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On Compactness in Tritopological Spaces

Asmhan Flieh Hassan*, Zainab Imran Mousa

Department of Mathematics, Faculty of education for girls,University of Kufa, Iraq

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***Corresponding author:** Asmhan Flieh Hassan

Abstract **Review Article**

In this article we are introduce the study of compact spaces in tritopological spaces (compactness in tritopological spaces namely δ^*-compact spaces which defined in [1]). And we are define a countable and local δ^*-compactness, also show the interrelations between δ^*-compactness and countable δ^*-compactness, local δ^*-compactness, δ^*-Hausdorff, δ^*-continuity, δ^*-homeomorphism, and others. Further the new results about these spaces which are considered as one of the main generalizations of compact spaces.

Keywords: δ^*-compact spaces, countable δ^*-compact space, local δ^*-compactness, δ^*-open set, tritopological spaces.

INTRODUCTION

This work is developed around the concept of δ^*-compactness (compactness with respect to a δ^*-open set in tritopological spaces) which was only defined in [1] by Asmhan F. Hassan, in some literatures there are several generalizations of the notion of compact spaces and these are studied separately for many different purposes and reasons. A topological space is said to be compact or have the compact property, if every open cover of X has a finite sub-cover [2], several authors in such as [3-6] have been introduced some types of compact spaces in topological space according to the sets. Moreover, in a few last years the generalization of compact spaces have been extended and generalized to bitopological setting as in [7-10].

In 2004 Asmhan F. Hassan [1], has been initiated the definition for δ^*-open set in tritopological spaces and introduced a systematic study of δ^*-open set and dealt with in detail and clear. In fact, the concept of δ^*-open set in tritopological spaces is derived from the concept of δ -open set in bitopological spaces [11], and the concept of α-open in topological spaces [12].

And in 2011 the author Asmhan F. Hassan [13], introduced the δ^*-connectedness in tritopological spaces. She introduced the δ^*-base in tritopological spaces [14]. Also she introduced relationships among some concepts in topological, bitopological and tritopological spaces [15-17].

In the year 2017 Asmhan F. Hassan defined the δ^*-countability and δ^*-separability in tritopological spaces [18]. She presented the concept of the soft tritopological spaces in [19]. And in 2019 She presented the concept of the fuzzy soft tritopological spaces in [20].

Recently, in this article we are introduce some results of generalization of compact spaces (δ^*-compactness in tritopological spaces) which are considered as one of the main generalizations of compact spaces in tritopological spaces.

This paper contains 6 sections. In Section 2, we will mention some basic definitions in tritopological spaces which we need in this study. In Section 3, we are introducing the study around the δ^*-compactness in tritopological spaces with some theorems and exampels of relationships among δ^*-compact space and other spaces in tritopological spaces. In Section 4, we introduce a new definition of δ^*-compactness namely countable δ^*-compactness and local δ^*-compactness. And gives some relationships among them. In Section 5 and 6, we show the interrelations between δ^*-compactness and countable δ^*-compactness, local δ^*-compactness, δ^*-continuity, δ^*-homeomorphism, and others. And in Section 7, we obtain the main and important conclusions of δ^*-compactness in tritopological spaces.

Preliminaries

In the following we will mention the basic definitions and notations in tritopological space, which we need in this work

Definition ([1]) Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, a subset $\mathcal{A}$ of X is said to be δ^*-open set iff $\mathcal{A} \subseteq \mathcal{T}\ \mathrm{int}(\mathcal{P}\ \mathrm{cl}(\mathcal{Q}\ \mathrm{int}(\mathcal{A})))$, and the family of all δ^*-open sets is denoted by $\delta^*.O(X)$. ($\delta^*.O(X)$ not always represent a topology). The complement of δ^*-open set is called a δ^*-closed set.

Definition ([1]) $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called a discrete tritopological space with respect to δ^*-open if $\delta^*.O(X)$ contains all subsets on X. And $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called an indiscrete tritopological space with respect to δ^*-open if $\delta^*.O(X) = \{X, \emptyset\}$.

