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Some Aspects of the Geometry for Conharmonic Curvature Tensor of the Locally Conformal Kahler Manifold

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Abstract: In this research, calculated components conharmonic curvature tensor in some aspects Hermitian manifold in particular of the Locally Conformal Kahler manifold. And proved that this tensor possesses the classical symmetry properties of the Riemannian curvature. Also, establish relationships between the components of the tensor in this manifold.
Keywords: Conharmonic curvature tensor, Locally Conformal Kahler Manifold.

INTRODUCTION

Conformal transformations of Riemannian structures are the important object of differential geometry, where this "transformations which keeping the property of smooth harmonic function". It is known, that such transformations have tensor in variant so-called conharmonic curvature tensor, in this paper we investigated the "conharmonic curvature tensor of locally conformal Kahler manifold".

Preliminaries

Let M –"smooth manifold of dimension 2n", and let g and $\tilde{g}$ be two Riemannian metrics on smooth manifold M, we say that on M given a conformal transformation metric if there is a smooth function $f \in C^{\infty}(M)$ such that $\tilde{g} = e^{2f}g$.

Let $\{M, J, g = \langle , \rangle\}$ be an AH-manifold, if there exists a conformal transformation of the metric g into the metric $\tilde{g}$, then $\{M, J, \tilde{g} = e^{2f}g\}$ will be AH $-$manifold, In this case we say that on smooth manifold M given conformal transformation of AH-structure, denoted by $\widetilde{M}_f$.

Definition1.1 [2]

An AH -manifold is called a locally conformal Kahler manifold, if for each point $m \in M$ there exist an open neighborhood U of this point and there exists $f \in C^{\infty}(M)$ such that $\widetilde{U}_f$ is Kahler manifold. We will denoted to the locally conformal Kahler manifold by L.C.K.

Definition 1.2 [2]

Let M be an AH-manifold, the form which is given by the relation

$\alpha = \frac{-1}{n-1} S\,\Omega \circ J$ is called a Lie form, where S represents the coderivative. If Ω is $r-$form. then its coderivative is $(r-1)-$form, and its dual is a vector

which is called a Lie vector.

Remark 1.3 [3]

By the Banaru's classification of AH-manifold, the L.C.K- manifold satisfies the following conditions:

$$B^{abc} = 0 \;, B^{ab}_{c} = \alpha^{[a}\delta^{b]}_{c}$$

Theorem 1.4 [4]

The structure equations of L.C.K- manifold in the adjoint$G-$structure space is given by the following forms:

1.$d\omega^a = \omega^a_b \wedge \omega^b + B^{ab}_c \omega^c \wedge \omega_b$

2.$d\omega_a = -\omega^b_a \wedge \omega_b + B^c_{ab}\omega_c \wedge \omega^b$

3.$d\omega^a_b = \omega^a_c \wedge \omega^c_b + A^{ad}_{bc}\omega^c \wedge \omega_d + \{\frac{1}{2}\alpha^{a[c}\delta^{d]}_b + \frac{1}{4}\alpha^a\alpha^{[c}\delta^{d]}_b\}\omega_c \wedge \omega_d$

Theorem 1.5 [4]

In the adjoint$G$ −structure spaace, the component of Riemannian curvature tensor of L.C.K- manifold are given by the following forms :

1. $R^{a}_{bcd} = \alpha_{a[c}\delta^{b}_{d]} + \frac{1}{2}\alpha_a\alpha_{[c}\delta^{b}_{d]}$
2. $R^{\hat{a}}_{\hat{b}\hat{c}\hat{d}} = -\alpha^{a[c}\delta^{d]}_{b} - \frac{1}{2}\alpha^a\alpha^{[c}\delta^{d]}_{b}$
3. $R^{a}_{\hat{b}cd} = -2\alpha^{[a}_{[c}\delta^{b]}_{d]}$
4. $R^{\hat{a}}_{b\hat{c}\hat{d}} = 2\alpha^{[c}_{[a}\delta^{d]}_{b]}$
5. $R^{a}_{bc\hat{d}} = A^{ad}_{bc} - \alpha^{[a}\delta^{h]}_{c}\alpha_{[h}\delta^{d}_{b]}$
6. $R^{\hat{a}}_{\hat{b}\hat{c}d} = -A^{bc}_{ad} + \alpha^{[h}\delta^{b]}_{d}\alpha_{[a}\delta^{c}_{h]}$
7. $R^{a}_{b\hat{c}d} = A^{ad}_{bc} - \alpha^{[a}\delta^{h]}_{d}\alpha_{[b}\delta^{c}_{h]}$
8. $R^{\hat{a}}_{\hat{b}c\hat{d}} = -A^{bc}_{ad} + \alpha^{[b}\delta^{h]}_{c}\alpha_{[a}\delta^{d}_{h]}$
9. $R^{a}_{b\hat{c}\hat{d}} = \alpha^{a[c}\delta^{d]}_{b} + \frac{1}{2}\alpha^a\alpha^{[c}\delta^{d]}_{b}$
10. $R^{\hat{a}}_{\hat{b}cd} = -\alpha_{a[c}\delta^{b}_{d]} - \frac{1}{2}\alpha_a\alpha_{[c}\delta^{b}_{d]}$
11. $R^{a}_{\hat{b}\hat{c}d} = -\alpha^{[a|c|}\delta^{b]}_{d} + \alpha^{[a}\delta^{b]}_{h}\alpha^{[h}\delta^{c]}_{d}$
12. $R^{\hat{a}}_{bc\hat{d}} = \alpha_{[a|c|}\delta^{d}_{b]} - \alpha_{[a}\delta^{h}_{b]}\alpha_{[h}\delta^{d}_{c]}$
13. $R^{a}_{\hat{b}c\hat{d}} = -\alpha^{[a|d|}\delta^{b]}_{c} + \alpha^{[a}\delta^{b]}_{h}\alpha^{[h}\delta^{d]}_{c}$
14. $R^{\hat{a}}_{b\hat{c}d} = \alpha_{[a|d|}\delta^{c}_{b]} - \alpha_{[a}\delta^{h}_{b]}\alpha_{[h}\delta^{c}_{d]}$
15. $R^{a}_{\hat{b}\hat{c}\hat{d}} = 0$
16. $R^{\hat{a}}_{bcd} = 0$

