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Existence of Random Attractors for Nonlinear Plate Equations with Thermal Memory Effect and Additive Noise

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Abstract **Review Article**

In this paper, the long-time dynamics of the nonlinear plate equations with thermal memory effect and additive noise caused by non-Fourier heat flow law is studied. We study the existence of random attractors for stochastic dynamical systems related to equations.

Keywords: Nonlinear plate equations, thermal memory, stochastic dynamic system, random attractor, additive noise.

INTRODUCTION

In this paper, the research methods in the literature [4-6] consider the nonlinear Plate equation with thermal memory effect caused by the non-Fourier heat flow law with additive noise and has a random attractor in the stochastic dynamic system. The study equation is as follows

$$\begin{cases} \theta_t - \Delta u_t + \int_0^{\infty} k(s)[-\Delta\theta(t-s)]ds = \sum_{j=1}^{m} \phi_j dw_j & t \in R^+, x \in U \\ u_{tt} - \Delta u_t + \Delta(\Delta u + \theta) + f(u) = 0 \end{cases} \tag{1.1}$$

Boundary condition

$$\begin{aligned} &u(t) = \Delta u(t) = 0 && t \ge 0, x \in \Gamma \\ &\theta(t) = 0 && t \in R, x \in \Gamma \end{aligned}$$

Initial value condition

$$\begin{aligned} &u(0) = u_0 && x \in U \\ &u_t(0) = v_0 && x \in U \\ &\theta(0) = \theta_0 && x \in U \\ &\theta(-s) = \Psi(s) && (x,s) \in U \times R^+ \end{aligned}$$

Among them, $\sum_{j=1}^{m} \phi_j dw_j = \sum_{j=1}^{m} \phi^j w^j dt$. U is an appropriate smooth boundary Γ in R^2 , $u = u(t,x)$ is a real-valued function on $U \times [0,+\infty)$, θ is the temperature change of the self-balanced reference value, u is the vertical displacement of the plate, and function $\Psi : R^+ \times U \mapsto R$ is called the initial history of temperature, assuming the memory kernel function $k : R^+ \mapsto R$ .It is a positive bounded convex function and disappears at infinity. For random terms $\phi_j(x) \in H_0^2(U) \bigcap H^3(U)$ $(j = 1,2,\cdots,m)$, $\{\omega_j\}_{j=1}^{m}$ is an independent bilateral real-valued Wiener process on probability space (Ω, F, P) . Among them,

$$\Omega=\{\omega=(\omega_1,\omega_{2,}\cdots,\omega_m)\in C(R,R^m):\omega(0)=0\}.$$

Given a tight open topology, F is a P-complete Borel σ -algebra on Ω .

There is a lot of research work on the thermoelastic plate equation in the literature. The Plate equation is derived from the elastic vibration equation established by Woinowsky-Krieger ([12]) and Berger ([13]). For the Plate equation, the latest study of the attenuation estimation and asymptotic curve of the Plate equation with memory (cf[20]); in [23], the quasi-stability properties of the system are established, and the existence of global attractors and exponential attractors with finite fractal dimensions is obtained; studied the precise controllability of the thermoelastic plate equation with memory (cf[21]);obtained the general stability results of the viscoelastic plate equation with past history and general kernel (cf[22]), etc. In these studies, we have noticed that in recent decades, the long-term behavior of the deterministic Plate equation or the solution of a single Plate equation has been studied by many scholars. Such as, the author of the literature [14] in 2005 considered the following evolution problem in the two-dimensional theory of linear genetic thermoelasticity,

$$\begin{cases} u_{tt}(t)+\Delta(\Delta u(t)+\vartheta(t))=0 \\ \vartheta_t(t)+\int_0^\infty k(s)[c\vartheta(t-s)-\Delta\vartheta(t-s)]ds-\Delta u_t(t)=0 \end{cases} \tag{1.2}$$

which simulates Kirchhoff thermoelastic thin plate with heat conduction memory effect. The exponential stability of the solution of the equation is considered.

In 2008, a nonlinear Plate equation with thermal memory effect was considered in [7].

$$\begin{cases} \vartheta_t-\Delta u_t+\int_0^\infty k(s)[-\Delta\vartheta(t-s)]ds=0 \\ u_{tt}-\Delta u_t+\Delta(\Delta u+\vartheta)+f(u)=0 \end{cases} \tag{1.3}$$

The existence and uniqueness of the global solution and the existence of the global attractor are proved.

However, in real life, many systems will inevitably be affected by random factors. Therefore, this paper studies the existence of random attractors of stochastic Plate equations in stochastic dynamic systems based on [7]. This article is organized as follows. In the next section, we will review some basic concepts and properties of general stochastic dynamic systems. In Section 3, we provide some basic settings related to formula (1.1) and show that it generates a stochastic dynamic system in the appropriate function space. Section 4 is devoted to proving the existence of the unique random attractor of the stochastic dynamic system.

BASIC DEFINITION THEOREM

Definition 2.1 ([8]) The random dynamic system $(\Omega,F,P,(\vartheta_t)_{t\in R})$ is called RDS φ .

$$\varphi:\ T\times X\times\Omega\to X$$
$$(t,x,\omega)\mapsto\varphi(t,\omega)x,$$

where identity map $\varphi(0,\omega)=id$ on X ,and has the following relationship

$$\varphi(t+s,\omega)=\varphi(t,\vartheta_s\omega)\circ\varphi(s,\omega), \tag{2.1}$$

among them $t,s\in T$, $\omega\in\Omega$, $\circ$ represent combinations. A cluster of maps $\varphi(t,\omega)$ satisfying (2.1) is called a cocycle, and (2.1) is also the nature of cocycle. The inverse of $\varphi(t,\omega)$ is $\varphi(t,\omega)^{-1}=\varphi(-t,\vartheta_t\omega)$.

Definition 2.2 ([8]) Random set A attracts another random set B .if

$$d(\varphi(t,\vartheta_{-t}\omega)B(\vartheta_{-t}\omega),A(\omega))\xrightarrow[n\to\infty]{}0,$$

then call A as an attractive set.

Definition 2.3 ([8]) Random set K attracts another random set B, if $t_B(\omega)$ exists, for all t $\geq t_B(\omega)$ there is

$$\varphi(t,\vartheta_{-t}\omega)B(\vartheta_{-t}\omega)\subset K(\omega),$$

$t_B(\omega)$ is called absorption time. Then K is called absorption set.

Theorem 2.1 ([8] [9]) Let Φ be the measurable power system $(\Omega, F, P, (\vartheta_t)_{t\in R})$ and a continuous random power system in space X. If there is a random compact set $K(\omega)$, such that for any nonrandom bounded set $B \subset X$, there is

$$\lim d(\Phi(t, \vartheta_{-t}\omega)B, K(\omega)) = 0, t \to +\infty,$$

then Φ has a random attractor

$$A = \{A(\omega)\}_{\omega\in\Omega} = \overline{\bigcup_{B\subset X} \Lambda_B(\omega)},$$

where B is taken through the bounded subset of X, $\Lambda_B(\omega)$ is the ω-limit set of B, ie

$$\Lambda_B(\omega) = \bigcap_{s\geq 0}\overline{\bigcup_{t\geq s}\Phi(t, \vartheta_{-t}\omega)B}.$$

Theorem 2.2 ([15]) Suppose X_0, X_1, X are three Banach spaces, of which X_0, X are reflexive and satisfy

$$X_0 \to X \to X_1 \quad \text{(Embed)},$$

here embedding $X_0 \to X$ is tight. If $Y \subset L^2_\mu(R^+, X)$ meets the following conditions:

(i) Y is bounded in $L^2_\mu(R^+, X_0) \cap H^1_\mu(R^+, X_1)$;

(ii) For $\forall s \in R^+$, there is $K_0 > 0$, so that $\sup_{\eta\in Y} \|\eta(s)\|^2_X \leq K_0$;

then Y is relatively tight in $L^2_\mu(R^+, X)$.

