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Stability Analysis of Worker Retraining Model with Double Time Delay

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Abstract

In this paper, the stability of retraining system for the same workers in enterprises with double time-delay is investigated. Sufficient conditions about the local asymptotic stability and global stability of the positive equilibrium and the non-negative equilibrium are derived by using characteristic value method and Hurwitz criterion. Finally, the retraining system was numerically simulated with Matlab by taking appropriate parameters and different time delay values, and diagrams of all components change and solution curves were given around the critical value.

Keywords: simple balance, non-trivial equilibrium, local stability, global stability

Mathematics Subject Classification 37C20 37C75 O175.13.

INTRODUCTION

It is an urgent problem to study the law of technology renewal and communication of the same workers and put forward the control policy of retraining workers so as to maximize the benefits of enterprises and workers.

In recent years, with the deep development of the research on mathematical model and dynamic model, researchers find that the state of the system at a certain time is affected by the relationship among the various groups at that time, also by Environment and time factors. At present, scholars have consider the time delay of various mathematical models in the simulation of population change, transmission of infectious diseases, biological science, physics, control theory and other fields, so as to study the equilibrium stability of models with time delay(see[1]-[8]). Literature [9] studied the stability of a smoking cessation model under the influence of public health education, and provided the local stability of smokeless equilibrium point and smoking equilibrium point; In reference [10], a time-delay SEIR computer virus propagation model was studied. The incubation period of computer virus was taken as the bifurcation parameter, and the local asymptotic stability of the model was discussed; Literature [11] studied the stability of SIR infectious disease model with immunisation and population size change; Literature [12] studies the stability analysis of enterprise competition model with double time delay. Brauer and Carlos [13] first proposed the retraining system for skilled workers in the same sector of private enterprises, and iterature [14] divided the system population into interrelated compartments based on the characteristics of the dynamic model, and qualitatively analyzed it, proving that under certain conditions, non-negative equilibrium points and positive equilibrium points have local and global asymptotic stability.

$$\begin{cases} P' = qK - \dfrac{\beta}{K}PM + \delta R - \mu P \\ R' = (1-q)K - (\delta+\mu)R - aR \\ M' = \dfrac{\beta}{K}PM - (r+\mu)M \\ U' = -\mu U + \alpha R \\ I' = rM - \mu I \end{cases}$$

Where $P(t), M(t), R(t), U(t), I(t)$ denote the number of technical workers, regular workers, returning workers, foreman, non-expendable workers (injured or pregnant) and sabotage workers at time t, respectively. The parameters $\alpha, \beta, \mu, r, q, K, \delta$ are positive constants, in which $\frac{\beta}{K}$ is the contact rate between foreman and regular worker; q is the transformation rates from the training workers into regular workers ; μ is the removal rate of all kinds of skilled workers; δ is the transformation rates from the returning workers into regular workers; α is the transformation rates from the returning workers into non-expendable workers; r is the transformation rates from the foreman into sabotage workers; K is the constant.

When the model was established, it did not take into account the influence of environment and time among various groups of workers. Since the non-expendable workers include the workers who are injured and pregnant workers at time t, they can return to formal workers and receive training after the time delay τ ; sabotage workers returns to regular workers after conversion time due to their own needs. When time pass τ, some of the non-expendable $aR(t-\tau)e^{-\mu\tau}$ will return to receive retraining after time; some of the sabotage workers $\theta M(t-\tau)e^{-\mu\tau}$ returned to retraining over time.

So this article consider non-expendable and sabotage workers after a delay return to ordinary workers, in the model has been added to the conversion time, through the analysis of the characteristic equation method and Hurwitz theorem, the local stability and global stability of non-negative equilibrium point, the positive equilibrium point global stability and the sufficient condition of locally asymptotically stable, and USES Matlab to validate related conclusion.

