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R-T Stability of a Stratified Maxwell Viscoelastic Fluid with Dust Particles in a Porous medium

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Abstract **Review Article**

The present paper deals with the Rayleigh - Taylor stability of a stratified Maxwell viscoelastic fluid with suspended dust particles through a porous medium in presence of a horizontal magnetic field. Consider the case of exponentially varying density, viscosity, magnetic field and suspended particles a dispersion relation has been derived and found that for the unstable stratification the system is unstable in the absence of a magnetic field and is also unstable if $k_x^2 V_A^2 < \frac{g\beta k^2}{L}$ however, the system can be completely stabilized by a large magnetic field if $V_A^2 > \frac{g\beta k^2}{k_x^2 L}$. The viscoelasticity, the medium permeability and the suspended particles do not have any qualitative effect on the nature of stability.

Keywords: Rayleigh-Taylor Instability, Stratified Fluid, Maxwell Viscoelastic Fluid, Porous Medium, Horizontal Magnetic Field (or Hydromagnetics).

1. INTRODUCTION

The stability of an incompressible heavy fluid of variable density was first investigated by Lord Rayleigh and is now known as the Rayleigh–Taylor instability. Several authors have extensively analyzed the Rayleigh–Taylor instability of Newtonian fluids under various assumptions of hydrodynamics and hydromagnetics, and a detailed account of these studies is presented in the classical monograph by Chandrasekhar [1]. Chin [2] examined wave propagation phenomena in porous media relevant to petroleum engineering applications. In geophysical situations, the fluids involved are often not pure but contain suspended or dust particles, which alter their dynamical behavior. Therefore, the investigation of Rayleigh–Taylor instability in fluids through porous media holds substantial importance in geophysics, soil science, groundwater hydrology, and astrophysics. The role of porosity in astrophysical bodies such as comets, meteorites, and interplanetary dust has been emphasized by McDonnell [3].

In stellar interiors and atmospheres, variable and non-uniform magnetic fields act and significantly influence the stability characteristics of such fluids. With the growing industrial and technological applications of non-Newtonian fluids, considerable attention has been directed toward their stability behavior. Sunil and Chand [4] investigated the Rayleigh–Taylor instability of a plasma in the presence of a variable magnetic field and suspended particles in porous media. Sharma and Kumar [5] extended this analysis to Maxwell viscoelastic fluids under the influence of a variable magnetic field in porous media.

The stability of shear flow in heterogeneous or stratified fluids has also been a subject of intensive research. Sunil, Sharma, and Chandel [6] analyzed the stability of stratified Rivlin–Ericksen viscoelastic fluids with suspended particles in a porous medium, whereas Sunil, et.al. [7] examined the stability of stratified Walters (Model B′) viscoelastic fluids in stratified porous media. Rayleigh–Taylor instability in porous media with rotation was examined by G.A. Hoshoudy [8]. R.P. Prajapati [9] has studied the stability analysis of dusty plasma in porous media. Recently Nonlinear instability of an Oldroydian viscoelastic fluid in porous media have been investigated by G.M. Moatimid, Y.O. El-Dib [10]. Sharma *et al.* [11] analyzed the Rayleigh–

Taylor instability in dusty magnetized fluids flowing through a porous medium.

The stability of stratified Maxwell viscoelastic fluid in the presence of suspended particles and variable magnetic field in porous medium has been investigated here. So keeping in mind the importance of non-Newtonian fluids in modern technology and their various applications mentioned above the present paper is devoted to the consideration of the stability of stratified Maxwell viscoelastic fluid in the presence of suspended particles and variable magnetic field through a porous medium.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS:

Let T_{ij}, τ_{ij}, e_{ij}, δ_{ij}, P, q_i, x_i, μ, and λ denote, respectively, the total stress tensor, shear stress tensor, rate of strain tensor, Kronecker delta, scalar pressure, velocity, position vector, viscosity and stress relaxation time. Then the Maxwell viscoelastic fluid is described by the constitutive relations

$$T_{ij} = -P\delta_{ij} + \tau_{ij},$$

$$\left(1+\lambda\frac{d}{dt}\right)\tau_{ij} = 2\mu e_{ij} \quad ...(1)$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_x}\right),$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{q}.\nabla$$

is the 'convective derivative'. Relations of the type (1) were proposed and studied by Maxwell. When a fluid flows through a homogeneous and isotropic porous medium, the gross effect is represented by Dracy's law. As a result, the usual viscous term is replaced by the resistance term