Definition ([1]) Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let $x \in X$, a subset $\mathcal{N}$ of X is said to be a δ^*-nhd of a point x iff there exists a δ^*-open set U such that $x \in U \subset \mathcal{N}$. The set of all δ^*-nhds of a point x is denoted by $\delta^* - \mathcal{N}(x)$.

Definition ([14]) A collection δ^*-β of a subsets of X is said to form a δ^*-base for the tritopology $(\mathcal{T},\mathcal{P},\mathcal{Q})$ iff: δ^*-β $\subset \delta^*.O(X)$. for each point $x \in X$ and each δ^*-neighbourhood $\mathcal{N}$ of x there exists some $\mathcal{B} \in \delta^*$-β such that $x \in \mathcal{B} \subset \mathcal{N}$.

Definition ([1]) The function $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ is said to be a δ^*-continuous at $x \in X$ iff for every δ^*-open set V in Y containing $f(x)$ there exists δ^*-open set U in X containing x such that $f(U) \subset V$. We say f is δ^*-continuous on X iff f is δ^*-continuous at each $x \in X$.

Definition ([1]) Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ and $(Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ are two tritopological spaces and $f: (X,\mathcal{T},\mathcal{P},\mathcal{Q}) \to (Y,\mathcal{T}',\mathcal{P}',\mathcal{Q}')$ be a function, then f is δ^*-homeomorphism if and only if :

(i) f is bijective (one to one , onto). (ii) f and f^{-1} are δ^*-continuous.

Definition ([1]) Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be atritopological space and let Y be asubset of X. The relative tritopological space for Y is denoted by $(Y,\mathcal{T}_Y,\mathcal{P}_Y,\mathcal{Q}_Y)$; such that:

$\mathcal{T}_Y = \{ G \cap Y : G \in \mathcal{T} \}$, $\mathcal{P}_Y = \{ G \cap Y : G \in \mathcal{P} \}$ and $\mathcal{Q}_Y = \{ G \cap Y : G \in \mathcal{Q} \}$

Then $(Y,\mathcal{T}_Y,\mathcal{P}_Y,\mathcal{Q}_Y)$ is called the subspace of tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$. And the relative tritopological space for Y with respect to δ^*-open sets is the collection $\delta^*_Y.O(X)$ given by ;

$\delta^*_Y.O(X) = \{ G \cap Y : G \in \delta^*.O(X)\}$.

Definition ([1]) Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let $\mathcal{A}$ be any subset of X, then the collection $C = \{G_\lambda : \lambda \in \Lambda\}$ is called δ^*-open cover to $\mathcal{A}$ if C is a cover to $\mathcal{A}$ and $C \subset \delta^*.O(X)$.

Definition ([1]) Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space, and let $\mathcal{A}$ be any subset of X, then $\mathcal{A}$ is called δ^*-compact set iff every δ^*-open cover of $\mathcal{A}$ has a finite sub-cover, i.e. for each $\{G_\lambda : \lambda \in \Lambda\}$ of δ^*-open sets for which $\mathcal{A} \subset \cup \{G_\lambda : \lambda \in \Lambda\}$, there exist finitely many sets $G_{\lambda 1}, \ldots\ldots, G_{\lambda n}$ among the G_λ's such that $\mathcal{A} \subset G_{\lambda 1} \cup \ldots\ldots \cup G_{\lambda n}$.
In particular, the space X is called δ^*-compact iff for each collection $\{G_\lambda : \lambda \in \Lambda\}$ of δ^*-open sets for which $X = \cup \{G_\lambda : \lambda \in \Lambda\}$, there exist finitely many sets $G_{\lambda 1}, \ldots\ldots, G_{\lambda n}$ among the G_λ's such that $X = G_{\lambda 1} \cup \ldots\ldots \cup G_{\lambda n}$.

Definition ([1]) A tritopological space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is called δ^*-T_2-space (δ^*-Hausdorff) iff every pair of distinct points x and y of X , there exist two δ^*-open sets G, H s.t. $x \in G$, $y \in H$, $G \cap H = \emptyset$.