The model is established as follows:

$$\begin{cases} \dfrac{dP(t)}{dt} = qK - \beta P(t)\dfrac{M(t)}{K} + \delta R(t) - \mu P(t) \\ \dfrac{dR(t)}{dt} = (1-q)K - (\delta + \mu + a)R(t) + \theta M(t-\tau)e^{-\mu\tau} + aR(t-\tau)e^{-\mu\tau} \\ \dfrac{dM(t)}{dt} = \beta P(t)\dfrac{M(t)}{K} - (r+\mu)M(t) \\ \dfrac{dU}{dt} = -\mu U - aR(t-\tau)e^{-\mu\tau} + aR(t) \\ \dfrac{dI}{dt} = rM(t) - \mu I - \theta M(t-\tau)e^{-\mu\tau} \end{cases} \tag{1}$$

According to the system (1), the total number of workers can be obtained to satisfy the equation:

$$N' = K - \mu N. \tag{2}$$

Where $N(t) = P(t) + R(t) + M(t) + U(t) + I(t)$,

$$N(t) = \frac{K}{\mu} - (\frac{K}{\mu} - N(0))e^{-\mu t}$$

According to the comparison theorem [15], there is $T > 0$, and when $t > T$, then, $0 < N(t) \le K/\mu$.

Stability of the equilibrium points

Existence of equilibrium point

The first three equations of model (1) do not contain variables U and I, therefore, only the first three equations in the model need to be discussed later, and the following model can be obtained:

$$\begin{cases} \dfrac{dP(t)}{dt} = qK - \beta P(t)\dfrac{M(t)}{K} + \delta R(t) - \mu P(t) \\ \dfrac{dR(t)}{dt} = (1-q)K - (\delta + \mu + a)R(t) + \theta M(t-\tau)e^{-\mu\tau} + aR(t-\tau)e^{-\mu\tau} \\ \dfrac{dM(t)}{dt} = \beta P(t)\dfrac{M(t)}{K} - (r+\mu)M(t) \end{cases} \quad (3)$$

Based on the practical significance of model (3), it is assumed that it has initial conditions

$$P(t) = \phi_1(t) \in C\{[-\tau,0],R_+\}, \quad R(t) = \phi_2(t) \in C\{[-\tau,0],R_+\}$$

$$M(t) = \phi_3(t) \in C\{[-\tau,0],R_+\}, \quad \phi_1(0) \geq 0, \quad \phi_2(0) \geq 0, \phi_3(0) \geq 0 \quad (4)$$

It is well known by the fundamental theory of functional differential equations [26] that system (2.4) has a unique solution $P(t), R(t), M(t)$ satisfying initial conditions (3). The equilibrium $E_0 = (P_0, R_0, M_0)$ and $E^* = (P^*, R^*, M^*)$ of system (3) satisfy the

Following combined equations.

$$\begin{cases} qK - \beta P(t)\dfrac{M(t)}{K} + \delta R(t) - \mu P(t) = 0 \\ (1-q)K - (\delta + \mu + a)R(t) + \theta M(t-\tau)e^{-\mu\tau} + aR(t-\tau)e^{-\mu\tau} = 0 \\ \beta P(t)\dfrac{M(t)}{K} - (r+\mu)M(t) = 0 \end{cases} \quad (5)$$

Let $\varepsilon = \mu + \delta + a$, $A = \varepsilon - ae^{-\mu\tau}$, $d = r + \mu$, From (5) we obtain

$$E_0 = (P_0, R_0, M_0) = \left(\frac{1}{\mu}[qK + \frac{(1-q)\delta K}{A}], \frac{(1-q)K}{A}, 0 \right)$$

That $\bar{R} = \dfrac{\beta q A + (1-q)\beta\delta}{A\mu d}$, when $\bar{R} > 1, \varepsilon > ae^{-\mu\tau}$ 且 $\dfrac{\delta\theta e^{-\mu\tau}}{Ad} < 1$, the unique positive equilibrium point of system (3) can be obtained.

$$E^* = (P^*, R^*, M^*) = \left(\frac{Kd}{\beta}, \frac{(1-q)K + \theta M^* e^{-\mu\tau}}{A}, \frac{\mu K(\bar{R}-1)}{\beta(1 - \frac{\delta\theta e^{-\mu\tau}}{Ad})} \right).$$

Theorem 1 If $\bar{R} > 1$, there exist a non-negative equilibrium $E_0 = (P_0, R_0, M_0)$ and a unique positive equilibrium $E^* = (P^*, R^*, M^*)$ for system (3).

