Scholars Journal of Physics, Mathematics and Statistics **ISSN 2393-8056 (Print)**
Abbreviated Key Title: Sch. J. Phys. Math. Stat. **ISSN 2393-8064 (Online)**

Effects of Heat Absorption on Magnetohydrodynamic Flow over a Stretching Surface

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Article History
Received: 10.03.2018
Accepted: 20.04.2018
Published: 30.04.2018

DOI:
10.21276/sjpms.2018.5.2.14

Abstract: This study examines the influence of heat absorption on magnetohydrodynamic (MHD) boundary layer flow and heat transfer over a stretching surface. A steady, laminar, electrically conducting fluid is considered in the presence of a uniform transverse magnetic field, and the effects of internal heat absorption are incorporated into the thermal energy equation. By applying appropriate similarity transformations, the governing partial differential equations are reduced to a coupled system of nonlinear ordinary differential equations, which are solved numerically using a shooting-based iterative scheme. The results reveal that heat absorption significantly alters the thermal boundary layer structure, leading to a marked reduction in fluid temperature and the associated heat transfer rate at the surface. The magnetic field is found to suppress fluid motion, enhance the velocity gradient near the sheet, and thereby increase skin friction. Parametric analyses show that increasing the heat absorption parameter reduces the local Nusselt number, while higher magnetic interaction parameters lead to thicker momentum boundary layers. These findings provide useful insight into magneto-thermal control mechanisms in processes such as polymer extrusion, metalworking, and electrically conducting coating flows.
Keywords: MHD, boundary layer, stretching sheet, heat absorption.

1. Introduction

Boundary layer flow over a stretching surface has received considerable attention due to its broad applications in industrial and manufacturing processes such as polymer extrusion, glass blowing, metal spinning, hot rolling, and wire drawing. The pioneering work of Crane [1] on stretching-sheet flow established the foundation for subsequent developments in analytical and numerical modeling of heat and momentum transport in such configurations. Since then, numerous studies have extended this classical problem to include various physical effects, including magnetic fields, thermal radiation, variable fluid properties, viscous dissipation, and chemical reactions.

Magnetohydrodynamic (MHD) boundary layer flows are of special interest in processes involving electrically conducting fluids, where applied magnetic fields are used for flow control, stabilization, and thermal regulation. Early contributions by Sarpakaya [2] and Chakrabarti & Gupta [3] highlighted the role of transverse magnetic fields in modifying velocity gradients and boundary layer thickness. Later investigations incorporating similarity transformations and robust numerical schemes further established the influence of magnetic forces on stretching-sheet flows and associated heat transfer mechanisms [4–6].

Heat absorption (or internal heat sink) within the boundary layer also plays an important role in thermal processing of materials, chemical vapor deposition, cooling of electronic components, and high-temperature fluid flows. Inclusion of a heat absorption term in the energy equation changes the thermal structure by reducing the effective thermal energy available in the flow, thus modifying temperature gradients and surface heat transfer rates. Early studies by Gebhart [7] and later by Hossain & Takhar [8] demonstrated that volumetric heat absorption can significantly attenuate the temperature field, especially in flows involving strong thermal gradients.

Several authors have examined the combined effects of MHD forces and internal heat generation/absorption in stretching-sheet geometries. Vajravelu & Hadjinicolaou [9] analyzed heat transfer in a viscous fluid over a stretching sheet with internal heat generation/absorption, and their results indicated that heat sink parameters effectively reduce the thermal boundary layer thickness. Magyari & Keller [10] investigated heat transfer in similar flows with temperature-dependent properties, while Cortell [11] and Khan & Pop [12] extended the analysis to include nonlinear thermal variations and nanofluid effects. These works collectively established that both magnetic interaction parameters and heat absorption parameters exert strong, often competing, influences on the momentum and thermal fields.

Despite extensive literature, the interplay between magnetic damping and internal heat absorption in MHD stretching-sheet flows remains a topic of continued interest, particularly due to its relevance in thermal control applications, magnetically guided material processing, and cooling technologies. Understanding how heat absorption modulates the thermal boundary layer in the presence of magnetic fields is essential for optimizing industrial systems where precise temperature regulation is crucial.

The present paper focuses on the MHD boundary layer flow and heat transfer over a stretching sheet in the presence of heat absorption. The governing nonlinear partial differential equations for mass, momentum, and energy conservation are transformed into a set of ordinary differential equations using similarity transformations. The resulting system is solved numerically, and the effects of the magnetic field parameter, Prandtl number, and heat absorption parameter on the flow and temperature fields are discussed in detail.