$$-\left(\frac{\mu}{k_1}\right)\vec{v},$$

where k_1 and $\vec{v}$ denote, respectively, the medium permeability and the filter velocity. Here we consider a static state in which an incompressible, electrically conducting Maxwell fluid layer of variable density is arranged in horizontal strata in the presence of suspended particles and a variable horizontal magnetic field. The pressure P and density ρ are functions of vertical z-coordinate only. The character of the equilibrium of this initial state is determined by supposing that the system is slightly disturbed and then by following its further evolution. The fluid is under the action of the gravity $\vec{g}$ (0, 0, $-g$) and the magnetic field $\vec{H}$ = (H_0(z),0,0). This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of the porosity ε and medium permeability k_1.

Let P, ρ, μ and $\vec{v}$ (u, v, w) denote, respectively, the pressure, the density, the viscosity and the velocity of pure fluid, $\vec{u}(\bar{x},t)$ and N($\bar{x}$,t) denote the filter velocity and the number density of the suspended particles. $K = 6\pi\mu\eta$, where η is the particle radius, is the Stokes drag coefficient. $\vec{u} = (l,r,s)$) and $\bar{x} = (x,y,z)$. Let g, ε and k_1 stand for acceleration due to gravity, medium porosity, medium permeability. Let μ_e denote magnetic permeability. Then the equations of motion and continuity for Maxwell viscoelastic fluid with suspended particle and horizontal variable magnetic field in porous medium are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}\right]$$

$$= \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla P + \vec{g}\rho + \frac{\mu e}{\pi}\left\{(\nabla\times\vec{H})\times\vec{H}\right\} + \frac{KN}{\varepsilon}(\vec{u}-\vec{v})\right]$$

$$-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\vec{v}. \quad \ldots(2)$$

$$\nabla.\vec{v} = 0, \quad \ldots(3)$$

$$\nabla.\vec{H} = 0, \quad \ldots(4)$$

$$\varepsilon\frac{\partial\vec{H}}{\partial t} = (\vec{H}.\nabla)\vec{v} - (\vec{v}.\nabla)\vec{H} \quad \ldots(5)$$

Since the density of a fluid particle remains unchanged as we follow it with its motion, we have

$$\varepsilon \frac{\partial \rho}{\partial t} + (\vec{v}.\nabla)\rho = 0 \qquad ...(6)$$

In the equation of motion (2), assuming uniform particle size, spherical shape and small relative velocities between the fluid and particle, the presence of particle adds an extra force term, in the equation of motion (2) proportional to the velocity difference between particles and fluid.

The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Interparticle reactions are ignored for we assume that the distances between the particles are large compared with their diameters. If *mN* is the mass of the particles per unit volume, then the equations of motions and continuity for the particles are

$$mN\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right] = KN(\vec{v} - \vec{u}), \qquad ...(7)$$

$$\frac{\partial N}{\partial t} + \nabla(N\vec{u}) = 0 \qquad ...(8)$$

Let $\delta\rho$, δP, $\vec{v}$ (u,v,w) $\vec{u}$ (l.r.s) and $\vec{h}$ (h_x, h_y, h_z) denote, respectively, the perturbation in the fluid density ρ, the fluid pressure *P*, the fluid velocity (0,0,0), the particle velocity (0,0,0) and the magnetic field $\vec{H}(H_0(z),0,0)$; then the linearized hydromagnetic perturbation equations of Maxwell viscoelastic fluid in porous medium with suspended particles are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \vec{v}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta P + \vec{g}\delta\rho + \frac{\mu e}{4\pi}\left\{(\nabla\times\vec{h})\times\vec{H}\right.\right.$$

$$= \left.\left.(\nabla\times\vec{H})\times\vec{H}\right\} + \frac{KN}{\varepsilon}(\vec{u}-\vec{v})\right] - \frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\vec{v} \qquad ...(9)$$

$$\nabla.\vec{v} = 0, \qquad ...(10)$$

$$\nabla.\vec{h} = 0, \qquad ...(11)$$

$$\varepsilon\frac{\partial \vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{v} - (\vec{v}.\nabla)\vec{H} \qquad ...(12)$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho = -w(D\rho), \qquad ...(13)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\vec{u} = \vec{v} \qquad ...(14)$$

$$\frac{\partial M}{\partial t} + \nabla.\vec{u} = 0 \qquad ...(15)$$

where:

$$M = \frac{\varepsilon N}{N_0},$$

N_0 and N stand for initial uniform number density and perturbation in number density, and