We need the components of Ricci tensor of L.C.K- manifold, so we compute it as the following.

Theorem 1.6 [5]

In the adjoint G −structure space, the component of Ricci of L.C.K- manifold are given by the following forms:

1. $r_{ab} = \alpha_{c[b}\delta^{a}_{c]} + \frac{1}{2}\alpha_c\alpha_{[b}\delta^{a}_{c]} + \alpha_{[c|b|}\delta^{c}_{a]} - \alpha_{[c}\delta^{h}_{a]}\alpha_{[h}\delta^{c}_{b]}$
2. $r_{\hat{a}\hat{b}} = -\alpha^{c[b}\delta^{c]}_{a} - \frac{1}{2}\alpha^c\alpha^{[b}\delta^{c]}_{a} - \alpha^{[c|b|}\delta^{a]}_{c} + \alpha^{[c}\delta^{a]}_{h}\alpha^{[h}\delta^{b]}_{c}$
3. $r_{a\hat{b}} = 2\alpha^{[b}_{[c}\delta^{c]}_{a]} + A^{cc}_{ab} - \alpha^{[c}\delta^{h]}_{c}\alpha_{[a}\delta^{b}_{h]}$
4. $r_{\hat{a}b} = -2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ab}_{cc} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}$

Remark 1.7 [6]

The value of Riemannian metric g is define by the form

1. $g_{ab} = g_{\hat{a}\hat{b}} = 0$
2. $g_{\hat{a}b} = \delta^{a}_{b}$
3. $g_{a\hat{b}} = \delta^{b}_{a}$

Definition 1.8

Suppose (M, J, g) is a AH-manifol, the conharmonic curvature of the $(L.C.K)$ difine as tensor $K = \{K^{i}_{jkl}\}$of type (3,1) by the form:

$$K^{i}_{jkl} = R^{i}_{jkl} - \frac{1}{2(n-1)}[r_{il}g_{jk} + r_{jk}g_{il} - r_{jl}g_{ik} - r_{ik}g_{jl}]$$

Where R Riemannian curvature tensor is $.\, r$ is Ricci tensor and g is Riemannian metric.

Theorem1.9

In the adjoint G-structure space, the components of the conharmonic tensor of the $L.C.K$ manifold are given by the following forms:

1)$K^{a}_{bcd} = R^{a}_{bcd}$
2)$K^{a}_{\hat{b}cd} = R^{a}_{\hat{b}cd}$
3)$K^{a}_{\hat{b}\hat{c}d} = R^{a}_{\hat{b}\hat{c}d}$
4)$K^{a}_{bc\hat{d}} = R^{a}_{bc\hat{d}}$
5)$K^{a}_{b\hat{c}\hat{d}} = R^{a}_{b\hat{c}\hat{d}}$
6)$K^{a}_{\hat{b}\hat{c}\hat{d}} = R^{a}_{\hat{b}\hat{c}\hat{d}}$

7) $K^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}c\hat{d}} - \frac{1}{(1-n)}(r^{[d}{}_{[a}\delta^{b]}{}_{c]})$

8) $K^{a}_{\hat{b}\hat{c}d} = R^{a}_{\hat{b}\hat{c}d} + \frac{1}{(1-n)}(r^{[b}{}_{[d}\delta^{c]}{}_{a]})$

And the other are conjugate of them .

Proof:-

1) put, $i = a, j = b, k = c, l = d$,

$$K^{a}_{bcd} = R^{a}_{bcd} - \frac{1}{2(n-1)}[r_{ad}g_{bc} + r_{bc}g_{ad} - r_{bd}g_{ac} - r_{ac}g_{bd}]$$

$$K^{a}_{bcd} = R^{a}_{bcd} - \frac{1}{2(n-1)}[r_{ad}(0) + r_{bc}(0) - r_{bd}(0) - r_{ac}(0)]$$

$$K^{a}_{bcd} = R^{a}_{bcd}$$

2) put, $i = a, j = \hat{b}, k = c, l = d$,

$$K^{a}_{\hat{b}cd} = R^{a}_{\hat{b}cd} - \frac{1}{2(n-1)}[r_{ad}g_{\hat{b}c} + r_{\hat{b}c}g_{ad} - r_{\hat{b}d}g_{ac} - r_{ac}g_{\hat{b}d}]$$

$$K^{a}_{\hat{b}cd} = R^{a}_{\hat{b}cd} - \frac{1}{2(n-1)}[r_{ad}g_{\hat{b}c} + r_{\hat{b}c}(0) - r_{\hat{b}d}(0) - r_{ac}g_{\hat{b}d}]$$