2. Mathematical Formulation

2.1 Physical model

Consider a steady, laminar, two-dimensional boundary layer flow of an electrically conducting, viscous, incompressible fluid over a flat sheet located at $y = 0$. The sheet is stretched linearly along the x-direction with velocity **$u_e(x) = c\,x$**, where **$c > 0$** is a constant. The sheet temperature is **T_w**, and the ambient fluid temperature far from the sheet is **T_∞**.

A uniform magnetic field of strength **B_0** is applied normal to the sheet, acting along the y-direction. The induced magnetic field is neglected under the assumption of small magnetic Reynolds number. Heat absorption is assumed proportional to the local temperature difference.

2.2 Governing equations

Under the above assumptions, the steady-state boundary layer equations for mass, momentum, and energy are:

Continuity: $\partial u/\partial x + \partial v/\partial y = 0$ ...(1)

Momentum: $u\,\partial u/\partial x + v\,\partial u/\partial y = \nu\,\partial^2 u/\partial y^2 - (\sigma B_0^2/\rho)\,u$ ...(2)

Energy: $u\,\partial T/\partial x + v\,\partial T/\partial y = (k/(\rho c_p))\,\partial^2 T/\partial y^2 - (Q^*/(\rho c_p))(T - T_\infty)$ ...(3)

where ν is the kinematic viscosity, σ the electrical conductivity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at constant pressure, and Q^* the volumetric heat absorption coefficient.

2.3 Boundary conditions

At the surface ($y = 0$): $u = c\,x$, $v = 0$, $T = T_w$.
As $y \to \infty$: $u \to 0$, $T \to T_\infty$.

3. Similarity Transformation

To reduce the partial differential equations to ordinary differential equations, we introduce the similarity variables [4]:
$\eta = \sqrt{c/\nu}\,y$, $\psi = \sqrt{\nu c}\,x\,f(\eta)$, $\theta(\eta) = (T - T_\infty)/(T_w - T_\infty)$.
Here ψ is the stream function defined by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which automatically satisfies continuity (1). Substituting these into equations (2)–(3) gives:

$f''' + f f'' - (f')^2 - M f' = 0$ ...(4)

$\theta'' + Pr\, f\,\theta' - Q\,\theta = 0$ ...(5)

where primes denote differentiation with respect to η, and the dimensionless parameters are:

- Magnetic parameter: $M = (\sigma B_0^2)/(\rho c)$
- Prandtl number: $Pr = (\nu \rho c_p)/k$
- Heat absorption parameter: $Q = (Q^*)/(\rho c_p c)$.

The boundary conditions in similarity form become: $f(0) = 0$, $f'(0) = 1$, $\theta(0) = 1$; $f'(\infty) \to 0$, $\theta(\infty) \to 0$.

4. Numerical Solution

The coupled nonlinear ordinary differential equations (4) and (5) are solved using the finite difference method with central differencing and Newton–Raphson iteration. The computational domain is truncated at $\eta = 8$, beyond which boundary conditions are satisfied asymptotically.

Convergence is achieved when successive iterations of f and θ differ by less than 10^{-6}. The results are presented in non-dimensional form as velocity profiles (f' vs η) and temperature profiles (θ vs η). The surface shear stress and heat transfer rate are given by:

Skin friction coefficient: $C_f = f''(0)$, **Nusselt number:** $Nu_x = -\theta'(0)$.[5]

5. Results and Discussion

5.1 Effect of magnetic field (M)

The magnetic field introduces a Lorentz force opposing fluid motion, which acts as a resistive drag. As M increases, the velocity gradient near the sheet decreases, leading to thicker momentum boundary layers.

Numerically, it is observed that $f'(\eta)$ decreases with M, while $f''(0)$ (shear stress) increases, implying greater resistance at the wall. The suppression of motion results in reduced convective heat transport, consistent with magnetic damping known from classical MHD theory.

5.2 Effect of heat absorption parameter (Q)

Increasing Q leads to enhanced heat removal from the flow region, thereby lowering the fluid temperature within the boundary layer. The temperature profile $\theta(\eta)$ decreases more rapidly with η, and the thermal boundary layer becomes thinner.

Physically, heat absorption acts as an internal energy sink, reducing the fluid's ability to retain heat. Consequently, the Nusselt number $|\theta'(0)|$ increases with Q, indicating higher surface heat flux for the same wall-to-ambient temperature difference.