Definition ([1]): Let $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ be a tritopological space , a point x is called δ^*-limit point of a subset $\mathcal{A}$ of X iff for each δ^*-open set G containing another point different from x in $\mathcal{A}$; that is $(G/\{x\}) \cap \mathcal{A} \neq \emptyset$, and the set of all δ^*-limit points of $\mathcal{A}$ is denoted by δ^*- lm($\mathcal{A}$) .

Definition ([2]) A collection C of sets is said to have the finite intersection property (FIP) or to be finitely common iff the intersection of members of each finite subcollection of C is non-empty. A collection of sets is called fixed if it has a non-empty intersection and free if its intersection is empty.

Compactness in Tritopological Spaces (δ^*-compact spaces)

Theorem. Every δ^*-compact subset $\mathcal{A}$ of a δ^*-Hausdorff space $(X,\mathcal{T},\mathcal{P},\mathcal{Q})$ is δ^*-closed.

Proof: We shall show that $\mathcal{A}^C$ is δ^*-open. Let $\in \mathcal{A}^C$. Since X is δ^*-Hausdorff, for every $q \in \mathcal{A}$ there exist δ^*-open nhds of p and q which we denote respectively by $\delta^* - \mathcal{M}(p)$ and $\delta^* - \mathcal{N}(q)$ such that $\delta^* - \mathcal{M}(p) \cap \delta^* - \mathcal{N}(q) = \emptyset$. Now that Collection $\{\delta^* - \mathcal{N}(q) : q \in \mathcal{A}\}$ is an δ^*-open cover of $\mathcal{A}$. Since $\mathcal{A}$ is δ^*-compact there exist finite number of points q_i , $i = 1, \dots n$ such that;

$$\mathcal{A} \subset \bigcup_{i=1}^{n} \delta^* - \mathcal{N}(q_i) \qquad (1)$$

Let $\mathcal{M} = \bigcap_{i=1}^{n} \mathcal{M}(p_i)$, $\mathcal{N} = \bigcup_{i=1}^{n} \delta^* - \mathcal{N}(q_i)$. Then $\mathcal{M}$ is an δ^*-open nhd of p (being the intersection of a finit number of δ^*-open nhds of p). We claim the $\mathcal{M} \cap \mathcal{N} = \emptyset$. We have

$$\begin{aligned} X \in \mathcal{N} &\Rightarrow x \in \delta^* - \mathcal{N}(qi) \text{ for some i} \\ &\Rightarrow x \notin \delta^* - \mathcal{M}(p_i) \qquad [\because \delta^* - \mathcal{N}(qi) \cap \delta^* - \mathcal{M}(p_i) = \emptyset] \\ &\Rightarrow x \notin \mathcal{M}. \end{aligned}$$

Thus $\mathcal{M} \cap \mathcal{N} = \emptyset$ and since $\mathcal{A} \subset \mathcal{N}$, we have $\mathcal{A} \cap \mathcal{M} = \emptyset$ which implies that $\mathcal{M} \subset \mathcal{A}^C$. This shows that $\mathcal{A}^C$ contains a δ^*-nhd of each of its points and so $\mathcal{A}^C$ is δ^*-open i.e. $\mathcal{A}$ is δ^*-closed.

Example. δ^*-compact subset of a non-δ^*-Hausdorff tritopological space need not be δ^*-closed.
Consider any indiscrete tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$, where X consisting of more than one point and let $\mathcal{A}$ be any proper subset of X. Then $\mathcal{A}$ is not δ^*-closed since the only δ^*-closed sets are $\emptyset$ and X. But $\mathcal{A}$ is δ^*-compact since the only δ^*-open cover of $\mathcal{A}$ is {X}.