If $c \leftrightarrow d$, then

$$K^{a}_{\hat{b}cd} = R^{a}_{\hat{b}cd} - \frac{1}{2(n-1)}[r_{ad}g_{\hat{b}c} - r_{ad}g_{\hat{b}c}]$$

$$K^{a}_{\hat{b}cd} = R^{a}_{\hat{b}cd}$$

3) put, $i = a$, $j = b$, $k = \hat{c}$, $l = d$

$$K^{a}_{b\hat{c}d} = R^{a}_{b\hat{c}d} - \frac{1}{2(n-1)}[r_{ad}g_{b\hat{c}} + r_{b\hat{c}}g_{ad} - r_{bd}g_{a\hat{c}} - r_{a\hat{c}}g_{bd}]$$

$$K^{a}_{b\hat{c}d} = R^{a}_{b\hat{c}d} - \frac{1}{2(n-1)}[r_{ad}g_{b\hat{c}} + r_{b\hat{c}}(0) - r_{bd}g_{a\hat{c}} - r_{a\hat{c}}(0)]$$

$$K^{a}_{b\hat{c}d} = R^{a}_{b\hat{c}d} - \frac{1}{2(n-1)}[r_{ad}g_{b\hat{c}} - r_{bd}g_{a\hat{c}}]$$

If $b \leftrightarrow a$ then

$$K^{a}_{b\hat{c}d} = R^{a}_{b\hat{c}d} - \frac{1}{2(n-1)}[r_{ad}g_{b\hat{c}} - r_{ad}g_{b\hat{c}}]$$

$$K^{a}_{b\hat{c}d} = R^{a}_{b\hat{c}d}$$

4) put, $i = a$, $j = b$, $k = c$, $l = \hat{d}$,

$$K^{a}_{bc\hat{d}} = R^{a}_{bc\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{bc} + r_{bc}g_{a\hat{d}} - r_{b\hat{d}}g_{ac} - r_{ac}g_{b\hat{d}}]$$

$$K^{a}_{bc\hat{d}} = R^{a}_{bc\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}(0) + r_{bc}g_{a\hat{d}} - r_{b\hat{d}}(0) - r_{ac}g_{b\hat{d}}]$$

$$K^{a}_{bc\hat{d}} = R^{a}_{bc\hat{d}} - \frac{1}{2(n-1)}[r_{bc}g_{a\hat{d}} - r_{ac}g_{b\hat{d}}]$$

If $a \leftrightarrow b$ then

$$K^{a}_{bc\hat{d}} = R^{a}_{bc\hat{d}} - \frac{1}{2(n-1)}[r_{bc}g_{a\hat{d}} - r_{bc}g_{a\hat{d}}]$$

$$K^{a}_{bc\hat{d}} = R^{a}_{bc\hat{d}}$$

5) put, $i = a$, $j = b$, $k = \hat{c}$, $l = \hat{d}$,

$$K^{a}_{b\hat{c}\hat{d}} = R^{a}_{b\hat{c}\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{b\hat{c}} + r_{b\hat{c}}g_{a\hat{d}} - r_{b\hat{d}}g_{a\hat{c}} - r_{a\hat{c}}g_{b\hat{d}}]$$

If $a \leftrightarrow b$ then

$$K^{a}_{b\hat{c}\hat{d}} = R^{a}_{b\hat{c}\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{b\hat{c}} + r_{b\hat{c}}g_{a\hat{d}} - r_{a\hat{d}}g_{b\hat{c}} - r_{b\hat{c}}g_{a\hat{d}}]$$

$$K^{a}_{b\hat{c}\hat{d}} = R^{a}_{b\hat{c}\hat{d}}$$

6) put, $i = a$, $j = \hat{b}$, $k = \hat{c}$, $l = \hat{d}$,

$$K^{a}_{\hat{b}\hat{c}\hat{d}} = R^{a}_{\hat{b}\hat{c}\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{\hat{b}\hat{c}} + r_{\hat{b}\hat{c}}g_{a\hat{d}} - r_{\hat{b}\hat{d}}g_{a\hat{c}} - r_{a\hat{c}}g_{\hat{b}\hat{d}}]$$

$$K^{a}_{\hat{b}\hat{c}\hat{d}} = R^{a}_{\hat{b}\hat{c}\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}(0) + r_{\hat{b}\hat{c}}g_{a\hat{d}} - r_{\hat{b}\hat{d}}g_{a\hat{c}} - r_{a\hat{c}}(0)]$$

$$K^{a}_{\hat{b}\hat{c}\hat{d}} = R^{a}_{\hat{b}\hat{c}\hat{d}} - \frac{1}{2(n-1)}[r_{\hat{b}\hat{c}}g_{a\hat{d}} - r_{\hat{b}\hat{d}}g_{a\hat{c}}]$$

If $\hat{d} \leftrightarrow \hat{c}$ then

$$K^{a}_{\hat{b}\hat{c}\hat{d}} = R^{a}_{\hat{b}\hat{c}\hat{d}} - \frac{1}{2(n-1)}[r_{\hat{b}\hat{c}}g_{a\hat{d}} - r_{\hat{b}\hat{c}}g_{a\hat{d}}]$$
$$K^{a}_{\hat{b}\hat{c}\hat{d}} = R^{a}_{\hat{b}\hat{c}\hat{d}}$$