Lemma 2.1 ([19]) Let X be a Banach space and $Z \in C([0,\infty), X)$. Let $E: X \to R$ be a function bounded from below such that $E(Z(0)) \leq M$ for $Z \in X$. If

$$\frac{d}{dt}E(Z(t)) + \sigma\|Z(t)\|^2_X \leq \chi,$$

for some $\sigma \geq 0, \chi \geq 0$ independent of Z, then for all $\varepsilon_1 > 0, t_0 = t_0(M, \varepsilon_1) > 0$, such that

$$E(Z(t)) \leq \sup_{\xi\in X}\{E(\xi) : \sigma\|\xi\|^2_X \leq \chi + \varepsilon_1\}, \forall t \geq t_0.$$

Lemma 2.2 For $m < \varepsilon < l, g = 0,1, \forall B_i \in V^0, i = 1,2,$ then

(1) $-\varepsilon(Au, B_1) \geq -\dfrac{\alpha\varepsilon}{4}\|u\|^2_2 - \dfrac{\varepsilon}{\alpha}\|B_1\|^2$,

(2) $-\int_U B_1 Au dx \geq -\dfrac{\alpha}{4}\|B_1\|^2 - \dfrac{1}{\alpha}\|u\|^2_2$,

(3) $|\pm(A^g B_1, B_2)| \leq \|A^g B_1\|\|B_2\| \leq \dfrac{1}{\alpha}\|A^g B_1\|^2 + \dfrac{\alpha}{4}\|B_2\|^2$,

(4) $|\pm\int_0^\infty \varepsilon\mu(s) < A^g B_1, \eta^t(s) > ds| \leq \dfrac{\varepsilon\alpha k_0}{4}\|A^{g/2}B_1\|^2 + \dfrac{\varepsilon}{\alpha D_1^{g/2}}\|\eta^t\|^2_{M^1}$.

Basic Settings

Assume that the memory kernel function $\mu(s)$ and the nonlinear term $f(u)$ satisfy the following conditions:

(1) For any $s \in R^+$, there is $\delta > \dfrac{23D_1 + 3}{6(D_1 - 2)} > 0$, such that

$$\mu(s) \in C^1(R^+) \cap L^1(R^+), \mu(s) \geq 0, \mu'(s) + \delta\mu(s) \leq 0. \tag{3.1}$$

(2) Assuming that k is zero at infinity, in addition, define $\mu(s)=-k'(s)$.In recent years, various development equations under the non-Fourier heat flow law have aroused the interest of many mathematicians. Let q be the heat flow vector. According to Gurtin-Pinkin theory (cf.[16]), the linear constitutive equation of q is given by

$$q(t)=-\int_0^\infty k(s)\nabla\theta(t-s)ds,$$

where k is the thermal conductivity relaxation kernel, and the existence of the convolution term in the above formula means the finite propagation velocity of thermal disturbance, so in this case, the corresponding equation is hyperbolic. It is easy to see that if k is the Dirac mass at zero, the above formula can be simplified to the classic Fourier's law $q=-\nabla\theta$. In addition,

$$k(s)=\frac{1}{\sigma}e^{-\frac{s}{\sigma}},\sigma>0 \tag{3.2}$$

among them, $\frac{2}{3}<k_0<\min\{4,\frac{\delta}{2}-\frac{\delta}{D_1}-\frac{5}{4}-\frac{1}{4D_1},8-\frac{10}{D_1}\}$.

(within the limited range of each parameter, $\frac{\delta}{2}-\frac{\delta}{D_1}-\frac{5}{4}-\frac{1}{4D_1}$ and $8-\frac{10}{D_1}$ are less than $\frac{2}{3}$).

(3) The nonlinear function $f(u)$ satisfies:

$$f(0)=0,\left|f'(s)\right|\le C_0(1+|s|),\forall s\in R, \tag{3.3}$$

$$F(s)=\int_0^s f(r)dr\ge C_1(|s|^3-1),\forall s\in R, \tag{3.4}$$

$$\liminf_{|s|\to\infty}\frac{sf(s)-C_2F(s)}{s^2}\ge 0,\forall s\in R. \tag{3.5}$$

$$\liminf_{|s|\to+\infty}\frac{f(s)}{s}>-\frac{1}{C_U}. \tag{3.6}$$

among them, $0<C_2\le\frac{2C_0u^2+C_0|u|^3}{2C_1(|u|^3-1)}, C_i>0(i=0,1,\cdots)$ is a normal number, in this article the constant C changes, each row or even the same row of C is not equal. C_U is the best constant that depends only on U, then

$$\|u\|^2_{L^2(U)}\le C_U\|\mathrm{A}u\|^2_{L^2(U)} \tag{3.7}$$

Consider the positive operator A on $L^2(U)$, defined as $\mathrm{A}=-\Delta=\nabla^2$ and bounded by $D(\mathrm{A})=H^2(U)\cap H_0^1(U)$. For $r\in\mathbb{R}$, consider Hilbert space $v^r=D(\mathrm{A}^{r/2})$, define the inner product

$$(u_1,u_2)_{v^r}=(\mathrm{A}^{r/2}u_1,\mathrm{A}^{r/2}u_2),\forall u_1,u_2\in v^r,$$

$$\|u\|_r^2=(A^{r/2}u,A^{r/2}u),u\in v^r,$$

$$\|u\|^2=(u,u),u\in v^0,$$

where $(\cdot,\cdot)$ and $\|\cdot\|$ represent the inner product and norm in $v^0=L^2(U)$. Use $((\cdot,\cdot)),\|\cdot\|_2$ to denote the inner product and norm of $v^2=H_0^2(U)$, respectively. For $r_1>r_2$, embedding $v^{r_1}\to v^{r_2}$ is tight.

We introduce a weighted Hilbert memory space $\mathrm{M}^{1+r}=L^2_\mu(R^+;v^r),$, of which $r\in R$. Define the inner product as follows

$$\langle\eta_1,\eta_2\rangle_{\mathrm{M}^r}=\int_0^\infty\mu(s)\left\langle\mathrm{A}^{r/2}\eta_1(s),\mathrm{A}^{r/2}\eta_2(s)\right\rangle ds.\forall\eta_1,\eta_2\in\mathrm{M}^{1+r},$$

$$\|\eta\|^2_{\mathrm{M}^{1+r}}=\int_0^\infty\mu(s)\|\eta(s)\|_2^2ds,\forall\eta(s)\in\mathrm{M}^{1+r},$$

$$\|\eta\|^2_{\mathrm{M}^1}=\int_0^\infty\mu(s)\|\eta(s)\|^2_{\mathrm{M}^1}ds,\forall\eta(s)\in\mathrm{M}^1.$$

Here we notice that for $r_1 > r_2$, embedding $\mathrm{M}^{r_1} \to \mathrm{M}^{r_2}$ are continuous but not compact (cf.[17,18]).

Finally, we define the inner product Hilbert space

$$V^r = v^{2+r} \times v^r \times v^r \times \mathrm{M}^{1+r}, r \in R.$$

For all $z = (z_1, z_2, z_3, z_4)^T \in V^r$, there is norm

$$\|z\|_{V^r}^2 = \|\mathrm{A}^{(2+r)/2} z_1\|^2 + \|\mathrm{A}^{r/2} z_2\|^2 + \|\mathrm{A}^{r/2} z_3\|^2 + \|z_4\|_{\mathrm{M}^{1+r}}^2,$$

when r = 0, there is $V^0 = v^2 \times v^0 \times v^0 \times \mathrm{M}^1$.

The Existence of Random Attractors

It is convenient to work in the history space setting by introducing the so-called summed past history of θ which is defined as follows (cf.[7])

$$\eta^t(s) = \int_0^s \theta(t-y)dy, \quad (t,s) \in [0,\infty) \times R^+,$$

the variable η^t (formally) satisfies the linear equation

$$\eta_t^t(s) + \eta_s^t(s) = \theta(t), \ \text{in}\,\Omega, \ (t,s) \in R^+ \times R^+,$$

boundaries and initial conditions

$$\eta^t(0) = 0, \ \text{in}\,\Omega, \ t \geq 0,$$

$$\eta^0(s) = \eta_0(s) = \int_0^s \psi(y)dy, \ \text{in}\,\Omega, \ s \in R^+.$$

the linear operator T introduced in M^1 is defined as

$$T\eta = -\eta_s, \eta \in D(T),$$

its boundary is

$$D(T) = \{\eta \in \mathrm{M}^1 \mid \eta_s \in \mathrm{M}^1, \eta(0) = 0\}.$$

here and above η_s is the distribution derivative of η with respect to the internal variable s.