STABILITY OF NON-NEGATIVE EQUILIBRIUM POINTS

Theorem 2 If $0 < \bar{R} \leq 1$, $\tau \neq 0$, the non-negative equilibrium E_0 is local stability; if $\bar{R} > 1$, It is not stability.

Proof : The characteristic equation of A_{E_0} is

$$f(\lambda)=(\lambda+\mu)(\lambda+\varepsilon-ae^{-\mu\tau})(\lambda-\frac{\beta}{K}P_0+d) \qquad (6)$$

There are always two negative roots for the eigenvalues of A_{E_0} , $\lambda_1=-\mu<0$, and $\lambda_2=-A<0$, the other roots are determined by the following equation

$$\lambda-\frac{\beta}{K}P_0+d \;=0 \qquad (7)$$

That is

$$\lambda_3=\frac{\beta}{K}P_0-d=\frac{\beta}{K}\frac{1}{\mu}[qK+\frac{(1-q)\delta K}{\varepsilon-ae^{-\mu\tau}}]-d \;=\frac{1}{\mu A}[\beta\delta(1-q)+\mu\beta q(\varepsilon-ae^{-\mu\tau})]-d$$

$$=\frac{1}{\mu A}[dA\mu(\overline{R}-1)+A\beta q(\mu-1)] \;=\frac{1}{\mu}[d\mu(\overline{R}-1)+\beta q(\mu-1)]\leq 0$$

When $\overline{R}\leq 1$, all the characteristic roots of the characteristic equation A_{E_0} are negative, so E_0 is locally asymptotically stable. When $\overline{R}>1$, E_0 is unstable in region D. The proof is complete.

Theorem 3 IF $0<\overline{R}\leq 1$, and

$$\min\left\{\left(\frac{\beta P_0}{2K\mu}-\frac{\delta}{2\mu}\right),(1-\frac{(2a+\theta)e^{-\mu\tau}}{\varepsilon}-\frac{\delta}{2\mu}),(\frac{\beta P_0}{2k\mu}-\frac{\theta e^{-\mu\tau}}{\varepsilon})\right\}\geq 0,$$

The non-negative equilibrium E_0 is globally asymptotically stable.

Proof: Let $x=P-P_0$, $y=R-R_0$, $z=M-M_0$

Then model (1) can be deformed into

$$\begin{cases} x'(t)=-\mu x(t)+\delta y(t)-\frac{\beta P_0}{k}z(t) \\ y'(t)=-\varepsilon y(t)+ay(t-\tau)e^{-\mu\tau}+\theta e^{-\mu\tau}z(t-\tau) \\ z'(t)=-dz(t)+\frac{\beta P_0}{k}z(t) \end{cases} \qquad (8)$$

Construct the Lyapunov function

$$V_1(t)=\frac{x^2}{2\mu}+\frac{y^2}{2\varepsilon}+\frac{z^2}{2(d-\frac{\beta P_0}{K})} \qquad (9)$$

And derive the derivative of the orbit along the model (9)

$$\frac{dV_1}{dt}=-x^2(t)+\frac{\delta}{\mu}x(t)y(t)-\frac{\beta p_0}{k\mu)}x(t)z(t)$$

$$-y^2(t)+\frac{ae^{-u\tau}}{\varepsilon}y(t)y(t-\tau)+\frac{\theta e^{-u\tau}}{\varepsilon}y(t)z(t-\tau)-z^2$$

$$\frac{dV_1}{dt} \le \frac{\delta}{\mu} x(t)y(t) - \frac{\beta p_0}{k\mu)} x(t)z(t) - y^2(t) + \frac{ae^{-u\tau}}{\varepsilon} y(t)y(t-\tau) + \frac{\theta e^{-u\tau}}{\varepsilon} y(t)z(t-\tau)$$