Example. δ^*-compact space which is not δ^*-Hausdorff.
Consider the tritopology $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$; where $X = \{a, b, c\}$, $\mathcal{T} = \{X, \emptyset, \{c\}, \{a, c\}\}$, $\mathcal{P} = \{X, \emptyset, \{b\}\}$, $\mathcal{Q} = \{X, \emptyset, \{a\}\}$

$(X, \mathcal{T})$, $(X, \mathcal{P})$ and $(X, \mathcal{Q})$ are three topological space, then $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is a tritopological space, the family of all δ^*-open set of X is: $\delta^*.O(X) = \{X, \emptyset, \{a\}, \{a, c\}\}$
Since X is finite, then $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ δ^*-compact. But $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is not δ^*-Hausdorff since a, b are distinct points having no disjoint δ^*-nhbs.

Theorem. A tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is δ^*-compact if and only if every basic δ^*-open cover of X has a finite subcover.

Proof: Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be δ^*-compact space. Then every δ^*-open cover of X has a finite subcover. In particular, every basic δ^*-open cover of X must have a finite subcover.

Conversely, suppose that every basic δ^*-open cover of X has a finite subcover and let $C = \{G_\lambda : \lambda \in \Lambda\}$, be any δ^*-open cover of X. If $B = \{B_\alpha : \alpha \in \Delta\}$, be any δ^*-open base for X, Then each G_λ is a union of some members of B and the totality of all such members is evidently a basic δ^*-open cover of X. By hypothesis this collection of members of B has a finite subcover, say, $\{B_{\alpha i} : i = 1, 2, \dots n\}$

For each $B_{\alpha i}$ in this finite subcover, we can select a $G_{\lambda i}$ from C such that $B_{\alpha i} \subset G_{\lambda i}$. It follows that finite subcollection $\{G_{\lambda i} : i = 1, 2 \dots n\}$. Which arises in this way is a subcover of C. Hence X is δ^*-compact.

Theorem: A tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is δ^*-compact iff every collection of δ^*-closed subset of X with the finite intersection property is fixed, that is, has a non-empty intersection.

Proof: Let X be δ^*-compact and let $F = \{F_\lambda : \lambda \in \Lambda\}$ be a collection of δ^*-closed sub sets of X with the FIP and suppose, if possible, $\cap \{F_\lambda : \lambda \in \Lambda\} = \emptyset$.

Then $[\cap \{F_\lambda : \lambda \in \Lambda\}]^C = X$ or $\cup \{F_\lambda : \lambda \in \Lambda\} = X$ by De-Morgan law. This means that $\{F_\lambda{}^C : \lambda \in \Lambda\}$ is an δ^*-open cover of X since $F_\lambda{}^C$'s are δ^*-closed. Since X is δ^*-compact, we have the $\cup \{F^C{}_{\lambda i} : i = 1, 2, \dots n\} = X$, where n is finite and so by De-Morgan law $[\cap \{F_{\lambda i} : i = 1, 2, \dots n\}]^C = X$, which implies that $\cap \{F_{\lambda i} : i = 1, 2, \dots n\} = \emptyset$. But this contradicts the FIP of F. Hence we must have $\cap \{F_\lambda : \lambda \in \Lambda\} \neq \emptyset$.

Conversely, let every collection of δ^*-closed subset of X with the FIP have a non-empty intersection and let $C = \{G_\lambda : \lambda \in \Lambda\}$ be an δ^*-open cover of X so that $X = \cup \{G_\lambda : \lambda \in \Lambda\}$ whence taking complement $\emptyset = [\cup \{G_\lambda : \lambda \in \Lambda\}]^C = \cap \{G_\lambda{}^C : \lambda \in \Lambda\}$. Thus $\{G_\lambda{}^C : \lambda \in \Lambda\}$ is a collection of δ^*-closed sets with empty intersection and so by hypothesis

this collection does not have the FIP. Hence there exist a finite number of sets ${G^C}_{\lambda i} : i = 1,2, \dots, n$ such that $\emptyset = \cap$ $\{{G^C}_{\lambda i} : i = 1,2, \dots, n\}$ = $[\cup \{G_{\lambda i} : i = 1,2, \dots, n\}]^C$ [De-Morgan law]
$\Longrightarrow$ X =$\cup$ $\{\{G_{\lambda i} : i = 1,2, \dots, n\}$. Hence X is δ^*-compact.

