frac{1}{3}\boldsymbol{a}$, by the congruence of the triangles Bff′, Baa′, and BAB′.

Next, from the center O of the given equilateral triangle ABC, draw the segment Oa or Of. By Corollary 1 and Theorem 3, the radius r of the circle (O, r) is equal to Oa or Of.

Therefore, using a compass, we can certainly construct the circle (O, r) as required (see Figure 5 below).

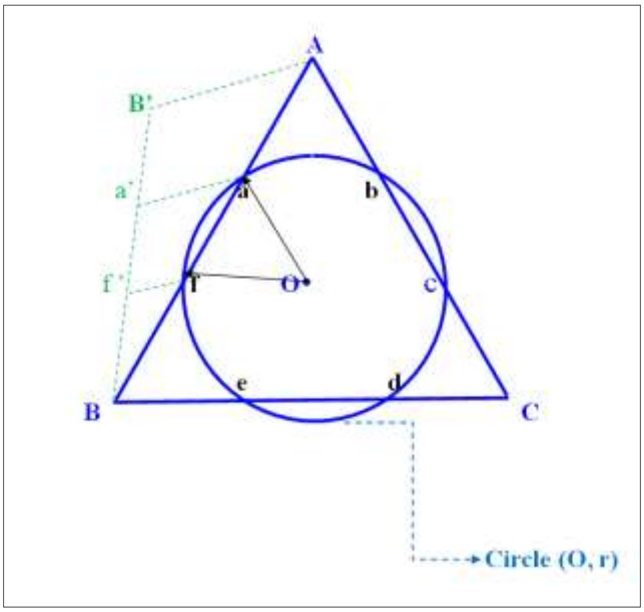


Figure 5: The figure illustrates the construction of the segments Aa = af = fB and the concentric circle (O, r).

II. DISCUSSION AND CONCLUSION

Can mathematicians construct a circle with the same area as a given equilateral triangle using only a straightedge and compass?

My previous articles presented the results of my independent research, which provided an exact solution to the ancient Greek challenge of "Squaring the Circle" and introduces the idea that if one can square a circle, then one can also circle a square.

Then, a construction was detailed in my paper "Regular Triangling a Circle with Straightedge and Compass in Euclidean Geometry" published in the Scholars Journal of Physics, Mathematics and Statistics on March 15, 2025 [Tran Dinh Son, DOI: https://doi.org/10.36347/sjpms.2025.v12i03.003].

This research demonstrates that it is indeed possible to construct, with only a straightedge and compass, a circle of radius r such that its area (πr^2) exactly equals the area of a given equilateral triangle.

Importantly, this method does not rely on approximation or on the numerical value of π. It is a purely geometric construction performed within the classical framework of Euclidean geometry.

The inspiration for this research came after the publication of my previous solution to the "Circling a Square" problem in SJPMS in 2024. If one can circle a square using only classical tools, then it raises the natural question: Why not "circle" other regular polygons such as an equilateral triangle, a regular pentagon and a regular hexagon, ..., independent of π, similar to how one can regularly triangle a circle?

In this article paper, the given equilateral triangle includes an inscribed regular hexagon. By extending three non-consecutive sides of this hexagon, a second equilateral triangle is formed. This new triangle defines the solution to the problem of *"Circling An Equilateral Triangle."* The key is to prove that the intersection of the two triangles matches the area of the regular hexagon, which in turn equals the area of the resulting circle.

Building on this method, a new geometric challenge emerges: *"Circling a Regular Hexagon"* using only a straightedge and compass. Specifically, given a regular hexagon with side length $R \in \mathbb{R}$, how can we construct a circle with exactly the same area—again, without invoking the numerical value of π? This remains an open question in Euclidean geometry.

Additionally, this research opens the door to a new inverse problem: *"Regular Hexagoning a Circle"*—constructing a regular hexagon with an area equal to that of a given circle, using only straightedge and compass.

Finally, all derivatives from the Squaring the Circle problem (ancient Greek challenge) for further researches, can be summarised in the following diagram:

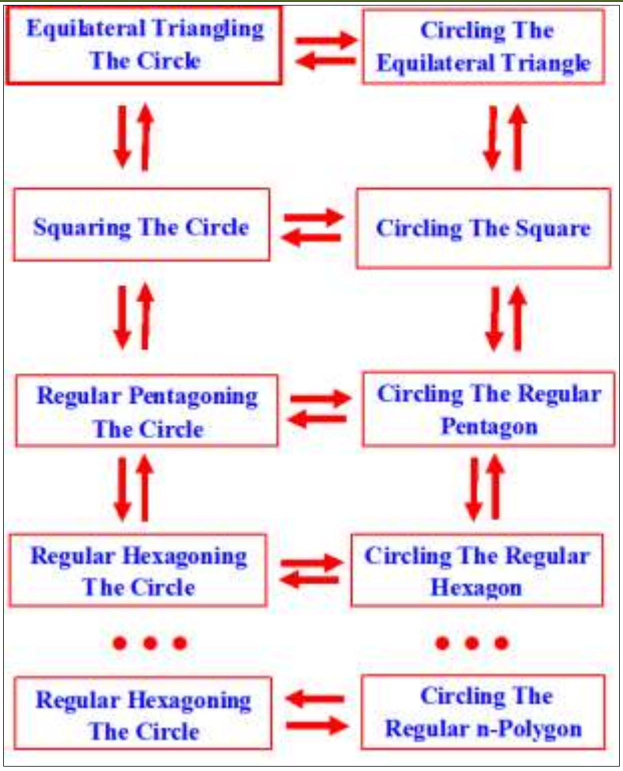


Diagram*: Relationships Between Classical and New Geometric Challenges.*

Among the ten problems illustrated above, I have solved six:

- Equilateral Triangling the Circle
- Circling An Equilateral Triangle
- Squaring the Circle
- Circling the Square
- Regular Pentagoning the Circle
- Circling the Regular Pentagon

Thus, only four remain unsolved, presenting exciting opportunities for further research.

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