And Construct function

$$V(t) = V_1(t) + \frac{ae^{-\mu\tau}}{2\varepsilon}\int_{t-\tau}^{t} y^2(u)du + \frac{\theta e^{-\mu\tau}}{2\varepsilon}\int_{t-\tau}^{t} z^2(u)du \qquad (10)$$

Because

$$\frac{dy}{dx}[\frac{ae^{-\mu\tau}}{2\varepsilon}\int_{t-\tau}^{t} y^2(u)du + \frac{\theta e^{-\mu\tau}}{2\varepsilon}\int_{t-\tau}^{t} z^2(u)du] =$$

$$\frac{ae^{-\mu\tau}}{2\varepsilon}[y^2(t) - y^2(t-\tau)] + \frac{\theta e^{-\mu\tau}}{2\varepsilon}[z^2(t) - z^2(t-\tau)]$$

We obtain

$$\frac{dV_1}{dt} \le \frac{\delta}{\mu} x(t)y(t) - \frac{\beta p_0}{k\mu} x(t)z(t) - y^2(t) + \frac{ae^{-u\tau}}{\varepsilon} y(t)y(t-\tau) + \frac{\theta e^{-u\tau}}{\varepsilon} y(t)z(t-\tau)$$

$$+\frac{ae^{-\mu\tau}}{2\varepsilon}[y^2(t) - y^2(t-\tau)] + \frac{\theta e^{-\mu\tau}}{2\varepsilon}[z^2(t) - z^2(t-\tau)]$$

$$\le -\left(\frac{\beta P_0}{2K\mu} - \frac{\delta}{2\mu}\right)x^2(t) - (1 - \frac{(2a+\theta)e^{-\mu\tau}}{\varepsilon} - \frac{\delta}{2\mu})y^2(t) - \left(\frac{\beta p_0}{2k\mu} - \frac{\theta e^{-\mu\tau}}{\varepsilon}\right)z^2(t)$$

When $\min\left\{\left(\frac{\beta P_0}{2K\mu} - \frac{\delta}{2\mu}\right), (1 - \frac{(2a+\theta)e^{-\mu\tau}}{\varepsilon} - \frac{\delta}{2\mu}), (\frac{\beta p_0}{2k\mu} - \frac{\theta e^{-\mu\tau}}{\varepsilon})\right\} \ge 0$, that is $\frac{dV}{dt} \le 0$。

According to LaSalle's invariance principle [15], the non-negative equilibrium point E_0 is globally asymptotically stable. The proof is complete.

STABILITY OF EQUILIBRIUM POINTS

Theorem 4 IF $\bar{R} > 1$, $\frac{P^*\beta}{2(M^*\beta + K\mu)} \ge \frac{\theta e^{-\mu\tau}}{2\varepsilon}$ and

$$\min\{-\frac{\delta}{2(\mu + \frac{Ad\mu(\bar{R}-1)}{Ad - \delta\theta e^{-u\tau}})} + \frac{Kd}{2(M^*\beta + K\mu)}, 1 - \frac{\delta}{2(\frac{M^*\beta}{K} + \mu)} - \frac{(2a+\theta)e^{-u\tau}}{2\varepsilon}\} \ge \frac{\delta}{2}$$

, the unique positive equilibrium E^* of system (3) is globally asymptotically stable.

Proof: Let $x = P - P^*$, $y = R - R^*$, $z = M - M^*$, then model (1) can be deformed into

$$\begin{cases} x'(t) = (-\dfrac{\beta M^*}{k} - \mu)x(t) + \delta y(t) - \dfrac{\beta P^*}{k} z(t) \\ y'(t) = -\varepsilon\, y(t) + ay(t-\tau)e^{-\mu\tau} + \theta e^{-\mu\tau} z(t-\tau) \\ z'(t) = \dfrac{\beta M^*}{k} x(t) + [\dfrac{\beta P^*}{k} - d]z(t) \end{cases} \qquad (11)$$

