Scholars Journal of Physics, Mathematics and Statistics

Abbreviated Key Title: Sch J Phys Math Stat
ISSN 2393-8056 (Print) | ISSN 2393-8064 (Online)
Journal homepage: https://saspublishers.com/sjpms/

Steady Generalized Plane Couette Flow of Viscous Incompressible Fluid between Two Porous Parallel Plates through Porous Medium with Magnetic Field

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DOI:10.36347/sjpms.2020.v07i08.004 | **Received:** 10.08.2020| **Accepted:** 18.08.2020| **Published:** 20.08.2020

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Abstract **Review Article**

In this paper, we have investigated the steady generalized plane Couette flow of viscous incompressible fluid between two porous parallel plates through porous medium with magnetic field. We have studied the velocity, average velocity, shear stress, skin frictions, the volumetric flow, drag coefficients & stream lines.

Keywords: Steady Couette flow, viscous parallel plates, incompressible fluid,porous medium, & magnetic field.

NOMENCLATURE

$u =$ Velocity component along x–axis
$v =$ Velocity component along y–axis
$t =$ The time
$\rho =$ The density of fluid
$P =$ The fluid pressure
$k =$ The thermal conductivity
$\mu =$ Coefficient of viscosity
$\upsilon =$ Kinematic viscosity
$Q =$ The volumetric flow

INTRODUCTION

We have investigated steady generalized plane Couette flow of viscous incompressible fluid between two porous parallel plates through porous mediumwith magnetic field. Attempts have been made by several researchers i.e. M. Aydin & R. T. Fenner [1] investigated boundary element analysis of driven cavity flow for low & moderate Reynolds number. N. Bahloul & Boutana & P. Vasseur [2] investigated double diffusive & sorbet induced convection in a shallow horizontal porous layer. V. et. al. Barbu [3] investigated exact controllability magneto hydrodynamic equations. D. Barkley & L. S. Tuckerman [4] investigated stability analysis of perturbed plane Couette flow. D. Barkley & L. S. Tuckerman [5] investigated turbulent laminar patterns in plane Couette flow. D. Barkley & L. S. Tuckerman [6] investigated mean flow of turbulent–laminar patterns in plane Couette flow. E. Barragy & G. F. Carey [7] investigated stream function vorticity driven cavity solutions using p finite elements. G. K. Batchelor [8] investigated a proposal concerning laminar wakes behind bluff body's at large Reynolds number. G. K. Batchelor [9] investigated on steady laminar flow with closed streamlines at large Reynolds number. C. Baytas & I. Pop [10] investigated free convection in oblique enclosures filled with a porous medium. R. M. Beam & R. F. warming [11] investigated an implicit factored scheme for the compressible Navier–Stokes equations. Beant Singh & Chanpreet Singh [12] investigated analysis of vortex motion in porous media. R. Šulc & P. Ditl [13] investigated local energy dissipation rate in an agitated vessel a comparison of evaluation methods. R. Šulc, P. Ditl, I. Fořt, D. Jašíkova, M. Kotek, V. Kopecký & B. Kysela [14] investigated the effect of particle image velocimetry setting parameters on local velocity measurements in an agitated vessel. G. G. Tsypkin & V. A. Shargatov [15] investigated influence of capillary pressure gradient on connectivity of flow through a porous medium. In this paperwe have investigated the velocity, average velocity, shear stress, skin frictions, the volumetric flow, drag coefficients and stream lines.

FORMULATION OF THE PROBLEM

Let us consider two infinite porous plates AB & CD separated by a distance 2h. The fluid enters in y direction. The velocity component along x-axis is a function of y only. The motion of incompressible fluid is in two dimension and is steady then

$$u = u(y),\quad w = 0 \;\;\&\;\; \frac{\partial}{\partial t} \equiv 0$$

The equation of continuity for incompressible fluid

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad put \quad w=0 \;\;\&\;\; \frac{\partial u}{\partial x}=0 \Rightarrow \frac{\partial v}{\partial y}=0$$

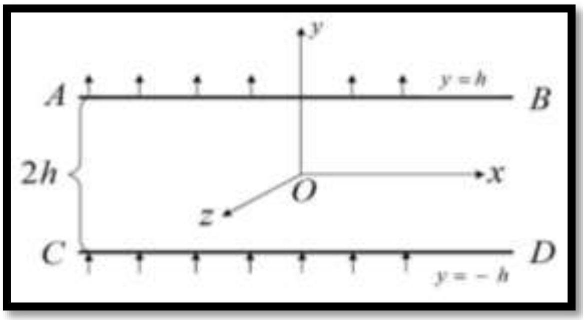


Figure-1

v Is independent of y but motion is along y-axis. So we can say that v is constant velocity *i.e.* $v = v_0$ or the fluid enters in flow region through one plate at the same constant velocity v_0 .

Also Navier–Stokes equations for incompressible fluid in the absence of body force when flow is steady

$$v_0\frac{du}{dy} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{d^2u}{dy^2}+\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)\upsilon u \text{ .... (1)} \;\;\&\;\; -\frac{1}{\rho}\frac{\partial p}{\partial y}=0 \text{ .... (2)}$$

SOLUTION OF THE PROBLEM

Equation (2) shows that the pressure does not depend on y hence p is a function of x only & so equation (1) reduces to

$$\frac{dp}{dx}=\rho\left(\upsilon\frac{d^2u}{dy^2}-v_0\frac{du}{dy}+\frac{\upsilon u}{k}+\frac{\sigma B_0^2\upsilon u}{\mu}\right)\Rightarrow \frac{d^2u}{dy^2}-\frac{v_0}{\upsilon}\frac{du}{dy}+\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)u=-\frac{P}{\rho\upsilon} \quad where \quad \frac{dp}{dx}=-P$$

$$A.E.\;\; m^2-\frac{v_0}{\upsilon}m+\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)=0 \Rightarrow m=\frac{\frac{v_0}{\upsilon}\pm\sqrt{\left(\frac{v_0}{\upsilon}\right)^2-4\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}}{2}=\frac{v_0}{2\upsilon}\pm\sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}$$

$$Let \quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}=A \quad , \quad \frac{1}{k}+\frac{\sigma B_0^2}{\mu}=B \quad \& \quad \left(\frac{v_0}{2\upsilon}\right)^2>\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)$$

$$\therefore C.F.=e^{\frac{v_0}{2\upsilon}y}\left[C_1 Cosh\,Ay+C_2 Sinh\,Ay\right] \;\;\&\;\; P.I.=-\frac{P}{\mu B}\Rightarrow u(y)=e^{\frac{v_0}{2\upsilon}y}\left[C_1 Cosh\,Ay+C_2 Sinh\,Ay\right]-\frac{P}{\mu B}$$

