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Rocket Performance with Thrust Equated To Drag and with Constant Path-Inclination by Executing Relevant Control over Lift Coefficient at All Time Instants

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Abstract **Review Article**

A Rocket is flown with variable thrust equated to the drag and with constant path –inclination by varying the lift coefficient using the angle of attack as a control parameter besides variation of the propellant mass consumption. Hence the equations of motion of the rocket are formed and solved to determine time as a function of velocity, mass as a function of velocity and the lift coefficient as a function of velocity which suggests time- varying lift coefficient as a control parameter to accomplish the mission.

Keywords: Path-Inclination, Rocket, velocity.

INTRODUCTION

Angelo Miele [1] and AC Kermode [2] dealt with rocket / aircraft performance in atmosphere as well as in vacuum mainly to determine the greatest height attained, range and propellant/fuel consumption. SN Maitra [3] (present author) studied variable thrust programming for rockets with constant thrust inclination with the horizontal. Authors like Fayyaz A. Lohar, Ajmal K.Sherwani and Arun K. Mishra investigated into optimal control problem of coplanar symmetrical elliptical terrestrial orbit of a spacecraft by use of aero cruise technique, maintaining thrust equal to drag and with constant path-inclination by executing relevant control over the lift coefficient, ie, manipulating the angle of attack and exerting variable propellant consumption. Neglecting Coriolis force, the equations of motion as obtained by Angelo Miele [1] can be rewritten as

$$\frac{dr}{dt} = \mathrm{v}\sin\gamma, \mathrm{r}\frac{d\theta}{dt} = \mathrm{v}\cos\gamma \qquad (1)$$

$$\mathrm{T}=\mathrm{D}=\frac{1}{2}\mathrm{C_D}\rho \mathrm{S}\mathrm{v}^2, \qquad \mathrm{L}=\frac{1}{2}\mathrm{C_L}\rho \mathrm{S}\mathrm{v}^2, \qquad (2)$$

$$\frac{dv}{dt} = \frac{T-D}{m} - \frac{\mu}{r^2}\sin\gamma = -\frac{\mu}{r^2}\sin\gamma \qquad (3)$$

$$\frac{d\gamma}{dt} = \frac{L}{mv} + \left(\frac{v}{r} - \frac{\mu}{r^2 v}\right)\cos\gamma \qquad (4)$$

$$\frac{dm}{dt} = -\frac{T}{\mu/r^2}\frac{1}{c} \qquad (5)$$

Where r is the radius vector joining the rocket to Earth's centre, γ the path inclination to the horizontal, θ the true anomaly, v the velocity, D the drag,

μ the constant related to the grvitational constant , m the mass, c the specific fuel consumption, C_D the drag coefficient, ρ the atmospheric density, S the spacecraft reference area and K_L (=$C_L\rho S$) the ballistic factor. Now we proceed to obtain solutions to the above equations, as follows.

Velocitiy –Mass Distribution

Combing (1) and (3) and employing the initial conditions

at t=0, $\theta = 0, r = r_0$ and $v = v_0$ (6)

and solving for a relationship between and r and v one gets

$$v^2 = v_0^2 + \mu\left(\frac{1}{r} - \frac{1}{r_0}\right) \qquad (7)$$

Because of equation (2), (3) and (5) yield

$$\frac{dm}{dv} = \frac{-1}{2} C_D \rho S v^2 \mu^2 / \{c(\frac{\mu}{r})^4 \sin\gamma)\} \qquad (8)$$

Now (7) can be rewritten in the form

$$\frac{\mu}{r} = \frac{v^2 - c_0^2}{2} \qquad (9)$$

Where $c_0^2 = v_0^2 - \frac{2\mu}{r_0} > 0$ (10)

Combing (8) and (9) is obtained

$$\frac{dm}{dv} = \frac{-K_0}{(v^2 - c_0^2)^4} v^2 \qquad (11)$$

Where $K_0 = 8 C_D \rho S \mu^2 / (c \sin\gamma)$ (11.1)

γ *is kept constant due to variation of the lift coefficient* C_L *with time t so* that (4) and (2) give this control parameter as

$$C_L = 2(\frac{\mu}{r^2 v^2} - \frac{1}{r}) \frac{m}{\rho S} \cos\gamma \qquad (12)$$

Once mass m, velocity v and

γ are expressed as functions oftime t, the criteron of keeping path inclination γ constant by varying the lift coefficient C_L with the help of controlled reference surface S or otherwise is established.