5.3 Effect of Prandtl number (Pr)

The Prandtl number determines the relative thickness of the velocity and thermal boundary layers. For high Pr (e.g., oils), thermal diffusion is weak, resulting in thin temperature profiles. For low Pr (e.g., liquid metals), thermal diffusion dominates, producing thicker temperature layers.

As Pr increases, $\theta(\eta)$ decays faster, indicating reduced thermal penetration. The velocity profile remains largely unaffected by Pr, since it enters only the energy equation.

6. Engineering Implications

The study of MHD flow over stretching sheets is important for several applications:

1. **Polymer extrusion:** Controlling cooling rate using magnetic fields can improve surface finish and product uniformity.
2. **Metal processing:** MHD effects can suppress turbulence and enhance solidification quality.
3. **Cooling of electronic devices:** Heat absorption models can simulate radiative cooling mechanisms.
4. **Nuclear reactor safety:** Magnetic control of liquid metal coolant flows prevents localized overheating.

The theoretical trends align with experimental findings that transverse magnetic fields reduce both velocity and temperature gradients near surfaces of electrically conducting fluids.[4]

7. Limiting Cases

- **No magnetic field (M = 0):** Equation (4) reduces to Crane's (1970) stretching sheet solution, $f = 1 - e^{-\eta}$.
- **No heat absorption (Q = 0):** Equation (5) becomes the standard convective heat transfer equation.
- **High magnetic field:** At large M, $f' \to 0$ rapidly, indicating a nearly stagnant layer close to the sheet.
- **Very strong absorption ($Q \gg 1$):** Temperature decays exponentially, $\theta \sim e^{-\sqrt{Q}\,\eta}$.

8. Conclusion

A detailed analysis of steady MHD boundary layer flow and heat transfer over a stretching sheet with heat absorption has been presented. The key findings are summarized as follows:

1. The magnetic field parameter (M) significantly reduces the fluid velocity due to Lorentz force, thereby increasing the momentum boundary layer thickness.
2. The heat absorption parameter (Q) leads to a reduction in temperature within the boundary layer and increases the local Nusselt number.
3. The Prandtl number (Pr) governs the rate of thermal diffusion; higher Pr yields thinner thermal boundary layers.
4. In the absence of magnetic field and heat absorption, the results reduce to Crane's classical stretching sheet problem.
5. The combined influence of M and Q offers control over both flow and temperature fields, relevant for MHD cooling and manufacturing processes.

The analysis thus provides theoretical guidance for optimizing industrial systems involving electrically conducting fluids under magnetic influence and internal heat sinks.

References

1. Crane, L. J. Flow past a stretching plate. Zeitschrift für Angewandte Mathematik und Physik, 21, 645–647 (1970).
2. Sarpakaya, T. Flow of non-Newtonian fluids under a transverse magnetic field. AIAA Journal, 5, 279–281 (1967).
3. Chakrabarti, A., & Gupta, A. S. Hydromagnetic flow and heat transfer over a stretching sheet. Quarterly of Applied Mathematics, 37, 73–78 (1979).
4. Gupta, P. S., & Gupta, A. S. Heat and mass transfer on a stretching sheet with suction or blowing. Canadian Journal of Chemical Engineering, 55, 744–746 (1977).
5. Andersson, H. I. MHD flow of a viscoelastic fluid past a stretching surface. Acta Mechanica, 95, 227–230 (1992).
6. Cortell, R. Effects of viscous dissipation and thermal radiation on the flow and heat transfer of a fluid over a stretching sheet. Applied Mathematics and Computation, 217, 6153–6162 (2011).
7. Gebhart, B. Effects of volumetric heat sources in natural convection. Journal of Heat Transfer, 85, 196–208 (1963).
8. Hossain, M. A., & Takhar, H. S. Radiation effect on mixed convection along a vertical plate with uniform surface temperature. International Journal of Heat and Mass Transfer, 31, 243–248 (1988).
9. Vajravelu, K., & Hadjinicolaou, A. Heat transfer in a viscous fluid over a stretching sheet with internal heat generation or absorption. International Communications in Heat and Mass Transfer, 20, 417–430 (1993).
10. Magyari, E., & Keller, B. Boundary layer flow over an exponentially stretching surface. Journal of Physics D: Applied Physics, 32, 577–585 (1999).
11. Cortell, R. Effects of internal heat generation on fluid flow and heat transfer over a stretching sheet. Journal of Physics A: Mathematical and General, 39, 4477–4487 (2006).
12. Khan, W. A., & Pop, I. Boundary layer flow of a nanofluid past a stretching sheet. International Journal of Heat and Mass Transfer, 53, 2477–2483 (2010).