Using boundary conditions $\quad u=0 \;\; at \;\; y=-h \;\;\&\;\; u=U \;\; at \;\; y=h$

$$e^{-\frac{v_0}{2\upsilon}h}\left[C_1 Cosh\,Ah-C_2 Sinh\,Ah\right]-\frac{P}{\mu B}=0 \text{........ (3)} \;\;\&\;\; U=e^{\frac{v_0}{2\upsilon}h}\left[C_1 Cosh\,Ah+C_2 Sinh\,Ah\right]-\frac{P}{\mu B} \text{........ (4)}$$

$$\Rightarrow \frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}h}=C_1 Cosh\,Ah-C_2 Sinh\,Ah \;\;\&\;\; \left(U+\frac{P}{\mu B}\right)e^{-\frac{v_0}{2\upsilon}h}=C_1 Cosh\,Ah+C_2 Sinh\,Ah$$

$$C_1=\frac{1}{2\,Cosh\,Ah}\left[\left(U+\frac{P}{\mu B}\right)e^{-\frac{v_0}{2\upsilon}h}+\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}h}\right] \;\;\&\;\; C_2=\frac{1}{2\,Sinh\,Ah}\left[\left(U+\frac{P}{\mu B}\right)e^{-\frac{v_0}{2\upsilon}h}-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}h}\right]$$

$$u(y)=\frac{e^{\frac{v_0}{2\upsilon}y}Cosh\,Ay}{2Cosh\,Ah}\left\{\left(U+\frac{P}{\mu B}\right)e^{-\frac{v_0}{2\upsilon}h}+\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}h}\right\}+\frac{e^{\frac{v_0}{2\upsilon}y}Sinh\,Ay}{2Sinh\,Ah}\left\{\left(U+\frac{\rho}{\mu B}\right)e^{-\frac{v_0}{2\upsilon}h}-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}h}\right\}-\frac{P}{\mu B}$$

$$u(y)=\left(U+\frac{P}{\mu B}\right)\frac{e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\,A(y+h)}{2Sinh\,Ah\,Cosh\,Ah}-\frac{P}{\mu B}\frac{e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\,A(y-h)}{2Sinh\,Ah\,Cosh\,Ah}-\frac{P}{\mu B}$$

$$u(y)=\frac{1}{Sinh2Ah}\left[\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}SinhA(y+h)-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}SinhA(y-h)\right]-\frac{P}{\mu B}\quad\ldots\ldots\ldots(5)$$

THE SHEAR STRESS AT ANY POINT

$$\sigma_{xy}=\frac{\mu}{Sinh2Ah}\left[\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}\left\{\frac{v_0}{2\upsilon}Sinh\,A(y+h)+ACosh\,A(y+h)\right\}-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}\left\{\frac{v_0}{2\upsilon}Sinh\,A(y-h)+ACosh\,A(y-h)\right\}\right]\ldots\ldots(6)$$

THE SKIN FRICTIONS AT LOWER AND UPPER PLATE

$$\left(\sigma_{xy}\right)_{y=-h}=\frac{\mu}{Sinh\,2Ah}\left[A\left(U+\frac{P}{\mu B}\right)e^{-\frac{v_0}{\upsilon}h}+\frac{P}{\mu B}\left\{\frac{v_0}{2\upsilon}Sinh\,2Ah-ACosh\,2Ah\right\}\right]\ldots\ldots\ldots\ldots(7)$$

$$\left(\sigma_{xy}\right)_{y=h}=\frac{\mu}{Sinh\,2Ah}\left[\left(U+\frac{P}{\mu B}\right)\left(\frac{v_0}{2\upsilon}Sinh\,2Ah+ACosh\,2Ah\right)-\frac{PA}{\mu B}e^{\frac{v_0}{\upsilon}h}\right]\ldots\ldots\ldots\ldots\ldots(8)$$

THE AVERAGE VELOCITY DISTRIBUTION IN GENERALIZED PLANE COUETTE FLOW

$$(u)_{av}=\frac{1}{2h}\int_{-h}^{h}u(y)\,dy=\frac{1}{2h}\int_{-h}^{h}\left[\frac{1}{Sinh2Ah}\left\{\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\,A(y+h)-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\,A(y-h)\right\}-\frac{P}{\mu B}\right]dy$$

$$=\frac{\left(U+\frac{P}{\mu B}\right)}{4hSinh\,2Ah}\int_{-h}^{h}\left\{e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)}-e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}\right\}dy-\frac{P}{4h\mu B\,Sinh\,2Ah}\int_{-h}^{h}\left\{e^{\frac{v_0}{2\upsilon}(y+h)+A(y-h)}-e^{\frac{v_0}{2\upsilon}(y+h)-A(y-h)}\right\}dy-\frac{P}{\mu B}$$

$$=\frac{\left(U+\frac{P}{\mu B}\right)}{4hSinh\,2Ah}\left\{\frac{e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)}}{\left(\frac{v_0}{2\upsilon}+A\right)}-\frac{e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right\}_{-h}^{h}-\frac{P}{4\mu hB\,Sinh\,2Ah}\left\{\frac{e^{\frac{v_0}{2\upsilon}(y+h)+A(y-h)}}{\left(\frac{v_0}{2\upsilon}+A\right)}-\frac{e^{\frac{v_0}{2\upsilon}(y+h)-A(y-h)}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right\}_{-h}^{h}-\frac{P}{\mu B}$$

$$=\frac{\left(U+\frac{P}{\mu B}\right)}{4h\,Sinh\,2Ah}\left[\frac{\left(e^{2Ah}-e^{-\frac{v_0}{\upsilon}h}\right)}{\left(\frac{v_0}{2\upsilon}+A\right)}-\frac{\left(e^{-2Ah}-e^{-\frac{v_0}{\upsilon}h}\right)}{\left(\frac{v_0}{2\upsilon}-A\right)}\right]-\frac{P}{4\mu hB\,Sinh\,2Ah}\left[\frac{\left(e^{\frac{v_0}{\upsilon}h}-e^{-2Ah}\right)}{\left(\frac{v_0}{2\upsilon}+A\right)}-\frac{\left(e^{\frac{v_0}{\upsilon}h}-e^{2Ah}\right)}{\left(\frac{v_0}{2\upsilon}-A\right)}\right]-\frac{P}{\mu B}$$

$$=\frac{\left(U+\frac{P}{\mu B}\right)}{4h\,Sinh\,2Ah}\left[\frac{\left(\frac{v_0}{2\upsilon}-A\right)\left(e^{2Ah}-e^{-\frac{v_0}{\upsilon}h}\right)-\left(\frac{v_0}{2\upsilon}+A\right)\left(e^{-2Ah}-e^{-\frac{v_0}{\upsilon}h}\right)}{\left\{\left(\frac{v_0}{2\upsilon}\right)^2-A^2\right\}}\right]-\frac{P}{4\mu hB\,Sinh\,2Ah}\left[\frac{\left(\frac{v_0}{2\upsilon}-A\right)\left(e^{\frac{v_0}{\upsilon}h}-e^{-2Ah}\right)-\left(\frac{v_0}{2\upsilon}+A\right)\left(e^{\frac{v_0}{\upsilon}h}-e^{2Ah}\right)}{\left\{\left(\frac{v_0}{2\upsilon}\right)^2-A^2\right\}}\right]-\frac{P}{\mu B}$$