$$D = \frac{d}{dz}$$

3. DISPERSION RELATION:

Analyzing the perturbation into normal modes, we assume that the perturbation quantities have an *x, y* and *t* dependence of the form

$$\exp(ik_x x + ik_y y + nt) \qquad ...(16)$$

where k_x, k_y are the wave numbers along x and y directions, respectively, $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number of disturbance and n stands for the growth rate which is, in general, a complex constant. For perturbations of the form (16) equations (9)–(14), after eliminating $\vec{u}$, give

$$\frac{1}{\varepsilon}\left[\rho+\frac{mN}{\tau n+1}\right](1+\lambda n)nu$$
$$=(1+\lambda n)\left[-ik_x\delta P+\frac{\mu_e}{4\pi}h_z(DH_0)\right]-\frac{\mu}{k_1}(1+\lambda_0 n)u, \quad \ldots(17)$$

$$\frac{1}{\varepsilon}\left[\rho+\frac{mN}{\tau n+1}\right](1+\lambda n)nv$$
$$=(1+\lambda n)\left[-ik_y\delta P+\frac{\mu_e}{4\pi}H_0(ik_xh_y-ik_yh_x)\right]-\frac{\mu}{k_1}(1+\lambda_0 n)v, \quad \ldots(18)$$

$$\frac{1}{\varepsilon}\left[\rho+\frac{mN}{\tau n+1}\right](1+\lambda n)nw$$
$$=(1+\lambda n)\left[-D\delta P-g\delta\rho+\frac{\mu_e}{4\pi}(ik_xh_zH_0-H_0Dh_x-h_xDH_0)\right]$$
$$-\frac{\mu}{k_1}(1+\lambda_0 n)w, \quad \ldots(19)$$

$$ik_xu + ik_y + Dw = 0 \quad \ldots(20)$$

$$ik_xh_x + ik_yh_y + Dh_z = 0 \quad \ldots(21)$$

$$\left.\begin{aligned}\varepsilon nh_x &= ik_xH_0u-wDH_0\\ \varepsilon nh_y &= ik_xH_0v,\\ \varepsilon nh_z &= ik_xH_0w\end{aligned}\right\} \quad \ldots(22)$$

$$\varepsilon n\delta\rho=-wD\rho, \quad \ldots(23)$$

where □ = m/K

Eliminating □P, □□, u,v, h_x, h_y, h_z between (17) – (19) and using (20) - (23) we obtain

$$(1+\lambda n)\left[\left\{D(\rho Dw)-k^2\rho w\right\}+\frac{gk^2}{n^2}(D\rho)w-\frac{\mu_e}{4\pi n^2}H_0^2k_x^2k^2w\right.$$
$$\left.+\frac{\mu_e k_x^2}{4\pi n^2}D\left(H_0^2Dw\right)+\frac{1}{(\tau n+1)}\left\{D(mNDw)-k^2mNw\right\}\right]$$
$$+\frac{\varepsilon}{nk_1}(1+\lambda_0 n)\left[D(\mu Dw)k^2\mu w\right]=0 \quad \ldots(24)$$

4. THE CASE OF EXPONENTIALLY VARYING DENSITY, VISCOSITY, MAGNETIC FIELD AND SUSPENDED PARTICLES:

Here we consider the stratification in magnetic field, density, viscosity and suspended particles in the fluid of the depth d as

$$\mu=\mu_0e^{\beta z},\rho=\rho_0e^{\beta z},H^2=H_0^2e^{\beta z} \text{ and } N=N_0e^{\beta z} \quad \ldots(25)$$

where ρ_0, μ_0, H_0, N_0 and β are constants. Equations (25) imply that the coefficient of kinematic viscosity ν and the Alfven velocity V_A are constant everywhere.

Substituting the values of ρ, μ, H_0^2, N in (24) and neglecting the effect of heterogeneity on inertia and solving the equation we arrive at

$$\left(D^2-k^2\right)w+\frac{g\beta k^2}{n^2\left[1+\frac{(1+\lambda_0 n)}{n(1+\lambda n)}\frac{\varepsilon v_0}{k_1}+\frac{k_x^2V_A^2}{n^2}+\frac{mN_0/\rho_0}{(\tau n+1)}\right]}w=0 \qquad \ldots(26)$$

where

$$v_0=\frac{\mu_0}{\rho_0},\ V_A^2=\frac{\mu_e H_0^2}{4\pi\rho_0}\ \text{and}\ \tau=\frac{m}{K}$$

are constants.

Assume that the system is restricted by two planes $z = 0$ and $z = d$ and that both the boundaries are free.