Construct the Lyapunov function $V(t) = \dfrac{x^2}{2(\dfrac{\beta M^*}{k} + \mu)} + \dfrac{y^2}{2\varepsilon} + \dfrac{z^2}{2(d - \dfrac{\beta P^*}{k})}$, and derive the derivative of the orbit along the model (11)

$$V'(t) = \frac{xx'}{\dfrac{\beta M^*}{k} + \mu} + \frac{yy'}{\varepsilon} + \frac{zz'}{d - \dfrac{\beta P^*}{k}}$$

$$= \frac{x}{\dfrac{\beta M^*}{k} + \mu}[(-\frac{\beta M^*}{k} - \mu)x(t) + \delta y(t) - \frac{\beta P^*}{k} z(t)]$$

$$+ \frac{y}{\varepsilon}[-\varepsilon\, y(t) + ay(t-\tau)e^{-\mu\tau} + \theta e^{-\mu\tau} z(t-\tau)]$$

$$+ \frac{z}{d - \dfrac{\beta P^*}{k}}[\frac{\beta M^*}{k} x(t) + [\frac{\beta P^*}{k} - d]z(t)]$$

$$= -x^2(t) + \frac{\delta}{\dfrac{M^*\beta}{K} + \mu} x(t)y(t) + \delta x(t)y(t) - \frac{P^*\beta}{M^*\beta + K\mu} x(t)z(t-\tau)$$

$$-y^2(t) + \frac{ae^{-u\tau}}{\varepsilon} y(t)y(t-\tau) + \frac{\theta e^{-u\tau}}{\varepsilon} y(t)z(t-\tau) - z^2 + \frac{M^*\beta}{Kd - P^*\beta} x(t)z(t)$$

$$\le \frac{\delta}{\dfrac{M^*\beta}{K} + \mu} x(t)y(t) + \delta x(t)y(t) - \frac{P^*\beta}{M^*\beta + K\mu} x(t)z(t-\tau)$$

$$-y^2(t) + \frac{ae^{-u\tau}}{\varepsilon} y(t)y(t-\tau) + \frac{\theta e^{-u\tau}}{\varepsilon} y(t)z(t-\tau) + \frac{M^*\beta}{Kd - P^*\beta} x(t)z(t)$$

Construct the Lyapunov function

$$V(t) = V_1(t) + \frac{ae^{-\mu\tau}}{2\varepsilon}\int_{t-\tau}^{t} y^2(u)du + \frac{\theta e^{-\mu\tau}}{2\varepsilon}\int_{t-\tau}^{t} z^2(u)du$$

We obtain

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{ae^{-\mu\tau}}{2\varepsilon}[y^2(t) - y^2(t-\tau)] + \frac{\theta e^{-\mu\tau}}{2\varepsilon}[z^2(t) - z^2(t-\tau)]$$

$$\le \frac{\delta}{\dfrac{M^*\beta}{K} + \mu} x(t)y(t) + \delta x(t)y(t) - \frac{P^*\beta}{M^*\beta + K\mu} x(t)z(t-\tau) - y^2(t) + \frac{ae^{-u\tau}}{\varepsilon} y(t)y(t-\tau)$$

$$+\frac{\theta e^{-u\tau}}{\varepsilon}y(t)z(t-\tau)+\frac{M^*\beta}{Kd-P^*\beta}x(t)z(t)+\frac{ae^{-\mu\tau}}{2\varepsilon}\left[y^2(t)-y^2(t-\tau)\right]+\frac{\theta e^{-\mu\tau}}{2\varepsilon}[z^2(t)-z^2(t-\tau)]$$

$$\leq -(-\frac{\delta}{2(\frac{M^*\beta}{K}+\mu)}-\frac{\delta}{2}+\frac{P^*\beta}{2(M^*\beta+K\mu)}-\frac{M^*\beta}{2(Kd-P^*\beta)})x^2(t)$$

$$-(-\frac{\delta}{2(\frac{M^*\beta}{K}+\mu)}-\frac{\delta}{2}+1-\frac{(2a+\theta)e^{-u\tau}}{2\varepsilon})y^2(t)-(\frac{P^*\beta}{2(M^*\beta+K\mu)}-\frac{M^*\beta}{2(Kd-P^*\beta)}-\frac{\theta e^{-\mu\tau}}{2\varepsilon})z^2(t)$$