We note that the time-segment integration of the convolution that occurs in the equation θ results in

$$\begin{cases} \theta_t - \Delta u_t - \int_0^\infty \mu(s)\Delta\eta^t(s)ds = \sum_{j=1}^m \phi_j d\omega_j \\ u_{tt} - \Delta u_t + \Delta(\Delta u + \theta) + f(u) = 0 \end{cases} \tag{4.1}$$

import the vector

$$b(t) = (u(t), v(t), \theta(t), \eta^t)^T,$$

define the initial value by

$$b_0 = (u_0, v_0, \theta_0, \eta_0)^T \in V^0,$$

we can transform the studied equation into

$$\begin{aligned} &b_t = Lb + G(b,t,\omega), \\ &b(0) = b_0 = (u_0, u_1, \theta_0, \eta_0) \end{aligned} \tag{4.2}$$

among them, $G(b,t,\omega) = (0, -f(u), \sum_{j=1}^m \phi_j d\omega_j, 0)^T$, let $u_t = v$

$$Lb = L\begin{pmatrix} u \\ v \\ \theta \\ \eta \end{pmatrix} = \begin{pmatrix} v \\ -Av - A(Au - \theta) \\ -Av - \int_0^s \mu(s)A\eta^t(s)ds \\ \theta + T\eta \end{pmatrix} \tag{4.3}$$

$$D(L)=\left\{ b\in V^0 \left| \begin{array}{c} v, Au-\theta\in v^2 \\ \theta\in \mathrm{M}^1 \\ \int_0^s \mu(s)A\eta^t(s)ds\in v^0 \\ \eta\in D(T) \end{array} \right. \right\}.$$

The existence and uniqueness of the solution

Next, explain the existence, uniqueness, and continuous dependence of the solution on the initial value of the problem (4.2), and show that the solution generates a continuous random dynamic system on probability space $(\Omega, F, P, (\vartheta_t)_{t\in R})$. For this, we consider the one-dimensional Ornstein-Uhlenbeck equation

$$dz_j+z_jdt=d\omega_j(t),\ j=1,2,...,m,$$

its solution is the Ornstein-Uhlenbeck process

$$z_j(\vartheta_t\omega_j)=-\int_{-\infty}^0 e^s(\vartheta_t\omega_j)(s)ds, t\in R.$$

It is known from the literature [2] that the random variable $\left|z_j(\omega_j)\right|$ is harmonic, and there is a ϑ_t -measure invariant set of all measures, so that for all $\omega\in\Omega$ and j = 1,2, ..., m, $t\mapsto z_j(\vartheta_t\omega_j)$ are continuous with respect to time t . Therefore, it is known from the proposition 4.3.3 in [2] that there is a harmonic function $r(\omega)>0$ such that

$$\sum_{j=1}^m(\left|z_j(\omega_j)\right|+\left|z_j(\omega_j)\right|^2+\left|z_j(\omega_j)\right|^4+\left|z_j(\omega_j)\right|^6)\le r(\omega) \tag{4.4}$$

among them, $r(\omega)$ satisfies P-a.e. $\omega\in\Omega$

$$r(\vartheta_t\omega)\le e^{\frac{1}{2}|t|}r(\omega), t\in R, \tag{4.5}$$

known from (4.4) and (4.5), for P-a.e. $\omega\in\Omega$

$$\sum_{j=1}^m(\left|z_j(\vartheta_t\omega_j)\right|+\left|z_j(\vartheta_t\omega_j)\right|^2+\left|z_j(\vartheta_t\omega_j)\right|^4+\left|z_j(\vartheta_t\omega_j)\right|^6)\le e^{\frac{1}{2}|t|}r(\omega), t\in R \tag{4.6}$$

let

$$z(\vartheta_t\omega)=z(x,\vartheta_t\omega)=\sum_{j=1}^m \phi_j z_j(\vartheta_t\omega_j),$$

This is a solution to the following equationon

$$dz+zdt=\sum_{j=1}^m\phi_j d\omega_j, \tag{4.7}$$

Theorem 4.1 ([10]) Variables ρ , $\rho^{(l)}:\Omega\mapsto R^+, l=\frac{1}{2},1$,for all $t\in R, \omega\in\Omega$

$$\left\|z(\vartheta_t\omega)\right\|\le e^{\sigma|t|}r(\omega), e^{-\sigma|t|}r(\omega)\le r(\vartheta_t\omega)\le e^{\sigma|t|}r(\omega),$$
$$\left\|A^{(l)}z(\vartheta_t\omega)\right\|\le e^{\sigma|t|}r^{(l)}(\omega), e^{-\sigma|t|}r^{(l)}(\omega)\le r^{(l)}(\vartheta_t\omega)\le e^{\sigma|t|}r^{(l)}(\omega),$$

among them, $r^{(l)}(\omega)=\sum_{j=1}^m r_j(\omega_j)\left\|A^{(l)}h_j\right\|$.

Simplify (4.2) to the first-order evolution equation.

let

$$\omega=u_t+\varepsilon u-z(\vartheta_t\omega), \varphi=(u,\omega,\theta,\eta^t)^T,$$

Choose ε such that $0<\varepsilon<\min\{\rho_1,\rho_2,\rho_3\}$,

The problem (4.2) is equivalent to the following deterministic dynamic system with random parameters:

$$\dot{\varphi}+H(\varphi)=F(\varphi,\vartheta_t\omega,t),\ \ \varphi_0(\omega)=(u_0,u_1+\varepsilon u_0-z(\omega),\theta_0,\eta_0)^T, t\ge 0, \tag{4.8}$$

among them,

$$-H(\varphi)=\begin{pmatrix}\omega-\varepsilon u\\ -A(\omega-\varepsilon u)-A(Au-\theta)\\ -A(\omega-\varepsilon u)-\int_0^\infty \mu(s)A\eta^t(s)ds\\ \theta+T\eta\end{pmatrix}=T_\varepsilon LT_\varepsilon^{-1},$$

$$F(\varphi,\vartheta_t\omega)=\begin{pmatrix}z(\vartheta_t\omega)\\ -f(u)-Az(\vartheta_t\omega)\\ -Az(\vartheta_t\omega)+z(\vartheta_t\omega)\\ 0\end{pmatrix}.$$

It is known from the literature [3] that the operator L in (4.3) is the infinitesimal generator of the compressed C_0 semigroup $\{e^{Lt}\}$ in space V^0 ,because $-H=T_\varepsilon LT_\varepsilon^{-1}$ and L are isomorphic operators in V^0 , So operator $-H$ also generates a compressed C_0 semigroup $\{e^{-Ht}\}$ in V^0 .

From the conditions satisfied by the nonlinear function $f(u)$ and the embedding relationship $H_0^2(U)\to L^4(U)$, it is known that function $F(\varphi,\vartheta_t\omega,t)$ is locally Lipschitz continuous with respect to φ and $\omega\in\Omega$. Moreover, the time t is bounded on a finite interval, so $F(\varphi,\vartheta_t\omega,t)$ is continuous with respect to (φ,t) and F is measurable with respect to ω , and the uniqueness of the solution of the differential equation is considered by the classic operator semigroup theory (cf.[3]), We have the following theorem.

Theorem 4.2 If conditions (1)-(4) hold, then for any of $\omega\in\Omega$ and $\varphi_0\in V^0$, there is T>0, so that when the initial value meets $\varphi(0,\omega,\varphi_0)=\varphi_0$, the problem (4.8) has a unique mild solution $\varphi(t,\omega,\varphi_0)\in C([0,T);V^0)$ and $\varphi(t,\omega,\varphi_0)$ satisfies

$$\varphi(t,\omega,\varphi_0)=e^{-Ht}\varphi_0(\omega)+\int_0^t e^{-H(t-s)}F(s,\vartheta_s\omega,\varphi(s,\omega,\varphi_0))ds,$$

further, $\varphi(t,\omega,\varphi_0)$ is continuous with respect to t and φ_0 and measurable with respect to ω .

Knowing from Theorem 4.2 and the following Theorem 4.3, for any $t\in[0,\infty)$, the solution $\varphi(\cdot,\omega,\varphi_0)$ of equation (4.8) exists globally, therefore, $\varphi(\cdot,\omega,\varphi_0)\in C([0,+\infty);V^0)$ defines a continuous random dynamic system from R to $(\Omega,F,P,(\vartheta_t)_{t\in R})$:

$$\Phi(t,\omega):V^0\to V^0$$

$$\varphi_0(\omega)\mapsto\varphi(t,\omega)\varphi_0.$$

In order to prove that the solution of stochastic partial differential equation (1.1) and the solution of partial differential equation (4.8) are conjugated, homeomorphic mapping is introduced

$$R(\vartheta_t\omega)y=(y_1,y_2+\varepsilon y_1-z(\vartheta_t\omega),y_3,y_4)^T,\ y=(y_1,y_2,y_3,y_4)^T\in V^0,$$

its inverse mapping is

$$R^{-1}(\vartheta_t\omega)y=(y_1,y_2-\varepsilon y_1-z(\vartheta_t\omega),y_3,y_4)^T,$$

then

$$\Phi(t,\omega)=R(\vartheta_t\omega)S(t,\omega)R^{-1}(\vartheta_t\omega)\ .$$

This also determines a continuous random dynamic system.