7) put , $i = a$, $j = \hat{b}$, $k = c$, $l = \hat{d}$,

$$K^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}c\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{\hat{b}c} + r_{\hat{b}c}g_{a\hat{d}} - r_{\hat{b}\hat{d}}g_{ac} - r_{ac}g_{\hat{b}\hat{d}}]$$
$$K^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}c\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{\hat{b}c} + r_{\hat{b}c}g_{a\hat{d}} - r_{\hat{b}\hat{d}}(0) - r_{ac}(0)]$$
$$K^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}c\hat{d}} - \frac{1}{2(n-1)}[r_{a\hat{d}}g_{\hat{b}c} + r_{\hat{b}c}g_{a\hat{d}}]$$
$$K^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}c\hat{d}} - \frac{1}{2(n-1)}[r^{d}_{a}\delta^{b}_{c} + r^{b}_{c}\delta^{d}_{a}]$$
$$K^{a}_{\hat{b}c\hat{d}} = R^{a}_{\hat{b}c\hat{d}} - \frac{1}{(n-1)}(r^{[d}_{[a}\delta^{b]}_{c]})$$

8) put, $i = a$, $j = \hat{b}$, $k = \hat{c}$, $l = d$,

$$K^{a}_{\hat{b}\hat{c}d} = R^{a}_{\hat{b}\hat{c}d} - \frac{1}{2(n-1)}[r_{ad}g_{\hat{b}\hat{c}} + r_{\hat{b}\hat{c}}g_{ad} - r_{\hat{b}d}g_{a\hat{c}} - r_{a\hat{c}}g_{\hat{b}d}]$$
$$K^{a}_{\hat{b}\hat{c}d} = R^{a}_{\hat{b}\hat{c}d} - \frac{1}{2(n-1)}[r_{ad}(0) + r_{\hat{b}\hat{c}}(0) - r_{\hat{b}d}g_{a\hat{c}} - r_{a\hat{c}}g_{\hat{b}d}]$$
$$K^{a}_{\hat{b}\hat{c}d} = R^{a}_{\hat{b}\hat{c}d} + \frac{1}{2(n-1)}[-r^{b}_{d}\delta^{c}_{a} - r^{c}_{a}\delta^{b}_{d}]$$
$$K^{a}_{\hat{b}\hat{c}d} = R^{a}_{\hat{b}\hat{c}d} + \frac{1}{(n-1)}(r^{[b}_{[d}\delta^{c]}_{a]})$$

Proposition 1.10

The conharmonic curvature of (L. C. K) manifold satisfies all the properties the algebraic:
1) $K(X_1, X_2, X_3, X_4) = -K(X_2, X_1, X_3, X_4)$
2) $K(X_1 X_2, X_3, X_4) = -K(X_1, X_2, X_4, X_3)$
3) $K(X_1, X_2, X_3, X_4) + K(X_2, X_3, X_1, X_4) + K(X_3, X_1, X_2, X_4) = 0$
4) $K(X_1, X_2, X_3, X_4) = K(X_3, X_4, X_1, X_2)$
Where $X_i \in X(M), i = 1,2,3,4$
Prove: we shall prove just (1)
1) $K(X_1, X_2, X_3, X_4) = R(X_1, X_2, X_3, X_4) - \frac{1}{2(n-1)}\{g(X_1, X_3)r(X_2, X_4) + g(X_2, X_4)r(X_1, X_3) - g(X_1, X_4)r(X_2, X_3) - g(X_2, X_3)r(X_1, X_4)\}$

$$= -R(X_1, X_2, X_3, X_4) + \frac{1}{2(n-1)}\{g(X_1, X_3)r(X_2, X_4) + g(X_2, X_4)r(X_1, X_3) - g(X_1, X_4)r(X_2, X_3) - g(X_2, X_3)r(X_1, X_4)\}$$
$$= -K(X_2, X_1, X_3, X_4)$$

Properties are similarly proved:
2) $K(X_1, X_2, X_3, X_4) = -K(X_1, X_2, X_4, X_3)$
3) $K(X_1, X_2, X_3, X_4) + K(X_2, X_1, X_3, X_4) + K(X_3, X_1, X_2, X_4) = 0$
4) $K(X_1, X_2, X_3, X_4) = -K(X_3, X_4, X_1, X_2)$

$$X_i \in X(M), i = 1,2,3,4$$

,(1),(2),(3) and (4) is called an algebra curvature tensor of (L. C. K) manifolds
The conharmonic curvature of (L. C. K) manifolds looks like

$$K(X_1, X_2)X_3 = R(X_1, X_2)X_3 - \frac{1}{2(n-1)}\{< X_2, X_3 > X_1 r + < X_1, X_3 > X_2 r - < X_2, X_3 > QX_1 - < X_1, X_3 > QX_2\}$$

Where $Q = r$
By definition of a spectrum tensor

$$K(X_1, X_2)X_3 = K_0 < X_1, X_2 > X_3 + K_1 < X_1, X_2 > X_3 + K_2 < X_1, X_2 > X_3 + K_3 < X_1, X_2 > X_3 + K_4 < X_1, X_2 > X_3 + K_5 < X_1, X_2 > X_3 + K_6 < X_1, X_2 > X_3 + K_7 < X_1, X_2 > X_3$$

Tensor $K_0 < X_1, X_2 > X_3$ nonzero – the component have only components
Of the form:

Tensor $K_0 < X_1, X_2 > X_3 -$ components$\{K_{0\ bcd}^{a}, K_{0\ \hat{b}\hat{c}\hat{d}}^{\hat{a}}\} = \{K_{bcd}^{a}, K_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}\}$
Tensor $K_1 < X_1, X_2 > X_3 -$ components$\{K_{1\ bc\hat{d}}^{a}, K_{1\ \hat{b}\hat{c}d}^{\hat{a}}\} = \{K_{bc\hat{d}}^{a}, K_{\hat{b}\hat{c}d}^{\hat{a}}\}$
Tensor $K_2 < X_1, X_2 > X_3 -$ components$\{K_{2\ b\hat{c}d}^{a}, K_{2\ \hat{b}c\hat{d}}^{\hat{a}}\} = \{K_{b\hat{c}d}^{a}, K_{\hat{b}c\hat{d}}^{\hat{a}}\}$
Tensor $K_3 < X_1, X_2 > X_3 -$ components$\{K_{3\ \hat{b}cd}^{a}, K_{3\ b\hat{c}\hat{d}}^{\hat{a}}\} = \{K_{\hat{b}cd}^{a}, K_{b\hat{c}\hat{d}}^{\hat{a}}\}$
Tensor $K_4 < X_1, X_2 > X_3 -$ components$\{K_{4\ b\hat{c}\hat{d}}^{a}, K_{3\ \hat{b}cd}^{\hat{a}}\} = \{K_{b\hat{c}\hat{d}}^{a}, K_{\hat{b}cd}^{\hat{a}}\}$
Tensor $K_5 < X_1, X_2 > X_3 -$ components$\{K_{5\ \hat{b}\hat{c}\hat{d}}^{a}, K_{5\ b\hat{c}d}^{\hat{a}}\} = \{K_{\hat{b}\hat{c}\hat{d}}^{a}, K_{b\hat{c}d}^{\hat{a}}\}$
Tensor $K_6 < X_1, X_2 > X_3 -$ components$\{K_{6\ \hat{b}\hat{c}d}^{a}, K_{6\ bc\hat{d}}^{\hat{a}}\} = \{K_{\hat{b}\hat{c}d}^{a}, K_{bc\hat{d}}^{\hat{a}}\}$
Tensor $K_7 < X_1, X_2 > X_3 -$ components$\{K_{7\ \hat{b}\hat{c}\hat{d}}^{a}, K_{7\ bcd}^{\hat{a}}\} = \{K_{\hat{b}\hat{c}\hat{d}}^{a}, K_{bcd}^{\hat{a}}\}$
Tensors $K_0 = K_0 < X_1, X_2 > X_3, K_1 = K_1 < X_1, X_2 > X_3, \dots, K_7 = K_7 < X_1, X_2 > X_3$.
The basic invariants conharmonic$(L.C.K)$ manifold will be named.

Definition 1.11
L.C.K- manifold for which $K_i = 0$ is LCK- manifold of class $K_i, i = 0, 1, \dots, 7$,
The manifold of class K_0 characterized by a condition $K_{0\ bcd}^{a} = 0$, or
$K_{bcd}^{a} = 0$, $[K(\varepsilon_c, \varepsilon_d)\varepsilon_b]^a \varepsilon_a = 0$, As σ- a projector on $D_J^{\sqrt{-1}}$, that

$$\sigma \circ \{K(\sigma X_1, \sigma X_2)\sigma X_3 = 0.$$

i,e$\left(id - \sqrt{-1}j\right)\left\{K\left(X - \sqrt{-1}JX, Y - \sqrt{-1}JY\right)\left(Z - \sqrt{-1}JZ\right)\right\} = 0$
Removing the brackets can be received:

$$K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 - \sqrt{-1}\{K(X_1, X_2)JX_3 + K(X_1, JX_2)X_3 + K(JX_1, X_2)X_3 - K(J X_1, JX_2)JX_3\} - JK(X_1, X_2)X_3 - JK(X_1, JX_2)JX_3 - JK(JX_1, X_2)JX_3 + JK(JX_1, JX_2)X_3\} = 0.$$

i,e,
1) $K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 = 0.$
Thus LCK- manifold of class K_0 characterized by identity
2)$K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 = 0,$

$$X_1, X_2, X_3 \in X(M).$$

These equalities are equivalent; the second equality turns out from the first
Replacement X_3on JX_3.

$$K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 = 0.$$

$$X_1, X_2, X_3 \in X(M).$$

Similarly considering L.C.K- manifold of classes $K_1 - K_7$ can be receved the following theorem.

Theorem 1.12
1) L.C.K- manifold of class K_0 characterized by identity

$$K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 = 0,$$

$$X_1, X_2, X_3 \in X(M).$$

2) L.C.K- manifold of class K_1 characterized by identity

$$K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 = 0.$$

$$X_1, X_2, X_3 \in X(M).$$

3) $L.C.K -$ manifold of class K_2 characterized by identity

$$K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 = 0.$$

$$X_1, X_2, X_3 \in X(M).$$

4) $L.C.K -$ manifold of class K_3 characterized by identity

$$K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 + JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 = 0.$$

$$X_1, X_2, X_3 \in X(M).$$

5) L.C.K- manifold of class K_4 characterized by identity

$$K(X_1,X_2)X_3 + K(X_1,JX_2)JX_3 + K(JX_1,X_2)JX_3 - K(J\,X_1,JX_2)X_3 + JK(X_1,X_2)JX_3 - JK(X_1,JX_2)X_3 - JK(JX_1,X_2)X_3 - JK(JX_1,JX_2)JX_3 = 0.$$
$$X_1,X_2,X_3 \in X(M).$$