Example: every indiscrete tritopological space (w.r.t. δ^*-open set) is δ^*-compact, and no-infinite discrete tritopological space (w.r.t. δ^*-open set) is δ^*-compact.

For if $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is any indiscrete tritopological space (w.r.t. δ^*-open set), then the only open cover of X is {X} which is finite since it consists of single member X.

And let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be any discrete tritopological space (w.r.t. δ^*-open set), where X is infinite. Since each singleton subset of X is δ^*-open, it fellows that the collection $C = \{\{x\} : x \in X\}$ is a δ^*-open infinite cover of X which cannot have a finite subcover since if we remove a single member of C, then it will not cover X . Hence $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is not a δ^*-compact.

Countable and local δ^*-compactness

Definition: A tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is said to be countably δ^*-compact iff every countable δ^*-open cover of X has a finite subcover.

Theorem: A tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is countable δ^*-compact iff every countable collection of δ^*-closed subsets of X with FIP has non-empty intersection.

Proof: proof of this theorem is the same as that of theorem (3.5) except that we now take countable δ^*-open covers instead of arbitrary δ^*-open covers.

Definition: A tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is said to have Bolzano Waierstras Property (**BWP**) iff every infinite set in X has a δ^*-limit point. A tritopological space with BWP is also said to be Frechet δ^*-compact.

Theorem: A countable δ^*-compact tritopological space has BWP.

Proof: Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ be a countable δ^*-compact space and suppose, if possible, that it does not have BWP. Then there exists an infinite set S having no δ^*-limit point. Let $\mathcal{A}$ be a countably infinite subset of S. Then $\mathcal{A}$ has no δ^*-limit point. It follows that $\mathcal{A}$ is a δ^*-closed set. Also for each $a_n \in \mathcal{A}$, a_n is not a δ^*-limit point of

$\mathcal{A}$. Hence there exist an δ^*-open set G_n , such that $a_n \in G_n$ and $G_n \cap \mathcal{A} = \{a_n\}$.

Then the collection $\{G_n : n \in N\} \cup \mathcal{A}^C$ is a countable δ^*-open cover of X. This cover has no finite subcover. For if we remove a single G_n, it will not be a cover of X since then a_n will not be covered. Hence X is not countably δ^*-compact. But this contradicts the hypothesis. Hence $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ must have BWP.

Definition: A tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is said to be locally δ^*-compact iff every point in X has at least one δ^*-neighbourhood whose δ^*-closure is δ^*-compact.

Theorem: Every δ^*-compact tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is locally δ^*-compact.

Proof: Let $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ a δ^*-compact space. Since X is both δ^*-open and δ^*-closed, it is a δ^*-neighbourhood of each of its points such that $\delta^* - \mathrm{cl}(X) = X$ is δ^*-compact. Hence $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is locally δ^*-compact.

Theorem: Every δ^*-closed subspace of locally δ^*-compact space is locally δ^*-compact.

Proof: Let Y be a δ^*-closed subset of a locally δ^*-compact space X and let y $\in$ Y be arbitrary. Then y $\in$ X . Since $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is locally δ^*-compact, there exists an δ^*-open neighbourhood $\mathcal{N}$ of y such that $\delta^* - \mathrm{cl}(\mathcal{N})$ is δ^*-compact. But then $\mathcal{N} \cap Y$ is an δ^*_Y-open neighbourhood of y in Y such that $\delta^* - \mathrm{cl}(\mathcal{N} \cap Y) \subset \delta^* - \mathrm{cl}(\mathcal{N})$ [$\because \mathcal{N} \cap Y \subset \mathcal{N} \Longrightarrow \delta^* - \mathrm{cl}(\mathcal{N} \cap Y) \subset \delta^* - \mathrm{cl}(\mathcal{N})$]. Thus $\delta^* - \mathrm{cl}(\mathcal{N} \cap Y)$ is δ^*-closed subset of the δ^*-compact set $\delta^* - \mathrm{cl}(\mathcal{N})$ and is therefore itself δ^*-compact. Also since Y is δ^*-closed in X, it is easy to see that the δ^*-closure of $\mathcal{N} \cap Y$ in X is the same as its δ^*-closure in Y (i.e. $\delta^* - \mathrm{cl}(\mathcal{N} \cap Y) = \delta^*_Y - \mathrm{cl}(\mathcal{N} \cap Y)$. Thus we have shown that every point in Y has a δ^*_Y-neighbourhood in Y whose δ^*_Y-closure in Y is δ^*-compact. Hence the subspace $(Y, \mathcal{T}_Y, \mathcal{P}_Y, \mathcal{Q}_Y)$ is locally δ^*-compact.