On the other hand, if

$$a_1=u(t),a_2=u_t(t),a_3=\theta(t),a_4=\eta^t\ ,$$

and

$$a=\begin{pmatrix}a_1\\a_2\\a_3\\a_4\end{pmatrix},G(a,\omega)=\begin{pmatrix}0\\ -f(u)\\ \sum_{j=1}^m\phi_j d\omega_j\\ 0\end{pmatrix},$$

then the partial differential equation (1.1) can be written as

$$\dot{a}+H(a)=G(a,\omega),G(a,\omega)=(u_0,u_1,\theta_0,\eta_0)^T,$$

let $Ty=(y_1,y_2,y_3,y_4)^T$, $y=(y_1,y_2,y_3,y_4)^T\in V^0$, its inverse map is

$$T^{-1}y=(y_1,y_2,y_3,y_4)^T,$$

so

$$\bar{S}(t,\omega)=TS(t,\omega)T^{-1}.$$

This also determines a continuous random dynamic system $(\vartheta,\bar{S}(t,\omega))$. Therefore, $\bar{S}(t,\omega)$ and $\Phi(t,\omega)$ are equivalent.

Next, the existence of the random attractor of the stochastic dynamic system $\Phi(t,\omega)$ will be studied according to Theorem 2.1.

Theorem 4.3 Assuming that conditions (1)-(4) hold, there is a closed sphere (centered on 0, $r_0(\omega)$ is radius), so that for any non-random set $B=\{B(\omega)\}_{\omega\in\Omega}\in D$, there is a $T_{B(r_0(\omega))}>0$, so that for any $\varphi_0(\vartheta_{-t}\omega)\in B(\vartheta_{-t}\omega)$, the solution $\varphi(t,\vartheta_{-t}\omega,\varphi_0)=(u(t),\omega(t),\theta(t),\eta^t)^T$ of equation (4.8) satisfies P-a.e. $\omega\in\Omega$, has

$$\|\varphi(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega))\|_{V^0}^2\le r_0^2(\omega).$$

Proof: Use $\varphi(t)=(u(t),\omega(t),\theta(t),\eta^t)^T$ and equation (4.8) to make the inner product in V^0 ,

$$\begin{aligned}&\frac{d}{dt}(\frac{1}{2}\|\varphi\|_{V^0}^2)+\varepsilon\|u\|_2^2+\|\nabla\omega\|^2-\varepsilon(Au,\omega)-\varepsilon(Au,\theta)-\frac{1}{2}\int_0^\infty\mu'(s)\|A^{1/2}\eta^t(s)\|^2ds\\&=((z(\vartheta_t\omega),u))+(-f(u)-Az(\vartheta_t\omega),\omega)+(-Az(\vartheta_t\omega)+z(\vartheta_t\omega),\theta)+0\end{aligned}\tag{4.9}$$

among them,

$$-\frac{1}{2}\int_0^\infty\mu'(s)\|A^{1/2}\eta^t(s)\|^2ds\ge\frac{\delta}{2}\|\eta^t\|_{M^1}^2,$$

$$\|\nabla\omega\|^2\ge D_1\|\omega\|^2,2<D_1=\|A^{1/2}\|^2<\frac{17-\sqrt{65}}{2}.$$

We multiply the second expression in (1.1) by ε^2u and add (4.9), then

$$\begin{aligned}&\frac{d}{dt}(\frac{1}{2}\|\varphi\|_{V^0}^2+\frac{\varepsilon^2}{2}\|\nabla u\|^2+\varepsilon^2\int_U uu_tdx)-\varepsilon^2\|u_t\|^2+(\varepsilon+\varepsilon^2)\|u\|_2^2+\|\nabla\omega\|^2\\&-\varepsilon(Au,\omega)-\varepsilon(Au,\theta)-\frac{1}{2}\int_0^\infty\mu'(s)\|A^{1/2}\eta^t(s)\|^2ds+\varepsilon^2\int_U uf(u)dx-\varepsilon^2\int_U\theta Audx\\&=((z(\vartheta_t\omega),u))+(-f(u)-Az(\vartheta_t\omega),\omega)+(-Az(\vartheta_t\omega)+z(\vartheta_t\omega),\theta)\end{aligned}\tag{4.10}$$

substitute $\omega=u_t+\varepsilon u-z(\vartheta_t\omega)$ into the right side of equation (4.10) ,the inner product term on the left uses Lemma 2.2, taking the appropriate α has

$$\begin{aligned}&\frac{d}{dt}(\frac{1}{2}\|\varphi\|_{V^0}^2+\frac{\varepsilon^2}{2}\|\nabla u\|^2+\varepsilon^2\int_U uu_tdx-\varepsilon^2\|u\|^2+\int_U F(u)dx)+\varepsilon^2\|u\|_2^2+(D_1-\frac{\varepsilon}{2})\|\omega\|^2\\&-\frac{\varepsilon}{2}\|\theta\|^2-\frac{1}{2}\int_0^\infty\mu'(s)\|A^{1/2}\eta^t(s)\|^2ds-\varepsilon^2\int_U\theta Audx+\varepsilon^2\int_U uf(u)dx\\&\le((z(\vartheta_t\omega),u))-\varepsilon(f(u),u)+(f(u),z(\vartheta_t\omega))-(Az(\vartheta_t\omega),\omega)+(-Az(\vartheta_t\omega)+z(\vartheta_t\omega),\theta)\end{aligned}\tag{4.11}$$

To estimate the right side of (4.11), from Lemma 2.2(1), take $\alpha=\frac{1}{4}$ and have the following inequality:

$$((z(\vartheta_t\omega),u))\le\frac{1}{8}\|u\|_2^2+2\|z(\vartheta_t\omega)\|_2^2$$

$$\begin{aligned}(f(u),z(\vartheta_t\omega))&\le C_0\int_U(1+|u|^2)|z(\vartheta_t\omega)|dx\\&\le C_0\int_U|z(\vartheta_t\omega)|dx+C_0(\int_U|u|^3dx)^{\frac{2}{3}}\|z(\vartheta_t\omega)\|_{L^3}\end{aligned}$$

$$\leq C_0\int_U |z(\vartheta_t\omega)|dx + C_0C_1^{-\frac{2}{3}}[\frac{2}{3}(\varepsilon_1(\int_U (F(u)+C_1)dx)^{\frac{2}{3}})^{\frac{3}{2}} + \frac{1}{3\varepsilon_1^3}\|z(\vartheta_t\omega)\|_{L^3}^3$$

$$\leq C_0|z(\vartheta_t\omega)| + \kappa\int_U F(u)dx + \kappa C_1|U| + \tau\|z(\vartheta_t\omega)\|_{L^3}^3,$$

among them, $\kappa = \frac{2}{3}C_0C_1^{-\frac{2}{3}}\varepsilon_1^{\frac{3}{2}}, \tau = \frac{1}{3\varepsilon_1^3}$.

Knowing from the condition (3.5), if $N_1 > 0$ exists, then

$$(f(u),u) \geq C_2\int_U F(u)dx - \frac{\lambda_1}{8}\|u\|_2^2 - N_1.$$

Use (3.6) to know that there is $0 < \delta_1 < \frac{1}{2}, I = I(\delta_1) > 0$ and get $f(s)s \geq -\frac{1-\delta_1}{C_U}s^2, \forall |s| \geq N$. Then

$$\int_U f(u)udx \geq -(1-\delta_1)\|u\|_2^2 + C(|U|, f)$$

among them, $C(|U|, f) = |U|\min_{|s|\leq N} F(s)$.