6) L.C.K- manifold of class K_5 characterized by identity
$$K(X_1,X_2)X_3 - K(X_1,JX_2)JX_3 + K(JX_1,X_2)JX_3 + K(J\,X_1,JX_2)X_3 + JK(X_1,X_2)JX_3 + JK(X_1,JX_2)X_3 - JK(JX_1,X_2)X_3 + JK(JX_1,JX_2)JX_3 = 0.$$
$$X_1,X_2,X_3 \in X(M).$$

7) $\mathrm{L.C.K}-$ manifold of class K_6 characterized by identity
$$K(X_1,X_2)X_3 + K(X_1,JX_2)JX_3 - K(JX_1,X_2)JX_3 + K(J\,X_1,JX_2)X_3 + JK(X_1,X_2)JX_3 - JK(X_1,JX_2)X_3 + JK(JX_1,X_2)X_3 + JK(JX_1,JX_2)JX_3 = 0.$$
$$X_1,X_2,X_3 \in X(M).$$

8) $\mathrm{L.C.K}-$ manifold of class K_7 characterized by identity
$$K(X_1,X_2)X_3 - K(X_1,JX_2)JX_3 - K(JX_1,X_2)JX_3 - K(J\,X_1,JX_2)X_3 + JK(X_1,X_2)JX_3 + JK(X_1,JX_2)X_3 + JK(JX_1,X_2)X_3 - JK(JX_1,JX_2)JX_3 = 0.$$
$$X_1,X_2,X_3 \in X(M).$$

Theorem 1.13

The following inclusion relation has been found:

i) $K_0 = K_3$,ii) $K_1 = K_2$,iii) $K_4 = K_7$,iiii) $K_5 = K_6$

Prove: - We shall prove (i)

$$K_0 = K(X_1,X_2)X_3 - K(X_1,JX_2)JX_3 - K(JX_1,X_2)JX_3 - K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 - JK(X_1,JX_2)X_3 - JK(JX_1,X_2)X_3 + JK(JX_1,JX_2)JX_3 \quad (1)$$

$$K_0 = K(X_1,X_2)X_3 - K\big(X_1,-\sqrt{-1}JX_2\big)\big(-\sqrt{-1}\big)JX_3 - K\big(-\sqrt{-1}JX_1,X_2\big)\big(-\sqrt{-1}\big)JX_3 - K\big(-\sqrt{-1}J\,X_1,-\sqrt{-1}JX_2\big)X_3 - \big(-\sqrt{-1}\big)JK(X_1,X_2)\big(-\sqrt{-1}\big)JX_3 - \big(-\sqrt{-1}\big)JK\big(X_1,-\sqrt{-1}JX_2\big)X_3 - \big(-\sqrt{-1}\big)JK\big(-\sqrt{-1}JX_1,X_2\big)X_3 + \big(-\sqrt{-1}\big)JK\big(-\sqrt{-1}JX_1,-\sqrt{-1}JX_2\big)\big(-\sqrt{-1}\big)JX_3$$

$$K_0 = K(X_1,X_2)X_3 + K(X_1,JX_2)JX_3 + K(JX_1,X_2)JX_3 - K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 + JK(X_1,JX_2)X_3 + JK(JX_1,X_2)X_3 + JK(JX_1,JX_2)JX_3 \quad (2)$$

From (1) and (2) we get

$$K_0 = K(X_1,X_2)X_3 - K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 + JK(JX_1,JX_2)JX_3 \quad (3)$$

$$K_3 = K(X_1,X_2)X_3 + K(X_1,JX_2)JX_3 + K(JX_1,X_2)JX_3 - K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 + JK(X_1,JX_2)X_3 + JK(JX_1,X_2)X_3 + JK(JX_1,JX_2)JX_3 \quad (4)$$

$$K_3 = K(X_1,X_2)X_3 + K\big(X_1,-\sqrt{-1}JX_2\big)\big(-\sqrt{-1}\big)JX_3 + K\big(-\sqrt{-1}JX_1,X_2\big)\big(-\sqrt{-1}\big)JX_3 - K\big(-\sqrt{-1}J\,X_1,-\sqrt{-1}JX_2\big)X_3 - \big(-\sqrt{-1}\big)JK(X_1,X_2)\big(-\sqrt{-1}\big)JX_3 + \big(-\sqrt{-1}\big)JK\big(X_1,-\sqrt{-1}JX_2\big)X_3 + \big(-\sqrt{-1}\big)JK\big(-\sqrt{-1}JX_1,X_2\big)X_3 + \big(-\sqrt{-1}\big)JK\big(-\sqrt{-1}JX_1,-\sqrt{-1}JX_2\big)\big(-\sqrt{-1}\big)JX_3$$

$$K_3 = K(X_1,X_2)X_3 - K(X_1,JX_2)JX_3 - K(JX_1,X_2)JX_3 - K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 - JK(X_1,JX_2)X_3 - JK(JX_1,X_2)X_3 + JK(JX_1,JX_2)JX_3 \quad (5)$$

From (4) and (5) we get

$$K_3 = K(X_1,X_2)X_3 - K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 + JK(JX_1,JX_2)JX_3 \quad (6)$$