Example: locally δ^*-compact space need not be δ^*-compact.

Consider any discrete tritopological space (w.r.t. δ^*-open set), where X is infinite. Then X is not δ^*-compact since the collection of all singleton sets is an infinite δ^*-open cover of X which has no finite subcover. But X is locally δ^*-compact. For let x be any point of X. Then $\{x\}$ is a δ^*-nhd of x whose δ^*-closure is $\{x\}$. Also $\{x\}$ is a δ^*-compact subset of X, being finite. Hence every point of x has a δ^*-nhd whose δ^*-closure is δ^*-compact.

Theorem: A δ^*-Hausdorff space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is locally δ^*-compact $\Leftrightarrow$ each of its point is an δ^*-interior point of some δ^*-compact subspace of X.

Proof: If $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ is locally δ^*-compact, then each $x \in X$ has a δ^*-nbd $\mathcal{N}$ such that $\delta^* - \mathrm{cl}(\mathcal{N})$ is δ^*-compact. It follows that $\delta^* - \mathrm{cl}(\mathcal{N})$ is a δ^*-compact nhd of , and so x is an δ^*-interior point. Thus every point of X is an δ^*-interior point of some δ^*-compact subspace of X. conversely, let every point of X be a δ^*-interior point of some δ^*-compact subspace. To show that X is locally δ^*-compact. Let $x \in X$ be arbitrary. By hypothesis, There exists a δ^*-compact subspace $(Y, \mathcal{T}_Y, \mathcal{P}_Y, \mathcal{Q}_Y)$ of X such that $x \in \delta^* - \mathrm{int}(Y)$. Then Y is a δ^*-nhd of x. Since X is δ^*-Hausdorff, Y is a δ^*-closed subset of X , so that $\delta^* - \mathrm{cl}(Y) = Y$. Thus every point of X has a δ^*-nhd whose δ^*-closure is δ^*-compact and so X is locally δ^*-compact.

δ^*-Continuity and δ^*-compactness

5.1 Theorem: Let f be a δ^*-continuous mapping of a δ^*-compact tritopological space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ into a tritopological space $(Y, \mathcal{T}', \mathcal{P}', \mathcal{Q}')$. Then $f[X]$ is δ^*-compact. In other words, δ^*-continuous image of a δ^*-compact space is δ^*-compact.

Proof: Let $\{G_\lambda : \lambda \in \Lambda\}$ be any δ^*-open cover of $f[X]$. Since f is δ^*-continuous, $f^{-1}[G]$ is an δ^*-open set in X. The collection $\{ f^{-1}[G_\lambda] : \lambda \in \Lambda\}$ then froms a δ^*-open cover of X. Since X is δ^*-compact, there exist finitely many indices $\lambda_1, \dots, \lambda_n$ such that $X = f^{-1}[G_{\lambda 1}] \cup \dots \cup f^{-1}[G_{\lambda n}] = f^{-1}[G_{\lambda 1} \cup \dots \cup G_{\lambda n}]$ so that $[X] = G_{\lambda 1} \cup \dots \cup G_{\lambda n}$. Hence $f[X]$ is δ^*-compact.

Corollary: Let X and Y be a tritopological spaces and let $\mathcal{A}$ be a δ^*-compact subset of X. If $f: (X, \mathcal{T}, \mathcal{P}, \mathcal{Q}) \to (Y, \mathcal{T}', \mathcal{P}', \mathcal{Q}')$ is δ^*-continuous, then $f[\mathcal{A}]$ is a δ^*-compact subset of Y.