Synthesizing the above inequalities and using Lemma 2.2, take the appropriate α to get

$$\begin{aligned}&\frac{d}{dt}(\frac{1}{2}\|\varphi\|_{V^0}^2 + \frac{\varepsilon^2}{2}\|\nabla u\|^2 + \varepsilon^2\int_U uu_t dx - \varepsilon^2\|u\|^2 + \int_U F(u)dx) + (\varepsilon^2\delta_1 - \frac{\lambda_1\varepsilon+1}{8} - \frac{\varepsilon^2}{2})\|u\|_2^2 \\ &+(D_1 - \frac{\varepsilon}{2} - \frac{1}{8})\|\omega\|^2 - (\frac{\varepsilon}{2} + \frac{1}{4} + \frac{\varepsilon^2}{2})\|\theta\|^2 - \frac{1}{2}\int_0^\infty \mu'(s)\|A^{1/2}\eta^t(s)\|^2 ds + (\varepsilon C_2 - \kappa)\int_U F(u)dx \\ &\leq 6\|Az(\vartheta_t\omega)\|^2 + C_0|z(\vartheta_t\omega)| + \kappa C_1|U| + \tau\|z(\vartheta_t\omega)\|_{L^3}^3 + 2\|z(\vartheta_t\omega)\|^2 - \varepsilon^2 C(|U|, f) + \varepsilon N_1\end{aligned} \quad (4.12)$$

Let $J(t) = -\int_0^\infty \mu(s)\langle\theta(t),\eta^t(s)\rangle ds$, then $|J(t)| \leq \|\theta(t)\|\int_0^\infty \mu(s)\langle\eta^t(s),\eta^t(s)\rangle^{\frac{1}{2}} ds \leq C\|\varphi\|_{V^0}^2$.

$$\begin{aligned}&\frac{d}{dt}J(t) = -\int_0^\infty \mu(s)\langle\theta_t(t),\eta^t(s)\rangle ds - \int_0^\infty \mu(s)\langle\theta(t),\eta_t^t(s)\rangle ds \\ &= -\int_0^\infty \mu(s)\langle -A\omega,\eta^t(s)\rangle ds - \int_0^\infty \mu(s)\langle\varepsilon Au,\eta^t(s)\rangle ds + \int_0^\infty \mu(s)\langle Az(\vartheta_t\omega),\eta^t(s)\rangle ds \\ &+\int_0^\infty \mu(s)\left\langle\int_0^\infty \mu(s)A\eta^t(s)ds,\eta^t(s)\right\rangle ds - \int_0^\infty \mu(s)\langle z(\vartheta_t\omega),\eta^t(s)\rangle ds - k_0\|\theta\|^2 + \int_0^\infty \mu(s)\langle\theta(t),\eta_s^t(s)\rangle ds\end{aligned}$$

Estimation of the above formula has the following inequality

$$\int_0^\infty \mu(s)\left\langle\int_0^\infty \mu(s)A\eta^t(s)ds,\eta^t(s)\right\rangle ds \leq k_0\|\eta^t\|_{M^1}^2,$$

At the same time using Lemma 2.2 (3)(4), take the appropriate α to get

$$\begin{aligned}&\frac{d}{dt}J(t) + \frac{3k_0}{4}\|\theta\|^2 \leq \frac{k_0D_1}{4}\|\omega\|^2 + \frac{k_0\varepsilon}{4}\|u\|_2^2 + (\frac{1+4\varepsilon}{4D_1} + k_0 + \frac{5}{4})\|\eta^t\|_{M^1}^2 \\ &-\frac{1}{D_1}\int_0^\infty \mu'(s)\|A^{1/2}\eta^t(s)\|^2 ds + k_0\|\nabla z(\vartheta_t\omega)\|^2 + k_0\|z(\vartheta_t\omega)\|^2.\end{aligned} \quad (4.13)$$

Next, add (4.13) and (4.12) to get

$$\begin{aligned}&\frac{d}{dt}(\frac{1}{2}\|\varphi\|_{V^0}^2 + \frac{\varepsilon^2}{2}\|\nabla u\|^2 + \varepsilon^2\int_U uu_t dx - \varepsilon^2\|u\|^2 + \int_U F(u)dx + J(t)) + (D_1 - \frac{\varepsilon}{2} - \frac{k_0\varepsilon}{4} - \frac{1}{8})\|\omega\|^2 \\ &+(\varepsilon^2\delta_1 - \frac{\lambda_1\varepsilon+1}{8} - \frac{\varepsilon^2}{2} - \frac{k_0\varepsilon}{4})\|u\|_2^2 + (\frac{3k_0-1}{4} - \frac{\varepsilon+\varepsilon^2}{2})\|\theta\|^2 + (\varepsilon C_2 - \kappa)\int_U F(u)dx \\ &+(\frac{\delta}{2} - \frac{\delta+\varepsilon}{D_1} - \frac{5}{4} - \frac{1}{4D_1} - k_0)\|\eta^t\|_{M^1}^2 \\ &\leq k_0\|\nabla z(\vartheta_t\omega)\|^2 + 6\|Az(\vartheta_t\omega)\|^2 + C_0|z(\vartheta_t\omega)| + \kappa C_1|U| + \tau\|z(\vartheta_t\omega)\|_{L^3}^3 + (2+k_0)\|z(\vartheta_t\omega)\|^2 \\ &-\varepsilon^2 C(|U|, f) + \varepsilon N_1\end{aligned}$$

among them

$$-\int_0^\infty \mu'(s)\left\|A^{1/2}\eta^t(s)\right\|^2 ds \geq \delta\int_0^\infty \mu(s)\left\|A^{1/2}\eta^t(s)\right\|^2 ds = \delta\left\|\eta^t\right\|_{M^1}^2 .$$

Let

$$R_0(\vartheta_t\omega) = \left\|\nabla z(\vartheta_t\omega)\right\|^2 + \left|z(\vartheta_t\omega)\right| + \left\|z(\vartheta_t\omega)\right\|_{L^3}^3 + \left\|Az(\vartheta_t\omega)\right\|^2 + \left\|z(\vartheta_t\omega)\right\|^2 ,$$

$$G(t) = \frac{1}{2}\left\|\varphi\right\|_{V^0}^2 + \frac{\varepsilon^2}{2}\left\|\nabla u\right\|^2 + \varepsilon^2\int_U uu_t dx - \varepsilon^2\left\|u\right\|^2 + \int_U F(u)dx + J(t),$$

There is

$$\frac{d}{dt}G(t) + \beta\left\|\varphi\right\|_{V^0}^2 \leq \frac{d}{dt}G(t) + \beta\left\|\varphi\right\|_{V^0}^2 + (\varepsilon C_2 - \kappa)\int_U F(u)dx \leq \chi \tag{4.14}$$

among them,

$Q = \max\{6, C_0, 2+k_0, \tau\}$,

$\chi = QR_0(\vartheta_t\omega) + \kappa C_1\left|U\right| - \varepsilon^2 C(|U|, f) + \varepsilon N_1$, (within the

$\beta = \min\{\varepsilon^2\delta_1 - \frac{\lambda_1\varepsilon + 1}{8} - \frac{\varepsilon^2}{2} - \frac{k_0\varepsilon}{4}, D_1 - \frac{\varepsilon}{2} - \frac{k_0\varepsilon}{4} - \frac{1}{8}, \frac{3k_0 - 1}{4} - \frac{\varepsilon+\varepsilon^2}{2}, \frac{\delta}{2} - \frac{\delta+\varepsilon}{D_1} - \frac{5}{4} - \frac{1}{4D_1} - k_0\}$

limited range of each parameter, each item in β is greater than zero).

Let $\rho_1 = \min\{\frac{\lambda_1 + 2k_0 - \sqrt{\frac{\lambda_1^2}{64} + \frac{\lambda_1 k_0 + k_0^2}{16} + \frac{\delta_1}{2} - \frac{1}{4}}}{16\delta_1 - 8}, \frac{8D_1 - 1}{4(k_0+1)}, -\frac{1}{2} + \sqrt{\frac{1}{8} + \frac{3k_0}{2}}, \frac{\delta}{2}D_1 - \delta - \frac{5D_1 + 1}{4} - k_0 D_1\}$.

(within the limited range of each parameter, each item in ρ_1 is greater than zero).

By Theorem 4.1

$$\int_{-t}^0 R_0(\vartheta_s\omega)e^{\beta s}ds \leq \int_{-t}^0 \tilde{R}_0(s,\omega)e^{\beta s}ds \leq \int_{-\infty}^0 \tilde{R}_0(s,\omega)e^{\beta s}ds < +\infty,$$

among them,

$$\tilde{R}_0(s,\omega) = e^{2\sigma|t|}r^2(\omega) + e^{\frac{1}{2}|t|}r(\omega) + (e^{\sigma|t|}r(\omega))^3 + (e^{\sigma|t|}r^{(1)}(\omega))^2 + (e^{\sigma|t|}r^{(1/2)}(\omega))^2.$$

By Lemma 2.1 there is $G(\varphi(t,\omega,\varphi_0(\omega)) \leq \sup_{\xi\in V^0}\{E(\xi) : \beta\left\|\xi\right\|_{V^0}^2 \leq \chi + \varepsilon_1\}$.

Use $\vartheta_{-t}\omega$ instead of ω in the above formula to get

$$G(\varphi(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega)) \leq \sup_{\xi\in V^0}\{E(\xi) : \beta\left\|\xi\right\|_{V^0}^2 \leq \chi + \varepsilon_1\}.$$

Then

$$\left\|\varphi(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega)\right\|_{V^0}^2 \leq \frac{\chi + \varepsilon_1}{\beta} = r_0^2(\omega).$$

To sum up, the theorem is proved.