From (3) and (6) we get $K_0 = K_3$

Now we shall prove (ii)

$$K_1 = K(X_1,X_2)X_3 + K(X_1,JX_2)JX_3 - K(JX_1,X_2)JX_3 + K(J\,X_1,JX_2)X_3 + JK(X_1,X_2)JX_3 - JK(X_1,JX_2)X_3 - JK(JX_1,X_2)X_3 - JK(JX_1,JX_2)JX_3 \quad (7)$$

$$K_1 = K(X_1,X_2)X_3 + K\big(X_1,-\sqrt{-1}JX_2\big)\big(-\sqrt{-1}\big)JX_3 - K\big(-\sqrt{-1}JX_1,X_2\big)\big(-\sqrt{-1}\big)JX_3 + K\big(-\sqrt{-1}J\,X_1,-\sqrt{-1}JX_2\big)X_3 + \big(-\sqrt{-1}\big)JK(X_1,X_2)\big(-\sqrt{-1}\big)JX_3 - \big(-\sqrt{-1}\big)JK\big(X_1,-\sqrt{-1}JX_2\big)X_3 - \big(-\sqrt{-1}\big)JK\big(-\sqrt{-1}JX_1,X_2\big)X_3 - \big(-\sqrt{-1}\big)JK\big(-\sqrt{-1}JX_1,-\sqrt{-1}JX_2\big)\big(-\sqrt{-1}\big)JX_3$$

$$K_1 = K(X_1,X_2)X_3 - K(X_1,JX_2)JX_3 + K(JX_1,X_2)JX_3 + K(J\,X_1,JX_2)X_3 + JK(X_1,X_2)JX_3 + JK(X_1,JX_2)X_3 + JK(JX_1,X_2)X_3 - JK(JX_1,JX_2)JX_3 \quad (8)$$

From (7) and (8) we get

$$K_1 = K(X_1,X_2)X_3 + K(J\,X_1,JX_2)X_3 - JK(X_1,X_2)JX_3 - JK(JX_1,JX_2)JX_3 \quad (9)$$

$K_2 = K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 \quad (10)$

$$K_2 = K(X_1, X_2)X_3 - K\left(X_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3 + K\left(-\sqrt{-1}JX_1, -\sqrt{-1}X_2\right)(-\sqrt{-1})JX_3 + K\left(-\sqrt{-1}J X_1, -\sqrt{-1}JX_2\right)X_3 - (-\sqrt{-1})\, JK(X_1, X_2)(-\sqrt{-1})JX_3 - (-\sqrt{-1})JK\left(X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, X_2\right)X_3 - (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3$$

$K_2 = K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 + JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 \quad (11)$

From (10) and (11) we get

$$K_2 = K(X_1, X_2)X_3 + K(J X_1, JX_2)X_3 - JK(X_1, X_2)JX_3 - JK(JX_1, JX_2)JX_3 \quad (12)$$

From (9) and (12) we get $K_1 = K_2$

Now we shall prove (iii)

$K_4 = K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 \quad (13)$

$$K_4 = K(X_1, X_2)X_3 + K\left(X_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3 + K\left(-\sqrt{-1}JX_1, X_2\right)(-\sqrt{-1})JX_3 - K\left(-\sqrt{-1}J X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})JK(X_1, X_2)(-\sqrt{-1})JX_3 - (-\sqrt{-1})JK\left(X_1, -\sqrt{-1}JX_2\right)X_3 - (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, X_2\right)X_3 - (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3$$

$K_4 =$
$K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 + JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 \quad (14)$

From (13) and (14) we get

$$K_4 = K(X_1, X_2)X_3 - K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(JX_1, JX_2)JX_3 \quad (15)$$

$K_7 = K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 + JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 \quad (16)$

$$K_7 = K(X_1, X_2)X_3 - K\left(X_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3 - K\left(-\sqrt{-1}JX_1, X_2\right)(-\sqrt{-1})JX_3 - K\left(-\sqrt{-1}J X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})JK(X_1, X_2)(-\sqrt{-1})JX_3 + (-\sqrt{-1})JK\left(X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, X_2\right)X_3 - (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3$$

$K_7 = K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 - K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 - JK(JX_1, JX_2)JX_3 \quad (17)$

From (16) and (17) we get

$$K_7 = K(X_1, X_2)X_3 - K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(JX_1, JX_2)JX_3 \quad (18)$$

From (15) and (18) we get $K_4 = K_7$

Now we shall prove (iiii)

$K_5 =$
$K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 + JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 \quad (19)$

$$K_5 = K(X_1, X_2)X_3 - K\left(X_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3 + K\left(-\sqrt{-1}JX_1, X_2\right)(-\sqrt{-1})JX_3 + K\left(-\sqrt{-1}J X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})JK(X_1, X_2)(-\sqrt{-1})JX_3 + (-\sqrt{-1})JK\left(X_1, -\sqrt{-1}JX_2\right)X_3 - (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, X_2\right)X_3 + (-\sqrt{-1}]K\left(-\sqrt{-1}JX_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3$$

$K_5 =$
$K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 \quad (20)$

From (19) and (20) we get

$$K_5 = K(X_1, X_2)X_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 + JK(JX_1, JX_2)JX_3 \quad (21)$$

$K_6 = K(X_1, X_2)X_3 + K(X_1, JX_2)JX_3 - K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 - JK(X_1, JX_2)X_3 + JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 \quad (22)$

$$K_6 = K(X_1, X_2)X_3 + K\left(X_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3 - K\left(-\sqrt{-1}JX_1, X_2\right)(-\sqrt{-1})JX_3 + K\left(-\sqrt{-1}J X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})\, JK(X_1, X_2)(-\sqrt{-1})JX_3 - (-\sqrt{-1})JK\left(X_1, -\sqrt{-1}JX_2\right)X_3 + (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, X_2\right)X_3 + (-\sqrt{-1})JK\left(-\sqrt{-1}JX_1, -\sqrt{-1}JX_2\right)(-\sqrt{-1})JX_3$$