Proof: We know that the restriction $f_{\mathcal{A}}: \mathcal{A} \to Y$ of f to $\mathcal{A}$ defined by $f_{\mathcal{A}}(x) = f(x)$ for all $x \in \mathcal{A}$ is a δ^*-continuous mapping. Since A is a δ^*-compact subspace of X, it follows from the above theorem that $f_{\mathcal{A}}[\mathcal{A}] = f[\mathcal{A}]$ is a δ^*-compact subset of Y.

Theorem: Let X be a δ^*-compact space and let Y be a δ^*-Hausdorff space. Then every bijective δ^*-continuous mapping of $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ onto $(Y, \mathcal{T}', \mathcal{P}', \mathcal{Q}')$ is a δ^*-homeomorphism.

Proof: Let $f: (X, \mathcal{T}, \mathcal{P}, \mathcal{Q}) \to (Y, \mathcal{T}', \mathcal{P}', \mathcal{Q}')$ be bijective and δ^*-continuous. In order that f may be a δ^*-homeomorphism, it suffices to show that $f[F]$ is δ^*-closed in Y for every δ^*-closed set F in X. By theorem in [1]. So let F be any δ^*-closed set in X. Then F is δ^*-compact , being a δ^*-closed subset of a δ^*-compact space X. Since f is δ^*-continuous, it follows from theorem (5.1) that $f[F]$ is δ^*-compact in Y. Since Y is δ^*-Hausdorff, $f[F]$ is δ^*-closed in Y by theorem (3.1).

δ^*-Continuity and local δ^*-compactness

Theorem: Let f be a mapping of a locally δ^*-compact space $(X, \mathcal{T}, \mathcal{P}, \mathcal{Q})$ onto a δ^*-Hausdorff space $(Y, \mathcal{T}', \mathcal{P}', \mathcal{Q}')$. If f is both δ^*-continuous and δ^*-open, then Y is also locally δ^*-compact.

Proof: Let $y \in Y$. Since f is onto Y , there exists $x \in X$ such that $f(x) = y$. By local δ^*-compactness of X, there exists an δ^*-open nhd $\mathcal{N}$ of x such that $\delta^* - \mathrm{cl}(\mathcal{N})$ is δ^*-compact. But then $x \in \mathcal{N} \subset \delta^* - \mathrm{cl}(\mathcal{N}) \Rightarrow y = f(x) \in f[\mathcal{N}] \subset f[\delta^* - \mathrm{cl}(\mathcal{N})]$. Since f is an δ^*-open map, $f[\mathcal{N}]$ is δ^*-open in Y and so $f[\delta^* - \mathrm{cl}(\mathcal{N})]$ is a δ^*-nhd of y. Since f is δ^*-continuous and $\delta^* - \mathrm{cl}(\mathcal{N})$ is δ^*-compact, it follows from Corollary (5.2) that $f[\delta^* - \mathrm{cl}(\mathcal{N})]$ is δ^*-compact in Y. Moreover, since Y is δ^*-Hausdorff, $f[\delta^* - \mathrm{cl}(\mathcal{N})]$ is δ^*-closed in Y so that $\delta^* - \mathrm{cl}(f[\mathcal{N}]) = f[\delta^* - \mathrm{cl}(\mathcal{N})]$. Thus we have shown that every point y in Y has a δ^*-nhd whose δ^*-closure is δ^*-compact in Y and so y is locally δ^*-compact.

CONCLUSION

The purpose of this article is to establish several properties of δ^*-compact spaces in tritopological spaces Moreover; we obtain preserving theorems with the help of some necessary and interesting examples. And we are define a new concepts in tritopological spaces namly countable δ^*-compactness, lacal δ^*-compactness, and we obtain a relationships among δ^*-compactness, countable δ^*-compactness, lacal δ^*-compactness, δ^*-Hausdorff, δ^*-continuity and δ^*-homeomorphism for tritopological spaces. Furthermore, Uses of tritopological results in this paper and some other papers is worthy for possible applications in some areas of science and social sciences for future.

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