In order to obtain the regularity results of the stochastic dynamic system $\Phi(t,\omega)$, using a method similar to [1], the solution $\varphi = (u,\omega,\theta,\eta^t)^T$ of the system is decomposed into $\varphi = \varphi_L + \varphi_N$, where $\varphi_L = (u_L,\omega_L,\theta_L,\eta_L^t)^T$ and $\varphi_N = (u_N,\omega_N,\theta_N,\eta_N^t)^T$ are the solutions of the following equations

$$\dot{\varphi}_L + H(\varphi_L) = 0, \varphi_L(0,\omega) = (u_0, u_1 + \varepsilon u_0, \theta_0, \eta_0)^T, t \geq 0 \tag{4.15}$$

and

$$\dot{\varphi}_N + H(\varphi_N) = F_2(\omega), \varphi_N(0,\omega) = (0, -z(\omega), 0, 0)^T, t \geq 0 \tag{4.16}$$

among them,

$$F_2(\omega) = \begin{pmatrix} z(\vartheta_t\omega) \\ -f(u) - Az(\vartheta_t\omega) \\ z(\vartheta_t\omega) - Az(\vartheta_t\omega) \\ 0 \end{pmatrix}.$$

For equations (4.15) and (4.16) there are regular results as follows.

Theorem 4.4 Assuming conditions (1)-(4) hold, let $B \subset V^0$ be a non-random bounded set, then for any $\varphi_L(0,\omega) = (u_0, u_1 + \varepsilon u_0, \theta_0, \eta_0)^T \in B$, the following holds

$$\|\varphi_L(t, \vartheta_{-t}\omega, \varphi_L(0, \vartheta_{-t}\omega))\|_{V^0}^2 = \|u_L\|_2^2 + \|\omega_L\|^2 + \|\theta_L\|^2 + \|\eta_L^t\|_{M^1}^2 \to 0, (t \to \infty),$$

$\varphi_L = (u_L, \omega_L, \theta_L, \eta_L^t)^T$ meet equation (4.15).

Proof: In space V^0, use $\varphi_L = (u_L, \omega_L, \theta_L, \eta_L^t)^T$ and (4.15) as the inner product, where $\omega_L = u_{Lt} + \varepsilon u_L$, the corresponding initial value is $(u_0, u_1 + \varepsilon u_0, \theta_0, \eta_0)^T$.

$$\frac{1}{2}\frac{d}{dt}\|\varphi_L\|_{V^0}^2 + (H(\varphi_L), \varphi_L)_{V^0} = 0.$$

among them, $(H(\varphi_L), \varphi_L)_{V^0} \geq \frac{\varepsilon}{2}\|u_L\|^2 + (D_1 - \varepsilon)\|\omega_L\|^2 - \varepsilon\|\theta_L\|^2 + \frac{\delta}{2}\|\eta_L^t\|^2$.

Define the energy functional:

$$\mathrm{K}(t) = \int_U \omega_L(t) u_L(t) dx,$$

$$\mathrm{P}(t) = \int_U \theta_L(t) u_L(t) dx,$$

$$\Psi(t) = -\int_U \theta_L(t) \int_0^\infty \mu(s) \eta_L^t(s) ds dx.$$

Then derivate $\mathrm{K}(t)$, $\mathrm{P}(t)$) and $\Psi(t)$ with respect to time t, and use the Herder's inequality to get

$$\begin{aligned}
\frac{d}{dt}\mathrm{K}(t) &= \int_U \omega_{Lt}(t) u_L(t) dx + \int_U \omega_L(t) u_{Lt}(t) dx \\
&= \int_U (-A(\omega_L - \varepsilon u_L) - A(Au_L - \theta_L)) u_L dx + \|\omega_L\|^2 - \varepsilon \int_U \omega_L u_L dx \\
&\leq (2 - \frac{\varepsilon}{4})\|\omega_L\|^2 + (\frac{\varepsilon(D_1 - 1)}{D_1^2} - \frac{1}{2})\|u_L\|_2^2 + \frac{1}{2}\|\theta_L\|^2 + C.
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}\mathrm{P}(t) &= \int_U \theta_{Lt}(t) u_L(t) dx + \int_U \theta_L(t) u_{Lt}(t) dx \\
&= \int_U (-A(\omega_L - \varepsilon u_L) - \int_0^\infty \mu(s) A\eta_L^t(s) ds) u_L dx + \int_U \theta_L (\omega_L - \varepsilon u_L) dx \\
&\leq -\int_U A\omega_L u_L dx + \varepsilon D_1 \|u_L\|_2^2 - \int_0^\infty \mu(s) (\int_U A u_L \eta_L^t(s) dx) ds + \int_U \theta_L \omega_L dx - \varepsilon \int_U \theta_L u_L dx \\
&\leq \|\omega_L\|^2 + (\frac{1 + k_0}{2} + \frac{\varepsilon(D_1 + 1)}{D_1^2})\|u_L\|_2^2 + \frac{1}{2D_1}\|\eta_L^t\|_{M^1}^2 + \frac{2 + \varepsilon}{4}\|\theta_L\|^2 + C.
\end{aligned}$$

Then take the derivative of $\Psi(t)$,

$$\begin{aligned}
\frac{d}{dt}\Psi(t) &= -\frac{d}{dt}\int_U \theta_L(t) \int_0^\infty \mu(s) \eta_L^t(s) ds dx \\
&= -\int_U \theta_{Lt}(t) \int_0^\infty \mu(s) \eta_L^t(s) ds dx - \int_U \theta_L(t) \int_0^\infty \mu(s) \eta_{Lt}^t(s) ds dx \\
&= \int_U A\omega_L \int_0^\infty \mu(s) \eta_L^t(s) ds dx - \varepsilon \int_U A u_L \int_0^\infty \mu(s) \eta_L^t(s) ds dx \\
&\quad + \int_U \int_0^\infty \mu(s) A\eta_L^t(s) ds) \int_0^\infty \mu(s) \eta_L^t(s) ds dx - \int_U \theta_L(t) \int_0^\infty \mu(s) \eta_{Lt}(s) ds dx
\end{aligned}$$

among them,

$$|\int_U \int_0^\infty \mu(s) A\eta_L^t(s) ds) \int_0^\infty \mu(s) \eta_L^t(s) ds dx | \leq k_0 \|\eta_L^t\|_{M^1}^2,$$

$$-\int_U \theta_L(t)\int_0^\infty \mu(s)\eta_{Lt}^t(s)dsdx$$

$$\le -k_0\|\theta_L\|^2 + \int_U |\theta_L| (\int_0^{s_0} \frac{|\mu'(s)|}{\mu^{1/2}(s_0)}\mu^{1/2}(s)|\eta_L^t(s)|ds + M\int_0^\infty \mu(s)|\eta_L^t(s)|ds)dx$$

$$\le -(k_0-\frac{1}{4})\|\theta_L\|^2 + C\|\eta_L^t\|_{M^1}^2,$$

among them, $C=\frac{\sqrt{2}}{\lambda_1}\max\{\frac{1}{\mu^{1/2}(s_0)}(\int_0^{s_0}|\mu'(s)|^2 ds)^{1/2}, M\mu_0^{1/2}\}$.

Then,

$$\frac{d}{dt}\Psi(t)\le \frac{\varepsilon}{4}\|u_L\|_2^2 + \frac{1}{4}\|\omega_L\|^2 - (k_0-\frac{1}{4})\|\theta_L\|^2 + (C+D_1+\frac{\varepsilon}{D_1}+k_0)\|\eta_L^t\|_{M^1}^2.$$

Let, $L(t)=MY(t)+\gamma(\mathrm{K}(t)+\mathrm{P}(t))+\Psi(t)$,

among them

$\gamma<l$,

$$l=\min\{\frac{D_1(4k_0-1)-4k_0\varepsilon}{-2\varepsilon^2+\varepsilon(8+D_1)+4D_1}, \frac{D_1(4k_0-1-2\varepsilon)}{16\varepsilon+8k_0D_1+4D_1+\varepsilon D_1}, \frac{\varepsilon D_1(\delta-4k_0)-4\varepsilon(\varepsilon+D_1C+D_1^2)}{2\varepsilon+2k_0\delta D_1-8\delta\varepsilon}\},$$

$$\varsigma<M<\frac{4(k_0-\gamma)-1-\varepsilon\gamma}{4\varepsilon},$$

$$\varsigma=\max\{\frac{12\gamma+1-\varepsilon\gamma}{4(D_1-\varepsilon)}, \frac{2}{\delta}(C+D_1+k_0+\frac{\varepsilon}{D_1}+\frac{\gamma}{2D_1}), \frac{1}{2}+\frac{4\gamma}{D_1}+\frac{\gamma k_0}{\varepsilon}\}.$$