$K_6 = K(X_1, X_2)X_3 - K(X_1, JX_2)JX_3 + K(JX_1, X_2)JX_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 + JK(X_1, JX_2)X_3 - JK(JX_1, X_2)X_3 + JK(JX_1, JX_2)JX_3 \quad (23)$

From (22) and (23) we get

$$K_6 = K(X_1, X_2)X_3 + K(J X_1, JX_2)X_3 + JK(X_1, X_2)JX_3 + JK(JX_1, JX_2)JX_3 \quad (24)$$

From (21) and (24) we get $K_5 = K_6$

Definition 1.14

The manifold $(\mathrm{M},\mathrm{J},\mathrm{g})$refers to as manifold of class:

1)$\overline{\mathrm{K}}_1$if $< K(\mathrm{X}_1,\mathrm{X}_2)\mathrm{X}_3,\mathrm{X}_4 > = < K(\mathrm{X}_1,\mathrm{X}_2)\mathrm{JX}_3,\mathrm{JX}_4 >$

2) $\overline{\mathrm{K}}_2$if $< K(\mathrm{X}_1,\mathrm{X}_2)\mathrm{X}_3,\mathrm{X}_4 > = < K(\mathrm{JX}_1,\mathrm{JX}_2)\mathrm{X}_3,\mathrm{X}_4 > + < K(\mathrm{JX}_1,\mathrm{X}_2)\mathrm{JX}_3,\mathrm{X}_4 > + < K(\mathrm{JX}_1,\mathrm{X}_2)\mathrm{X}_3,\mathrm{JX}_4 >$

3) $\overline{\mathrm{K}}_1$if $< K(\mathrm{X}_1,\mathrm{X}_2)\mathrm{X}_3,\mathrm{X}_4 > = < K(\mathrm{JX}_1,\mathrm{JX}_2)\mathrm{JX}_3,\mathrm{JX}_4 >$

Theorem 1.15

Let $\mathrm{S} = (\mathrm{J},\mathrm{g} = <,>)$ is $\mathrm{L},\mathrm{C},\mathrm{K}$,then the following statement are equivalent:

1) S −structure of class $\overline{\mathrm{K}}_3$.

2) $\mathrm{K}_7 = 0$.

3) On space of the adjoint$\mathrm{G}$ −structure identities $\mathrm{K}^{\mathrm{a}}_{\hat{\mathrm{b}}\hat{\mathrm{c}}\hat{\mathrm{d}}} = 0$ are fair .

Prove

Let S −structure of class $\overline{\mathrm{K}}_3$. Obviously it is equivalent to identity

$\mathrm{K}(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{JK}(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c = 0$; $\varepsilon_a,\varepsilon_b,\varepsilon_c \in \mathrm{X}(\mathrm{M})$, By definition of a spectrum tensor

$$\mathrm{K}(\varepsilon_a,\varepsilon_b)\varepsilon_c = \mathrm{K}_0(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_1(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_2(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_3(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_4(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_5(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_6(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_7(\varepsilon_a,\varepsilon_b)\varepsilon_c \ ; \varepsilon_a,\varepsilon_b\varepsilon_c \in \mathrm{X}(\mathrm{M})$$

$$\begin{aligned}\mathrm{JK}(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c &= \mathrm{JK}_0(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c + \mathrm{JK}_1(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c + \mathrm{JK}_2(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c + \mathrm{JK}_3(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c + \mathrm{JK}_4(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c \\ &\quad + \mathrm{JK}_5(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c + \mathrm{JK}_6(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c + \mathrm{JK}_7(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c \\ &= \mathrm{K}_0(\varepsilon_a,\varepsilon_b)\varepsilon_c - \mathrm{K}_1(\varepsilon_a,\varepsilon_b)\varepsilon_c - \mathrm{K}_2(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_3(\varepsilon_a,\varepsilon_b)\varepsilon_c - \mathrm{K}_4(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_5(\varepsilon_a,\varepsilon_b)\varepsilon_c \\ &\quad + \mathrm{K}_6(\varepsilon_a,\varepsilon_b)\varepsilon_c - \mathrm{K}_7(\varepsilon_a,\varepsilon_b)\varepsilon_c\end{aligned}$$

The identity $\mathrm{K}(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{JK}(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c = 0$ is equivalent to that

$$\mathrm{K}_7(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_4(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_5(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{K}_6(\varepsilon_a,\varepsilon_b)\varepsilon_c = 0$$

And this identity is equivalent to $\mathrm{K}_7 = 0$

By virtue of materiality tensor K and its properties (1.10;4) received relation which are equivalent to relations $\mathrm{K}^{\mathrm{a}}_{\hat{\mathrm{b}}\hat{\mathrm{c}}\hat{\mathrm{d}}} = 0$; i.e. identity $\mathrm{K}_7(\varepsilon_a,\varepsilon_b)\varepsilon_c = 0$.

The opposite, according to $\mathrm{K}(\varepsilon_a,\varepsilon_b)\varepsilon_c + \mathrm{JK}(\mathrm{J}\varepsilon_a,\mathrm{J}\varepsilon_b)\mathrm{J}\varepsilon_c = 0$, obviously.

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