$$=\frac{\left(U+\frac{P}{\mu B}\right)}{4Bh\,Sinh\,2Ah}\left\{\frac{v_0}{2\upsilon}\left(e^{2Ah}-e^{-\frac{v_0}{\upsilon}h}-e^{-2Ah}+e^{-\frac{v_0}{\upsilon}h}\right)-A\left(e^{2Ah}-e^{-\frac{v_0}{\upsilon}h}+e^{-2Ah}-e^{-\frac{v_0}{\upsilon}h}\right)\right\}$$

$$-\frac{P}{4\mu hB^2\ Sinh\ 2Ah}\left\{\frac{v_0}{2\upsilon}\left(e^{\frac{v_0}{\upsilon}h}-e^{-2Ah}-e^{\frac{v_0}{\upsilon}h}+e^{2Ah}\right)-A\left(e^{\frac{v_0}{\upsilon}h}-e^{-2Ah}+e^{\frac{v_0}{\upsilon}h}-e^{2Ah}\right)\right\}-\frac{P}{\mu B}$$

Since $\sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}=A$, $\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)=B \Rightarrow \left(\frac{v_0}{2\upsilon}\right)^2-A^2=B$

$$=\frac{\left(U+\frac{P}{\mu B}\right)}{2Bh\ Sinh\ 2Ah}\left(\frac{v_0}{2\upsilon}Sinh\ 2Ah-ACosh\ 2Ah+Ae^{-\frac{v_0}{\upsilon}h}\right)-\frac{P}{2\mu hB^2 Sinh\ 2Ah}\left(\frac{v_0}{2\upsilon}Sinh\ 2Ah+ACosh\ 2Ah-Ae^{\frac{v_0}{\upsilon}h}\right)-\frac{P}{\mu B}$$

$$u_{av}=\frac{U}{2hB\ Sinh\ 2Ah}\left(\frac{v_0}{2\upsilon}Sinh\ 2Ah-ACosh\ 2Ah+Ae^{-\frac{v_0}{\upsilon}h}\right)-\frac{PA}{\mu hB^2 Sinh\ 2Ah}\left\{Cosh\ 2Ah-Cosh\left(\frac{v_0}{\upsilon}h\right)\right\}-\frac{P}{\mu B}\text{.......(9)}$$

THE VOLUMETRIC FLOW: $Q=2h(u)_{av}$

$$Q=\frac{U}{B\ Sinh\ 2Ah}\left(\frac{v_0}{2\upsilon}Sinh\ 2Ah-ACosh\ 2Ah+Ae^{-\frac{v_0}{\upsilon}h}\right)-\frac{2PA}{\mu B^2 Sinh\ 2Ah}\left\{Cosh\ 2Ah-Cosh\left(\frac{v_0}{\upsilon}h\right)\right\}-\frac{2hP}{\mu B}\text{.......(10)}$$

THE DRAG COEFFICIENTS: $(C_f)_{y=h}=\frac{(\sigma_{xy})_{y=h}}{\frac{1}{2}\rho u_{av}^2}$ & $(C_f^{/})_{y=-h}=\frac{(\sigma_{xy})_{y=-h}}{\frac{1}{2}\rho u_{av}^2}$

$$(C_f)_{y=h}=\frac{8\mu^3h^2B^4\ Sinh\ 2Ah\left\{\left(U+\frac{P}{\mu B}\right)\left(\frac{v_0}{2\upsilon}Sinh2Ah+ACosh2Ah\right)-\frac{PA}{\mu B}e^{\frac{v_0}{\upsilon}h}\right\}}{\rho\left[\mu UB\left(\frac{v_0}{2\upsilon}Sinh\ 2Ah-ACosh\ 2Ah+Ae^{-\frac{v_0}{\upsilon}h}\right)-2AP\left\{Cosh\ 2Ah-Cosh\left(\frac{v_0}{\upsilon}h\right)\right\}-2PBh\,Sinh\ 2Ah\right]^2}\text{.........(11)}$$

$$(C_f^{/})_{y=-h}=\frac{8\mu^3h^2B^4\ Sinh\ 2Ah\left\{A\left(U+\frac{P}{\mu B}\right)e^{-\frac{v_0}{\upsilon}h}+\frac{P}{\mu B}\left(\frac{v_0}{2\upsilon}Sinh2Ah-ACosh2Ah\right)\right\}}{\rho\left[\mu UB\left(\frac{v_0}{2\upsilon}Sinh\ 2Ah-ACosh\ 2Ah+Ae^{-\frac{v_0}{\upsilon}h}\right)-2AP\left\{Cosh\ 2Ah-Cosh\left(\frac{v_0}{\upsilon}h\right)\right\}-2PBh\,Sinh\ 2Ah\right]^2}\text{..........(12)}$$

THE STREAM LINE IN THE PLANE GENERALIZED COUETTE FLOW:

$$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}\quad where\ \ \bar{q}=u\hat{i}+v\hat{j}+w\hat{k}$$

$$\Rightarrow\frac{dx}{\left[\frac{1}{Sinh2Ah}\left\{\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}SinhA(y+h)-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}SinhA(y-h)\right\}-\frac{P}{\mu B}\right]}=\frac{dy}{v_0}=\frac{dz}{0}$$

Taking first two equations

$$v_0\int dx=\int\left[\frac{1}{Sinh2Ah}\left\{\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}SinhA(y+h)-\frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}SinhA(y-h)\right\}-\frac{P}{\mu B}\right]dy+C_1$$

$$\Rightarrow v_0 x - \frac{\left(U+\frac{P}{\mu B}\right)}{Sinh\,2Ah}\int e^{\frac{v_0}{2\upsilon}(y-h)}\left\{\frac{e^{A(y+h)}-e^{-A(y+h)}}{2}\right\}dy + \frac{P}{\mu B\,Sinh\,2Ah}\int e^{\frac{v_0}{2\upsilon}(y+h)}\left\{\frac{e^{A(y-h)}-e^{-A(y-h)}}{2}\right\}dy + \frac{P}{\mu B}y = C_1$$

$$\Rightarrow v_0 x - \frac{\left(U+\frac{P}{\mu B}\right)}{2\,Sinh\,2Ah}\int\left(e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)} - e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}\right)dy + \frac{P}{2\mu B\,Sinh\,2Ah}\int\left(e^{\frac{v_0}{2\upsilon}(y+h)+A(y-h)} - e^{\frac{v_0}{2\upsilon}(y+h)-A(y-h)}\right)dy + \frac{P}{\mu B}y = C_1$$