If $\min\{-\frac{\delta}{2(\mu+\frac{Ad\mu(\overline{R}-1)}{Ad-\delta\theta e^{-u\tau}})}+\frac{Kd}{2(M^*\beta+K\mu)},1-\frac{\delta}{2(\frac{M^*\beta}{K}+\mu)}-\frac{(2a+\theta)e^{-u\tau}}{2\varepsilon}\}\geq\frac{\delta}{2}$, and $\frac{P^*\beta}{2(M^*\beta+K\mu)}\geq\frac{\theta e^{-\mu\tau}}{2\varepsilon}$ hold, then $V'(z)<0$. When $\overline{R}>1$, According to the Lyapunov stability theorem [16, 17], the only positive equilibrium point E^* of model (1) is globally asymptotically stable. The proof is complete.

NUMERICAL SIMULATION

In the following, we select several sets of parameters listed in the table. The influence of each parameter on the equilibrium point can be seen. This is consistent with theorems.

(1) the global stability of E_0

Table-1: Effect of parameters on values P,R,U,M,I steady-state within system (3)

β	q	μ	δ	α	r	θ	P	R	$\overline{R}$
0.79	0.9	0.8	0.07	0.01	0.2	0.42	56.7529	5.7469	0.8967
0.69	0.9	0.8	0.07	0.01	0.2	0.42	56.7529	5.7469	0.7832
0.79	0.76	0.8	0.07	0.01	0.2	0.42	48.7068	13.7925	0.7696
0.59	0.77	0.6	0.12	0.2	0.5	0.02	66.1674	17.1488	0.7226
0.53	0.65	0.5	0.02	0.4	0.6	0.02	66.3436	33.5893	0.6393

Let $q=0.9$, $k=50$, $\delta=0.07$, $\beta=0.79$, $\mu=0.8$, $\alpha=0.01$, $r=0.2$, $\theta=0.42$, $T=0.005$, $\tau=3$, Then model (3) is

$$\begin{cases}\frac{dP(t)}{dt}=0.9K-0.0158P(t)M(t)+0.07R(t)-0.8P(t)\\ \frac{dR(t)}{dt}=0.1K-0.8700R(t)+0.42M(t-\tau)e^{-0.0040}+aR(t-\tau)e^{-0.0040}\\ \frac{dM(t)}{dt}=0.0158P(t)M(t)-M(t)\end{cases}$$

Then we have $\overline{R}=0.8967<1$, and $E_0=(56.7529,\ 5.7469,0,\ 0,0)$, Let $P(0)=30$, $R(0)=3$, $M(0)=2$, From Fig. 1, we can see that the component curve shows that the interior equilibrium E_0 is global asymptotically stable.

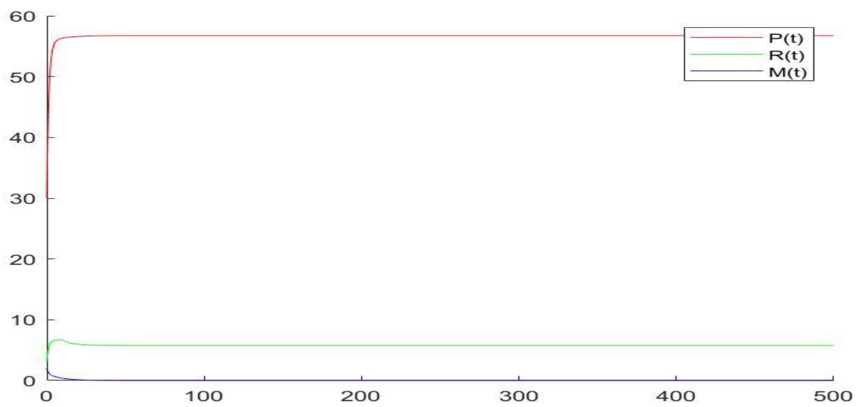

Fig-1: The global stability of non-negative equilibrium points

(2) the global stability of E^*

Table-2: Effect of parameters on values P, R, U, M, I steady-state within system (3)