The boundary conditions for the case of two free surfaces are
. $w = D^2w = 0$ at $z = 0$ and $z = d$ …(27)

The proper solution of equation (26) satisfying (27) is

$$w=w_0\sin\frac{m\pi z}{d} \qquad \ldots(28)$$

where w_0 is a constant and m is an integer. Substituting (28) in (26) and simplifying we obtain

$$\tau\lambda n^4+n^3\left[\tau+\lambda+\frac{\tau\lambda_0\varepsilon v_0}{k_1}+\frac{\lambda m N_0}{\rho_0}\right]+n^2\left[1+\left((\tau+\lambda_0)+\frac{\varepsilon v_0}{k_1}+\frac{mN_0}{\rho_0}\right)\right.$$
$$\left.+\tau\lambda\left(k_x^2V_A^2-\frac{g\beta k^2}{L}\right)\right]+n\left[\frac{\varepsilon v_0}{k_1}+(\tau+\lambda)\left\{k_x^2V_A^2-\frac{g\beta k^2}{L}\right\}\right]$$
$$\left[k_x^2V_A^2-\frac{g\beta k^2}{L}\right]=0 \qquad \ldots(29)$$

where

$$L=\frac{m^2\pi^2}{d^2}+k^2.$$

5. DISCUSSSION

For the stable stratifications (β < 0), equation (29) does not have any positive root of n implying thereby that the system is stable for disturbances of all wave numbers.

For the unstable stratifications (β > 0), the system is stable or unstable depending on whether $k_x^2V_A^2$ is greater than or less than $g\beta k^2/L$, i.e,

$$k_x^2V_A^2>\frac{g\beta k^2}{L}\ \text{or}\ k_x^2V_A^2<\frac{g\beta k^2}{L}$$

The system is clearly unstable for β > 0 in the absence of a magnetic field and is also unstable if

$$k_x^2V_A^2<\frac{g\beta k^2}{L}$$

However, the system can be completely stabilized by a large magnetic field as can be seen from equation (29) if

$$V_A^2>\frac{g\beta k^2}{k_x^2 L}$$

The magnetic field therefore succeeds in stabilizing wave numbers in the range

$$k^2>\frac{g\beta}{V_A^2}\sec^2\theta-\left(\frac{m\pi}{d}\right)^2 \qquad \ldots(30)$$

which are unstable in the absence of magnetic field. Here θ is the angle between k_x and k (i.e., $k_x = k\cos\theta$).

The viscoelasticity, the medium permeability and the suspended particles do not have any qualitative effect on the nature of stability.

REFRENCES

1. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, London, 1961.
2. D.C. Chin, Wave propagation in porous media, *J. Hydrol.* 64 (1983) 67–79.
3. J.A.M. McDonnell, On the structure and composition of cometary nuclei, *Space Sci. Rev.* 21 (1978) 355–418.
4. Sunil, D. Chand, Rayleigh–Taylor instability of plasma in the presence of suspended particles and variable magnetic field in porous media, *Astrophys. Space Sci.* 272 (2000) 523–533.
5. R.C. Sharma, P. Kumar, Rayleigh–Taylor instability of Maxwell viscoelastic fluids in porous media with variable magnetic field, *Acta Phys. Pol. A* 105 (2004) 247–255.
6. Sunil, R.C. Sharma, R.S. Chandel, Stability of stratified Rivlin–Ericksen viscoelastic fluid with suspended particles in a porous medium, *Int. J. Eng. Sci.* 44 (2006) 237–249.
7. Sunil, Divya, R.C. Sharma, V. Sharma, Stability of stratified Walters' (Model B′) viscoelastic fluid through porous medium, *ZAMM – J. Appl. Math. Mech.* 88 (2008) 385–396.
8. G.A. Hoshoudy, Rayleigh–Taylor instability in porous media with rotation and surface tension, *Appl. Math. Comput.* 217 (2011) 10108–10118.
9. R.P. Prajapati, Stability analysis of dusty plasma in porous media, *J. Porous Media* 13 (2010) 205–214.
10. G.M. Moatimid, Y.O. El-Dib, Nonlinear instability of an Oldroydian viscoelastic fluid in porous media, *Appl. Math. Comput.* 347 (2019) 147–160.
11. Praveen K. Sharma, Anita Tiwari, Ram P. Prajapati & Rajendra K. Chhajlani, "Rayleigh–Taylor instability in dusty magnetized fluids with surface tension flowing through porous medium" — *Thermal Science*, 2016.