$$\Rightarrow v_0 x - \frac{\left(U+\frac{P}{\mu B}\right)}{2\,Sinh\,2Ah}\left(\frac{e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)}}{\left(\frac{v_0}{2\upsilon}+A\right)} - \frac{e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right) + \frac{P}{2\mu B\,Sinh\,2Ah}\left(\frac{e^{\frac{v_0}{2\upsilon}(y+h)+A(y-h)}}{\left(\frac{v_0}{2\upsilon}+A\right)} - \frac{e^{\frac{v_0}{2\upsilon}(y+h)-A(y-h)}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right) + \frac{P}{\mu B}y = C_1$$

$$\Rightarrow v_0 x - \frac{\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}}{2B\,Sinh\,2Ah}\left\{\left(\frac{v_0}{2\upsilon}-A\right)e^{A(y+h)} - \left(\frac{v_0}{2\upsilon}+A\right)e^{-A(y+h)}\right\} + \frac{Pe^{\frac{v_0}{2\upsilon}(y+h)}}{2\mu B^2\,Sinh\,2Ah}\left\{\left(\frac{v_0}{2\upsilon}-A\right)e^{A(y-h)} - \left(\frac{v_0}{2\upsilon}+A\right)e^{-A(y-h)}\right\} + \frac{P}{\mu B}y = C_1$$

$$\Rightarrow v_0 x - \frac{\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}}{B\,Sinh\,2Ah}\left(\frac{v_0}{2\upsilon}Sinh\,A\,(y+h) - ACosh\,A\,(y+h)\right) + \frac{Pe^{\frac{v_0}{2\upsilon}(y+h)}}{\mu B^2\,Sinh\,2Ah}\left(\frac{v_0}{2\upsilon}Sinh\,A\,(y-h) - ACosh\,A\,(y-h)\right) + \frac{P}{\mu B}y = C_1 \ldots\ldots(13)$$

& *Second stream line is given by* $z = C_2$ .... (14)

$$curl\ \bar{q} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{pmatrix} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \left[\frac{1}{Sinh2Ah}\left\{\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}SinhA(y+h) - \frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}SinhA(y-h)\right\} - \frac{P}{\mu B}\right] & v_0 & 0 \end{pmatrix}$$

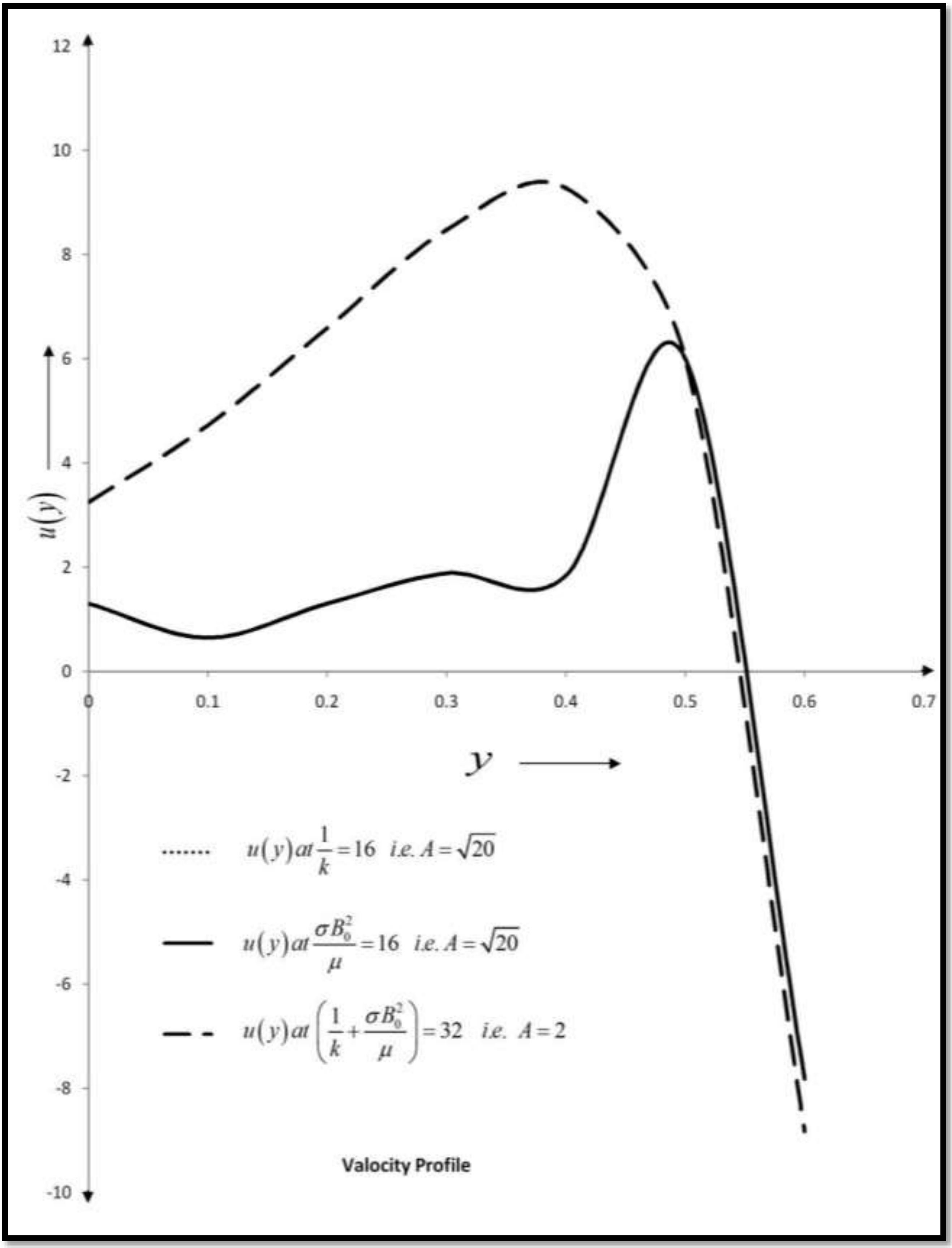
$$curl\ \bar{q} = -\frac{1}{Sinh\,2Ah}\left[\left(U+\frac{P}{\mu B}\right)e^{\frac{v_0}{2\upsilon}(y-h)}\left\{\frac{v_0}{2\upsilon}Sinh\,A(y+h) + ACosh\ A(y+h)\right\} - \frac{P}{\mu B}e^{\frac{v_0}{2\upsilon}(y+h)}\left\{\frac{v_0}{2\upsilon}Sinh\,A(y-h) + ACosh\ A(y-h)\right\}\right]\hat{k} \neq \bar{0}$$

$$\Rightarrow motion\ of\ the\ fluid\ is\ rotational.$$

Table for velocity: $P = 9$, $U = 6$, $\mu = .5$, $h = .5$, $\frac{v_0}{2\upsilon} = 6$ & $\frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16$, $\sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)} = 2$

Table-1: (for velocity)

	y	**0**	**.1**	**.2**	**.3**	**.4**	**.5**	**.6**
$\frac{1}{k} = 16$	**u(y)**	**1.3**	**.645**	**1.301**	**1.882**	**1.835**	**6**	**- 7.818**
$\frac{\sigma B_0^2}{\mu} = 16$	**u(y)**	**1.3**	**.645**	**1.301**	**1.882**	**1.835**	**6**	**- 7.818**
$\frac{1}{k} + \frac{\sigma B_0^2}{\mu} = 32$	**u(y)**	**3.246**	**4.726**	**6.592**	**8.473**	**9.276**	**6**	**- 8.82**