(within the limited range of each parameter, each item in ς is less than $\frac{4(k_0-\gamma)-1-\varepsilon\gamma}{4\varepsilon}$)

The inequalities obtained by integrating the results of the above three functions are

$$\frac{d}{dt}L(t)=M\frac{d}{dt}Y(t)+\gamma\frac{d}{dt}(\mathrm{K}(t)+\mathrm{P}(t))+\frac{d}{dt}\Psi(t)$$

$$\le(\varepsilon M-MD_1+3\gamma+\frac{1-\varepsilon\gamma}{4})\|\omega_L\|^2+(\frac{\varepsilon}{D_1}-\frac{\delta M}{2}+C+D_1+\frac{\gamma}{2D_1}+k_0)\|\eta_L^t\|_{M^1}^2$$

$$+(\frac{\varepsilon}{4}+\frac{2\varepsilon\gamma}{D_1}+\frac{\gamma k_0-\varepsilon M}{2})\|u_L\|_2^2+(\gamma+\varepsilon M-k_0+\frac{\gamma\varepsilon+1}{4})\|\theta_L\|^2,$$

let,

$$\beta_1=\min\{MD_1-\varepsilon M-3\gamma-\frac{1-\varepsilon\gamma}{4}, \frac{\varepsilon M}{2}-\frac{\varepsilon}{4}-\frac{2\varepsilon\gamma}{D_1}-\frac{\gamma k_0}{2}, k_0-\gamma-\varepsilon M-\frac{\gamma\varepsilon+1}{4}, \frac{\delta M}{2}-\frac{\varepsilon}{D_1}-C-D_1-\frac{\gamma}{2D_1}-k_0\},$$

$$\rho_2=\min\{D_1, \frac{72+D_1^2-15D_1}{4}, \frac{k_0\delta D_1}{4\delta-1}\}.$$

(within the limited range of each parameter, each item in β_1 and ρ_2 is greater than zero).

Then

$$\frac{d}{dt}L(t)+\beta_1 Y(t)\le 0,, \tag{4.17}$$

applying Gronwall's lemma to the above formula

$$L(t,\omega,\varphi_L(0,\omega))\le Y(0,\omega,\varphi_L(0,\omega)e^{-\beta_1 t},$$

use $\vartheta_{-t}\omega$ instead of ω in the above formula to get

$$L(t,\vartheta_{-t}\omega,\varphi_L(0,\vartheta_{-t}\omega))\le Y(0,\vartheta_{-t}\omega,\varphi_L(0,\vartheta_{-t}\omega)e^{-\beta_1 t},$$

So,

$$2Y(t,\vartheta_{-t}\omega,\varphi_L(0,\vartheta_{-t}\omega)\le 2CL(t,\vartheta_{-t}\omega,\varphi_L(0,\vartheta_{-t}\omega)))$$

$$\le 2CY(0,\vartheta_{-t}\omega,\varphi_L(0,\vartheta_{-t}\omega))e^{-\beta_1 t}$$

$$\le 2Ce^{-\beta_1 t}(\|u_0\|_2^2+\|u_1+\varepsilon u_0\|^2+\|\theta_0\|^2+\|\eta_0\|_{M^1}^2)\to 0.(t\to\infty).$$

To sum up, the theorem is proved.

Theorem 4.5 Assuming that conditions (1)-(4) hold, there is a random closed sphere $B=\{B(\omega)\}_{\omega\in\Omega}\in D$ centered on 0 and radiused $r_1(\omega)>0$, while $\bar{B}(\omega)\in D$, $\varphi_0(\omega)\in\bar{B}(\omega)$ and $T_{\bar{B}(r_1\omega)}>0$ exist, so that for any $\varphi_N=(u_N,\omega_N,\theta_N,\eta_N^t)^T$, the system (4.16) satisfies P-a.e. $\omega\in\Omega$,there is

$$\left\|A^{r/2}u_N\right\|_2^2+\left\|A^{r/2}\omega_N\right\|^2+\left\|A^{r/2}\theta_N\right\|^2+\left\|A^{r/2}\eta_N^t\right\|_{M^1}^2\le r_1^2(\omega).$$

Proof: Knowing from Theorem 4.3, Theorem 4.4 and $\varphi_N=\varphi-\varphi_L$, there is a positive random variable $M_3(\omega)>0$, making

$$\max\left\{\left\|\varphi(0,\vartheta_{-t}\omega,\varphi(0,\vartheta_{-t}\omega))\right\|_{V^0},\left\|\varphi_N(0,\vartheta_{-t}\omega,\varphi_N(0,\vartheta_{-t}\omega))\right\|_{V^0}\right\}\le M_3(\omega).$$

In space V^0, use $A^r\varphi_N=(A^ru_N,A^r\omega_N,A^r\theta_N,A^r\eta_N^t)^T$ and (4.16) as the inner product

$$\begin{aligned}&\frac{1}{2}(\left\|A^{r/2}u_N\right\|_2^2+\left\|A^{r/2}\omega_N\right\|^2+\left\|A^{r/2}\theta_N\right\|^2+\left\|A^{r/2}\eta_N^t\right\|_{M^1}^2)+(H(\varphi_N),A^r\varphi_N)_{V^0}\\&=((z(\vartheta_t\omega),A^ru_N))-(f(u),A^r\omega_N)-(Az(\vartheta_t\omega),A^r\omega_N)-(Az(\vartheta_t\omega),A^r\theta_N)+(z(\vartheta_t\omega),A^r\theta_N)\end{aligned}\tag{4.18}$$

It can be calculated by the similar theorem 4.4

$$(H(\varphi_N),A^r\varphi_N)_{V^0}\ge\frac{\varepsilon}{2}\left\|A^{r/2}u_N\right\|_2^2+(D_1-\varepsilon)\left\|A^{r/2}\omega_N\right\|^2-\varepsilon\left\|A^{r/2}\theta_N\right\|^2+\frac{\delta}{2}\left\|A^{r/2}\eta_N^t\right\|^2$$

Next estimate each item on the right side of (4.18), using the condition (3.6), $f(u)\ge-\frac{1-\delta_1}{C_U}u$, we get

$$|-(f(u),A^r\omega_N)|\le\frac{(1-\delta_1)^2C_s}{4C_U^2}r_0^2(\omega)+\left\|A^{r/2}\omega_N\right\|^2,$$

where $C_s>0$ is the embedding constant.

To sum up, using Lemma 2.2, take the appropriate α to get,

$$\begin{aligned}&\frac{d}{dt}\left\|A^{r/2}\varphi_N\right\|_{V^0}^2+(\varepsilon-\frac{1}{2})\left\|A^{r/2}u_N\right\|_2^2+(2D_1-2\varepsilon-\frac{5}{2})\left\|A^{r/2}\omega_N\right\|^2-(2\varepsilon+\frac{1}{2})\left\|A^{r/2}\theta_N\right\|^2+\delta\left\|A^{r/2}\eta_N^t\right\|^2\\&\le2\left\|A^{r/2}z(\vartheta_t\omega)\right\|_2^2+\frac{(1-\delta_1)^2C_s}{2C_U^2}r_0^2(\omega)+6\left\|A^{(r+2)/2}z(\vartheta_t\omega)\right\|^2+4\left\|A^{r/2}z(\vartheta_t\omega)\right\|^2\end{aligned}\tag{4.19}$$

Let $A^rJ_N(t)=-\int_0^\infty\mu(s)A^r\left\langle\theta_N,\eta_N^t(s)\right\rangle ds$, then

$$|A^rJ_N(t)|\le\left\|A^{r/2}\theta_N\right\|\int_0^\infty\mu(s)A^{r/2}\left\langle\eta_N^t(s),\eta_N^t(s)\right\rangle^{\frac{1}{2}}ds\le C_n\left\|A^{r/2}\varphi_N\right\|_{V^0}^2.$$

By Theorem 4.3 there

$$\begin{aligned}&\frac{d}{dt}A^rJ_N(t)+\frac{3k_0}{4}\left\|A^{r/2}\theta_N\right\|^2\le\frac{k_0D_1}{4}\left\|A^{r/2}\omega_N\right\|^2+\frac{k_0\varepsilon}{4}\left\|A^{r/2}u_N\right\|_2^2+(\frac{1+4\varepsilon+4\delta}{4D_1}+k_0+\frac{5}{4})\left\|A^{r/2}\eta_N^t\right\|_{M^1}^2\\&+k_0\left\|A^{(1+r)/2}z(\vartheta_t\omega)\right\|^2+k_0\left\|A^rz(\vartheta_t\omega)\right\|^2.\end{aligned}\tag{4.20}$$