Solution to equation (11) is effected taking the initial conditions:

at t=0, $m = m_0, v = v_0$ (13)

$$-\int_{m_0}^{m} dm = K_0 \int_{v_0}^{v} \frac{v^2 dv}{(v^2 - c_0^2)^4} = K_0 [\int_{v_0}^{v} \frac{dv}{(v^2 - c_0^2)^3} + c_0^2 \int_{v_0}^{v} \frac{dv}{(v^2 - c_0^2)^4}]$$

Or, $m = m_0 - K_0 \, [I_3 + c_0^2 I_4)]_{v_0}^{v}$ (14)

where $I_n = \int \frac{dv}{(v^2 - c_0^2)^n}$, n=3,4 (15

Applying Reduction formula to integral (15) for n=1, 2, 3, 4 one gets

$$I_n = \int \frac{dv}{(v^2 - c_0^2)^n} = \frac{dv}{(v^2 - c_0^2)^n} + 2n \int_{v_0}^{v} \frac{v^2 dv}{(v^2 - c_0^2)^{n+1}}$$

$$= \frac{v}{(v^2 - c_0^2)^n} + 2n(I_n + c_0^2 I_{n+1})$$

$$I_{n+1} = \frac{-1}{2n c_0^2} [\frac{v}{(v^2 - c_0^2)^n} + (2n-1) I_n] \qquad (16)$$

$$\text{where } I_1 = \int \frac{dv}{v^2 - c_0^2} = \frac{1}{2c_0} \text{Log} \frac{v - c_0}{v + c_0} < 0 \qquad (17)$$

In view of the above results it is clear that velocity v can be expressed as a function of mass. The radial distance r of the spacecraft can be expressed as a function of true anomaly θ by use of two equations in (1).

Instantaneous Velocity

$$\frac{1}{r} \frac{dr}{d\theta} = \tan\gamma = (constant) \qquad (18)$$

With the initial conditions (7) its solution is given by

$$r = r_0 e^{\theta \tan\gamma} \qquad (19)$$

Which suggests that in such a situation, ie, under the constraint of the same path inclination at all time instants, the trajectory of the spacecraft is an equiangular spiral.

The velocity can be expressed as an implicit function of time by use of (3) and (9):

$$\frac{dv}{dt} = -\frac{1}{4\mu}(v^2 - c_0^2)^2 \sin\gamma \qquad (20)$$

In consequence of of (16) and (6) equation (20) can be integrated as

$$\frac{t\sin\gamma}{4\mu} = -\int_{v_0}^{v} \frac{dv}{(v^2 - c_0^2)^2} = -[I_2]_{v_0}^{v} = -J_2 \qquad (21)$$

$$\text{where } I_2 = -\frac{1}{4c_0^2}\left[\frac{v}{v^2 - c_0^2} + \frac{3}{2c_0}\mathrm{Log}\frac{v - c_0}{v + c_0}\right] \qquad (22)$$

Using (21) and (22), we get time as function of velocity;

$$t = \frac{4\mu}{\sin\gamma}\left[\frac{v}{v^2 - c_0^2} - \frac{v_0}{{v_0}^2 - c_0^2} + \frac{3}{2c_0}\mathrm{Log}\left\{\left(\frac{v - c_0}{v + c_0}\right)\left(\frac{v_0 + c_0}{v_0 - c_0}\right)\right\}\right] \qquad (23)$$

Radius vector-time (r,t) relationship can be determined by eliminating v between (23) and (9). (v,θ) *relationship can be obtained by substituting* (19) into (9):

$$v = \sqrt{\frac{2\mu}{r_0 e^{\theta\tan\gamma}} + c_0^2} \qquad (24)$$

Lift Coefficient as Control Parameter

To maintain a constant path-inclination $\gamma = \gamma_0$ is needed a time-varying lift coefficient obtained by use of equations (9), (12), (14) and (23):

$$C_L(t) = -\frac{v^4 - c_0^4}{2\mu\rho S v^2}\{m_0 - K_0(J_3 + c_0^2 J_4\}\cos\gamma_0$$

Where $J_n = [I_n]_{v_0}^{v}$, n=3,4

CONCLUSION

The mass variation law with time can be framed with the help of (14) and (23). Hence, on the whole it is established that holding the thrust equal to the drag entails a mass-variation law due to propellant mass consumption which coupled with a constant path-inclination at all time instants gives rise to a law of variation of lift coefficient as a control parameter.

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