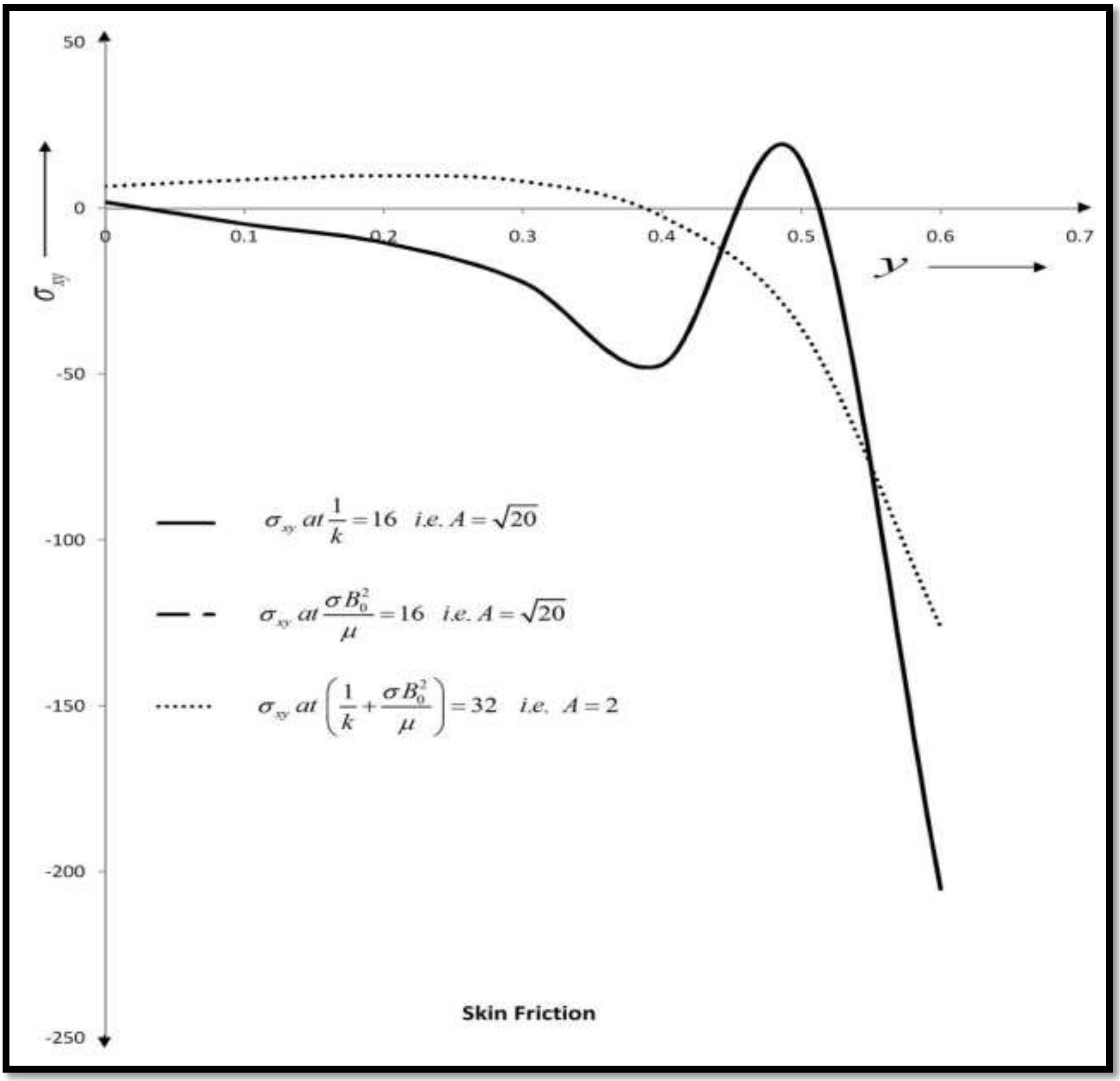


Graph of table-1

Table for skin friction: $P=9, U=6, \mu=h=.5, \dfrac{v_0}{2\upsilon}=6$ & $\dfrac{1}{k}=\dfrac{\sigma B_0^2}{\mu}=16, \sqrt{\left(\dfrac{v_0}{2\upsilon}\right)^2-\left(\dfrac{1}{k}+\dfrac{\sigma B_0^2}{\mu}\right)}=2$

Table-2: (for skin friction)

	y	0	.1	.2	.3	.4	.5	.6
$\frac{1}{k}=16$	σ_{xy}	**1.899**	**- 4.685**	**- 10.304**	**- 22.184**	**- 47.038**	**14.123**	**- 205.03**
$\frac{\sigma B_0^2}{\mu}=16$	σ_{xy}	**1.899**	**- 4.685**	**- 10.304**	**- 22.184**	**- 47.038**	**14.123**	**- 205.03**
$\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$	σ_{xy}	**6.632**	**8.572**	**9.845**	**8.138**	**- 2.436**	**- 36.07**	**- 126.012**