β	q	μ	δ	α	r	θ	P	R	M	$\overline{R}$
0.99	0.6	0.579	0.37	0.01	0.65	0.45	62.07	21.61	1.67	1.051
0.69	0.5	0.4	0.27	0.01	0.2	0.42	43.47	71.12	41.01	2.016
0.99	0.65	0.25	0.25	0.12	0.35	0.01	60.606	72.26	113.19	5.445
0.83	0.62	0.65	0.05	0.32	0.05	0.02	84.337	54.686	14.166	1.180
0.59	0.5	0.104	0.67	0.05	0.9	0.42	85.084	64.314	59.006	5.270

Let $q=0.5$, $k=50$, $\delta=0.67$, $\beta=0.59$, $\mu=0.104$, $\alpha=0.05$, $r=0.9$, $\theta=0.42, T=0.001$, $\tau=10$, Then model (4) is

$$\begin{cases} \dfrac{dP}{dt}=0.5K-0.0118PM+0.67R-0.104P \\ \dfrac{dR}{dt}= 0.5K- 0.8240R+0.05R(t-\tau)e^{-\ 1.0400\text{e-}04}+0.42R(t-\tau)e^{-\ 1.0400\text{e-}04} \\ \dfrac{dM}{dt}=0.0118PM-1.0040M \end{cases}$$

Then we have $\overline{R}=5.2708>1$, and $E^*=(\ 85.0847,\ 64.3148,\ 59.0061)$.Let $P(0)$=30, $R(0)$=3, $M(0)$=2. From Fig. 2, we can see that the component curve shows that the interior equilibrium E^* is global asymptotically stable.

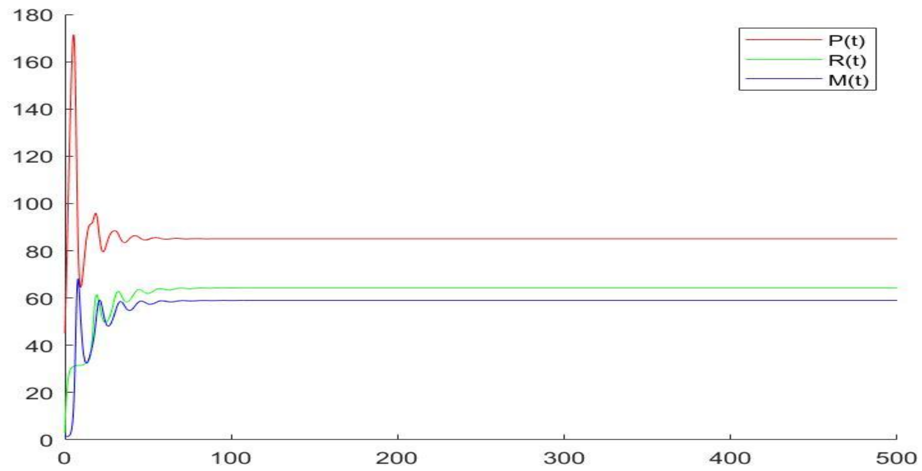

Fig-3: The global stability of a unique positive equilibrium point

CONCLUSION

In this paper, we analyze the stability of the retraining model of the same department workers with double time delays in private enterprises by using the method of characteristic equation analysis and Hurwitz decision theorem. The results show that there is a non-negative equilibrium point in the model when the number of workers returning to work is very large or the transfer rate of workers returning to work is large. When $\bar{R} = \frac{\beta q A + (1-q)\beta\delta}{A\mu d}$, $\varepsilon > ae^{-\mu\tau}$ and $\frac{\delta\theta e^{-\mu\tau}}{Ad} < 1$, training strategies have been around for a long time, which requires that in real life controls must be

Strengthened on the number of returning workers and the rate at which returning workers are transferred to non-expendable workers. Return to non-expendable workers and sabotage workers transfer rate, the greater the removed the less workers, do not eliminate a worker retraining the shorter the period of validity, will decline in the more unfavorable to retraining. Therefore, the control cannot be eliminated worker retraining when fewer people, must pay close attention to important influence workers technical level, take more to return the proportion of retraining workers, reduce the transfer rate of return to cannot eliminate workers.

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