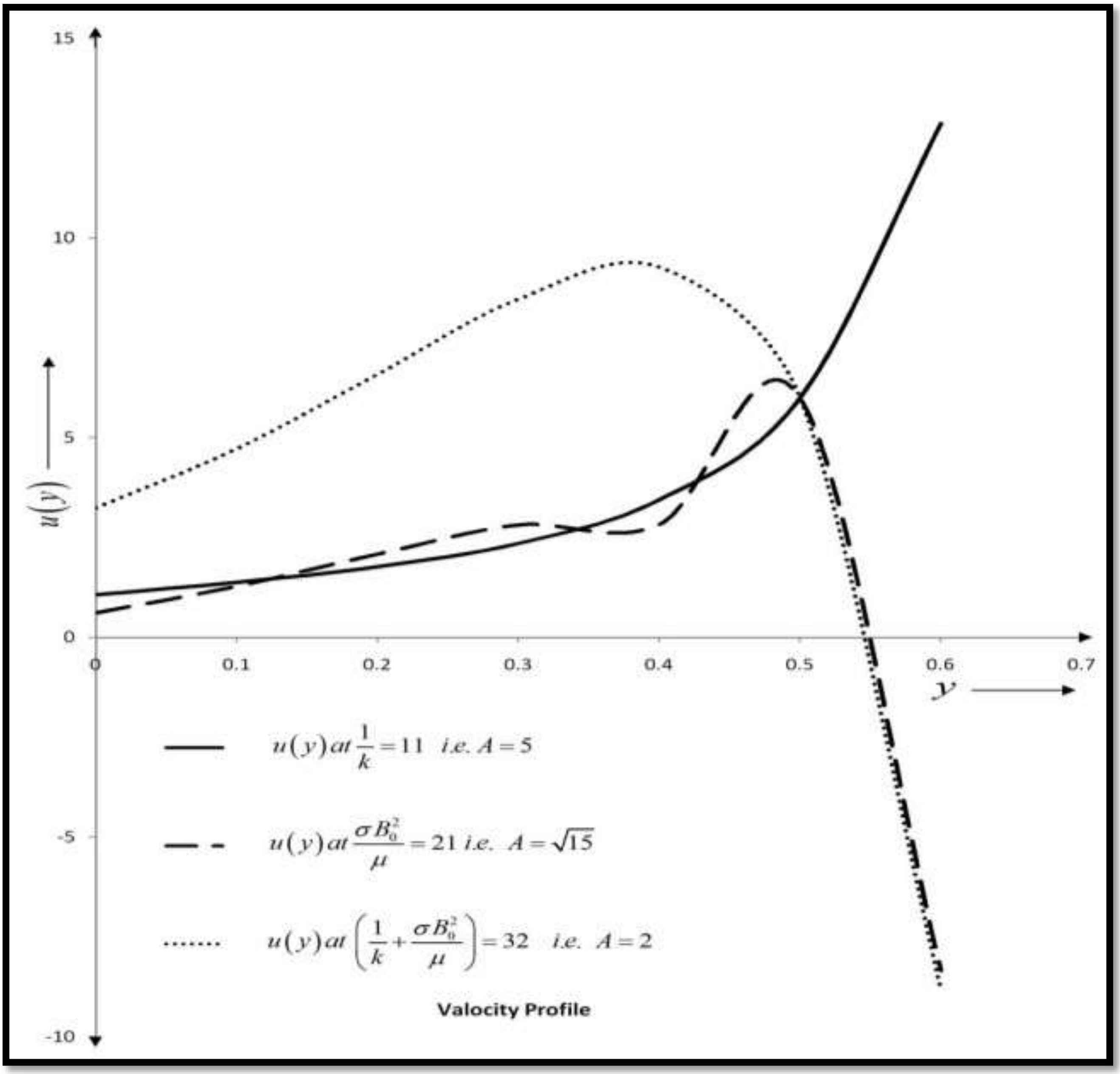


Graph of table-2

Table for velocity: $P = 9$, $U = 6$, $\mu = h = .5$, $\frac{v_0}{2\upsilon} = 6$, $\sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ $when$ $\frac{1}{k} < \frac{\sigma B_0^2}{\mu}$

Table-3: (for velocity)

	y	0	.1	.2	.3	.4	.5	.6
$\frac{1}{k} = 11$	**u(y)**	**1.074**	**1.384**	**1.776**	**2.359**	**3.45**	**6**	**12.858**
$\frac{\sigma B_0^2}{\mu} = 21$	**u(y)**	**.617**	**1.283**	**2.087**	**2.818**	**2.829**	**6**	**- 8.339**
$\frac{1}{k} + \frac{\sigma B_0^2}{\mu} = 32$	**u(y)**	**3.246**	**4.726**	**6.592**	**8.473**	**9.276**	**6**	**- 8.82**

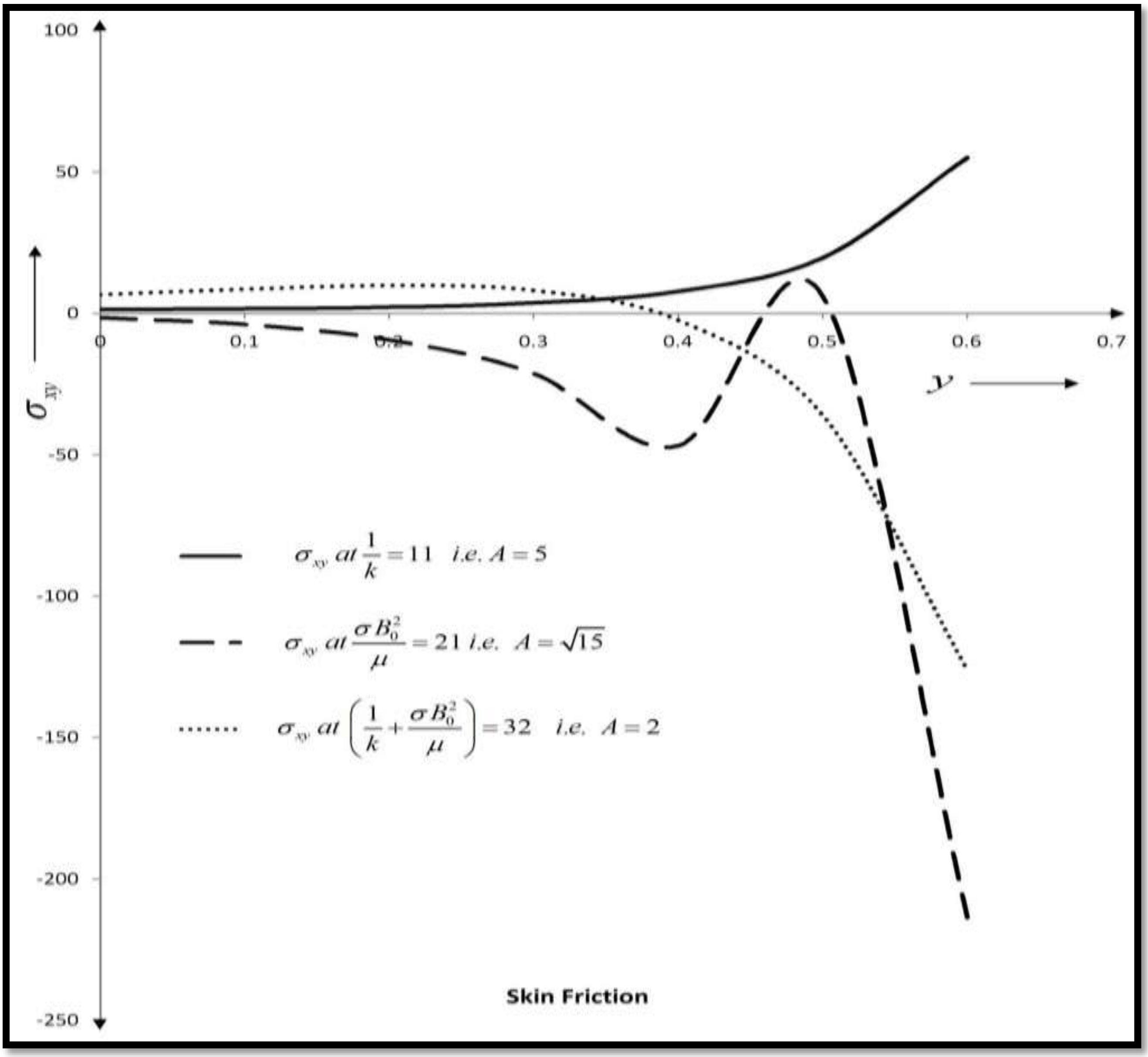


Graph of table-3

Table for skin friction: $P=9,\ U=6,\ \mu=h=.5,\ \frac{v_0}{2\upsilon}=6,\ \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}=2\ \ when\ \ \frac{1}{k}<\frac{\sigma B_0^2}{\mu}$

Table-4: (for skin friction)

	y	0	.1	.2	.3	.4	.5	.6
$\frac{1}{k}=11$	σ_{xy}	**1.421**	**1.706**	**2.294**	**3.764**	**7.85**	**19.76**	**55.14**
$\frac{\sigma B_0^2}{\mu}=21$	σ_{xy}	**- 1.515**	**- 3.923**	**- 9.372**	**- 21.269**	**- 46.78**	**5.998**	**- 213.56**
$\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$	σ_{xy}	**6.632**	**8.572**	**9.845**	**8.138**	**- 2.436**	**- 36.07**	**- 126.012**

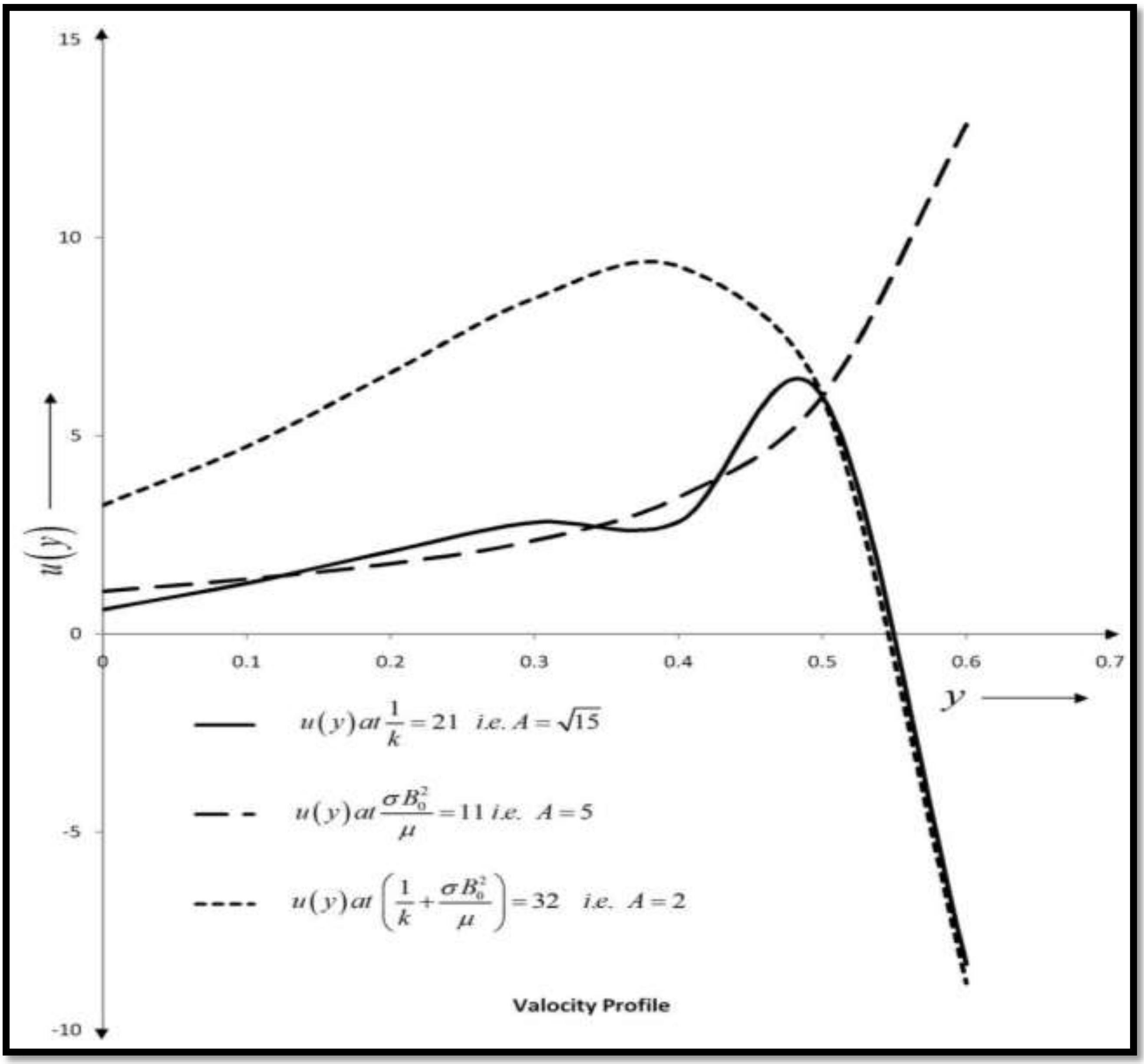


Graph of table-4

Table for velocity: $P=9, U=6,\ \mu=h=.5,\ \frac{v_0}{2\upsilon}=6,\ \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}=2\ when\ \frac{1}{k}>\frac{\sigma B_0^2}{\mu}$

Table-5: (for velocity)

	y	0	.1	.2	.3	.4	.5	.6
$\frac{1}{k}=21$	**u(y)**	**.617**	**1.283**	**2.087**	**2.818**	**2.829**	**6**	**- 8.339**
$\frac{\sigma B_0^2}{\mu}=11$	**u(y)**	**1.074**	**1.384**	**1.776**	**2.359**	**3.45**	**6**	**12.858**
$\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$	**u(y)**	**3.246**	**4.726**	**6.592**	**8.473**	**9.276**	**6**	**- 8.82**

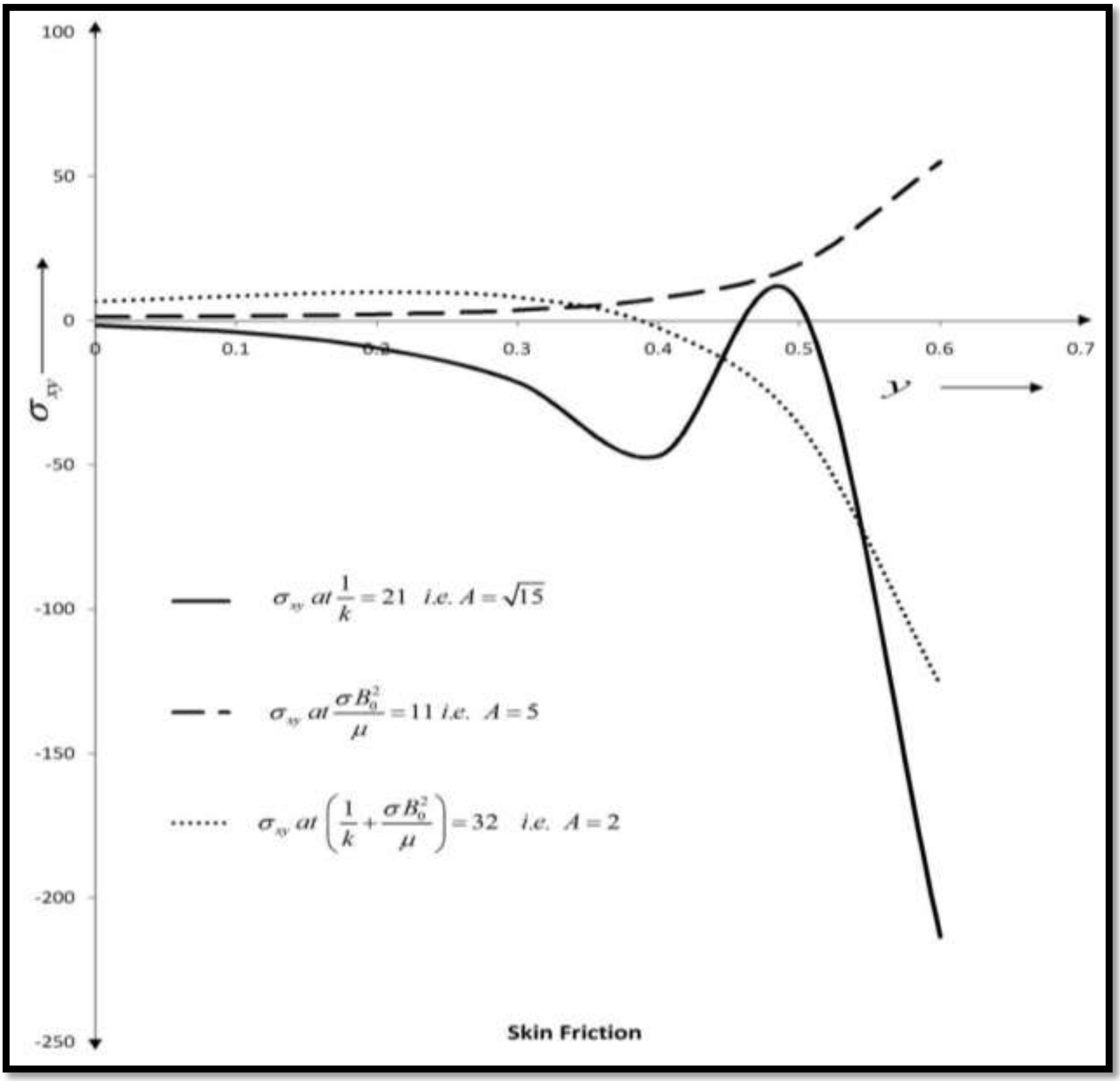


Graph of table-5

Table for Skin friction: $P=9,\ U=6,\ \mu=h=.5,\ \frac{v_0}{2\upsilon}=6,\ \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu}\right)}=2\ when\ \frac{1}{k}>\frac{\sigma B_0^2}{\mu}$

Table-6: (for skin friction)

	y	0	.1	.2	.3	.4	.5	.6
$\frac{1}{k}=21$	σ_{xy}	- 1.515	- 3.923	- 9.372	- 21.269	- 46.78	5.998	- 213.56
$\frac{\sigma B_0^2}{\mu}=11$	σ_{xy}	1.421	1.706	2.294	3.764	7.85	19.76	55.14
$\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$	σ_{xy}	6.632	8.572	9.845	8.138	- 2.436	- 36.07	- 126.012



Graph of table-6

CONCLUSION AND DISCUSSION

In this paper, we have investigated the velocity by the graph of table-1 of equation (5). The velocity in porous medium & magnetic field at $\frac{1}{k}=\frac{\sigma B_0^2}{\mu}=16$ is less than the corresponding value of velocity in porous with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0\le y\le 0.4$ & equal $\{u(y)=6\}$ in all medium at $y=0.5$. The value of velocity in porous medium and magnetic field at $\frac{1}{k}=\frac{\sigma B_0^2}{\mu}=16$ is negatively less than the correspondingly negative value of velocity in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ at $y=0.6$.

Again by the graph of table-3 of equation (5), the value of the velocity in porous medium at $\frac{1}{k}=11$ increases in the interval $0\le y\le 0.6$, velocity in magnetic field at $\frac{\sigma B_0^2}{\mu}=21$ increases in the interval $0\le y\le 0.5$ & the velocity in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ increases in the interval $0\le y\le 0.4$. The velocity is equal $\{u(y)=6\}$ in all medium at $y=0.5$ & velocity in porous medium at $\frac{1}{k}=11$ is positive while the velocity in magnetic field at $\frac{\sigma B_0^2}{\mu}=21$ & porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ is negative i.e. flow is in opposite direction at $y=0.6$.

Again by the graph of table-5 of equation (5), the value of the velocity in magnetic field at $\frac{\sigma B_0^2}{\mu}=11$ increases in the interval $0\le y\le 0.6$, velocity in porous medium at $\frac{1}{k}=21$ increases in the interval $0\le y\le 0.5$ & the velocity in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ increases in the interval $0\le y\le 0.4$. The velocity is equal $\{u(y)=6\}$ in all medium at $y=0.5$ & velocity in magnetic field at $\frac{\sigma B_0^2}{\mu}=11$ is positive while the velocity in porous medium at $\frac{1}{k}=21$ & porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ is negative i.e. flow is in opposite direction at $y=0.6$.

Again we have investigated the skin friction by the graph of table-2, of equation (6). The skin friction in porous medium & magnetic field at $\frac{1}{k}=\frac{\sigma B_0^2}{\mu}=16$ is positive at $y=0$ & $.5$ while the skin friction in porous medium & magnetic field at $\frac{1}{k}=\frac{\sigma B_0^2}{\mu}=16$ is negative at other values of y. Skin friction in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ is positive in the interval $0\le y\le 0.3$ while the skin friction in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ is negative in the interval $0.4\le y\le 0.6$.

Again by the graph of table-4 of equation (6), the skin friction in porous medium at $\frac{1}{k}=11$ is positive & increases in the interval $0\le y\le 0.6$ while skin friction in magnetic field at $\frac{\sigma B_0^2}{\mu}=21$ is negative excepts only at $y=.5$. The skin friction in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0\le y\le 0.3$ is positive while the skin friction in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ is negative in the interval $0.4\le y\le 0.6$.

Again by the graph of table-6 of equation (6), the skin friction in magnetic field at $\frac{\sigma B_0^2}{\mu}=11$ is positive & increases in the interval $0\le y\le 0.6$ while the skin friction in porous medium at $\frac{1}{k}=21$ is negative excepts only at $y=.5$. The skin friction in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0\le y\le 0.3$ is positive while the skin friction in porous medium with magnetic field at $\frac{1}{k}+\frac{\sigma B_0^2}{\mu}=32$ is negative in the interval $0.4\le y\le 0.6$.

Also we have investigated skin frictions, average velocity, the volumetric flow, drag coefficients & stream lines by the equations (7), (8), (9), (10), (11), (12), (13) & (14) respectively.

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Author Acknowledgement:

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