Combining (4.19) and (4.20)

$$\begin{aligned}&\frac{d}{dt}(\left\|A^{r/2}\varphi_N\right\|_{V^0}^2+A^rJ_N(t))+(\varepsilon-\frac{1}{2}-\frac{k_0\varepsilon}{4})\left\|A^{r/2}u_N\right\|_2^2+(2D_1-2\varepsilon-\frac{5}{2}-\frac{k_0D_1}{4})\left\|A^{r/2}\omega_N\right\|^2\\&+(\frac{3k_0}{4}-2\varepsilon-\frac{1}{2})\left\|A^{r/2}\theta_N\right\|^2+(\delta-\frac{1+4\varepsilon+4\delta}{4D_1}-k_0-\frac{5}{4})\left\|A^{r/2}\eta_N^t\right\|^2\\&\le2\left\|A^{r/2}z(\vartheta_t\omega)\right\|_2^2+\frac{(1-\delta_1)^2C_s}{2C_U^2}r_0^2(\omega)+6\left\|A^{(r+2)/2}z(\vartheta_t\omega)\right\|^2+(k_0+4)\left\|A^{r/2}z(\vartheta_t\omega)\right\|^2+k_0\left\|A^{(1+r)/2}z(\vartheta_t\omega)\right\|^2.\end{aligned}$$

Let $X(t)=\left\|A^{r/2}\varphi_N\right\|_{V^0}^2=\left\|A^{r/2}u_N\right\|_2^2+\left\|A^{r/2}\omega_N\right\|^2+\left\|A^{r/2}\theta_N\right\|^2+\left\|A^{r/2}\eta_N^t\right\|_{M^1}^2$

There is

$$(1-C_n)\frac{d}{dt}X(t)+\beta_2X(t)\le\frac{d}{dt}(\left\|A^{r/2}\varphi_N\right\|_{V^0}^2+A^rJ_N(t))+\beta_2X(t)\le PR_1(\vartheta_t\omega),\tag{4.21}$$

among them,

$$R_1(\vartheta_t\omega)=\left\|A^{(1+r)/2}z(\vartheta_t\omega)\right\|^2+\left\|A^{r/2}z(\vartheta_t\omega)\right\|_2^2+\left\|A^{(r+2)/2}z(\vartheta_t\omega)\right\|^2+\left\|A^{r/2}z(\vartheta_t\omega)\right\|^2+r_0^2(\omega),$$

$$P=\max\left\{6,\frac{(1-\delta_1)^2C_s}{2C_U^2},k_0+4\right\},$$

$$\beta_2=\min\{\varepsilon-\frac{1}{2}-\frac{k_0\varepsilon}{4},2D_1-2\varepsilon-\frac{5}{2}-\frac{k_0D_1}{4},\frac{3k_0}{4}-2\varepsilon-\frac{1}{2},\delta-\frac{1+4\varepsilon+4\delta}{4D_1}-k_0-\frac{5}{4}\},$$

$$\rho_3=\min\{\frac{2}{4-k_0},D_1-\frac{5}{4}-\frac{k_0D_1}{8},\frac{3}{8}k_0-\frac{1}{4}\}.$$

(within the limited range of each parameter, each item in β_2 and ρ_3 is greater than zero).

For (4.21), using Gronwall's inequality

$$X(t,\omega,\varphi_0(\omega))\le e^{-\frac{\beta_2}{1-C_n}t}X(0,\omega,\varphi_0(\omega))+\frac{P}{1-C_n}\int_0^t R_1(\vartheta_s\omega)e^{-\beta_2(t-s)}ds\,,$$

use $\vartheta_{-t}\omega$ instead of ω in the above formula to get

$$X(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega))\le e^{-\frac{\beta_2}{1-C_n}t}X(0,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega))+\frac{P}{1-C_n}\int_{-t}^0 R_1(\vartheta_s\omega)e^{\beta_2 s}ds\,,$$

by Theorem 4.1,there is

$$\int_{-t}^0 R_0(\vartheta_s\omega)e^{\beta_2 s}ds\le\int_{-t}^0\tilde{R}_1(s,\omega)e^{\beta_2 s}ds\le\int_{-\infty}^0\tilde{R}_1(s,\omega)e^{\beta_2 s}ds<+\infty\,,$$

among there,

$$\tilde{R}_1(s,\omega)=2(e^{\sigma|t|}r^{(1)}(A^{r/2}\omega))^2+(e^{\sigma|t|}r^{(1/2)}(A^{r/2}\omega))^2+(e^{\sigma|t|}r(A^{r/2}\omega))^2+r_0^2(\omega).$$

Because of $\varphi_0(\vartheta_{-t}\omega)\in\bar{B}(\vartheta_{-t}\omega)$, so there is $T_{\bar{B}(r_1(\omega))}>0$, so that for all $t\ge T_{\bar{B}(r_1(\omega))}$, there is

$$e^{-\frac{\beta_2}{1-C_n}t}X(0,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega))\le\frac{4P(1-C_n)}{\beta_2}r(A^{r/2}\omega)\,.$$

So,

$$\begin{aligned}X(t)&=\left\|A^{r/2}u_N\right\|_2^2+\left\|A^{r/2}\omega_N\right\|^2+\left\|A^{r/2}\theta_N\right\|^2+\left\|A^{r/2}\eta_N^t\right\|_{M^1}^2\\&\le\frac{4P(1-C_n)}{\beta_2}r(A^{r/2}\omega)+\frac{P}{1-C_n}\int_{-\infty}^0\tilde{R}_1(s,\omega)e^{\beta_2 s}ds=r_1^2(\omega).\end{aligned}$$

To sum up, the theorem is proved.

Consider the space M^{1+r} corresponding to variable η^t, refer to [9], note that for any $t\ge0,\omega\in\Omega$, there are

$$\eta_N^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega),s)=\begin{cases}u_N(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega))-u_N(t-s,\vartheta_{s-t}\omega,\varphi_0(\vartheta_{s-t}\omega)),s\le t\\ \eta_N^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega)),s\ge t.\end{cases}\tag{4.22}$$

$$\eta_{Ns}^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega),s)=\begin{cases}u_{Nt}(t-s,\vartheta_{s-t}\omega,\varphi_0(\vartheta_{s-t}\omega)),s\le t\\ 0,s\ge t.\end{cases}\tag{4.23}$$

Define collection $\tilde{B}(\omega)=\bigcup\varphi_0(\vartheta_{-t}\omega)\in\bar{B}(\vartheta_{-t}\omega)\bigcup_{t\ge0}\eta_N^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega),s)$,

where $\varphi=(u,\omega,\theta,\eta^t)^T$ is the solution of equation (4.8).Then theorem 4.5, (4.22) and (4.23) can be obtained

$$\max\left\{\left\|\eta_{Ns}^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega),s)\right\|_{\mathrm{M}^{1+r}}^2,\left\|\eta_N^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega),s)\right\|_{\mathrm{M}^{3+r}}^2\right\}\le2r_1^2(\omega),\forall s\ge0\,,$$

so $\tilde{B}(\omega)$ in $\mathrm{M}^{r+1}=L_\mu^2(R^+;\nu^{r+1})$ is bounded.

Know by Theorem 4.3 and Theorem 4.5

$$\sup_{\eta^t\in\tilde{B}(\omega),s\ge0}\|\eta^t(s)\|_2^2=\sup_{t\ge0}\sup_{\varphi_0(\vartheta_{-t}\omega)\in\tilde{B}(\vartheta_{-t}\omega)}\|\eta_N^t(t,\vartheta_{-t}\omega,\varphi_0(\vartheta_{-t}\omega),s)\|_2^2\le 2r_0^2(\omega).$$

Therefore, for any $\eta^t\in\tilde{B}(\omega)$,there is

$$\|\eta^t(s)\|_{\mathrm{M}^1}^2=\int_0^\infty\mu(s)\|\eta^t(s)\|_2^2ds\le 2r_1^2(\omega)\int_0^\infty\frac{1}{\sigma^2}e^{-\frac{s}{\sigma}}ds=\frac{2r_1^2(\omega)}{\sigma}.$$

That is, $\tilde{B}(\omega)\subset L_\mu^2(R^+;H^2(U))\cap H_0^1(U)$ is a bounded subset.

According to Theorem 2.2, $\tilde{B}(\omega)$ is tight in $L_\mu^2(R^+;H^2(U))\bigcap H_0^1(U)$.

The following proves that the random power system $\Phi(t,\omega)$ has a random attractor.

Theorem 4.6 Assuming conditions (1)-(4) hold, for any $\omega\in\Omega$, the random dynamic system $\Phi(t,\omega)$ generated by equation (4.8) has a compact random attractor set $\Lambda(\omega)\in V^0$, and has a random attractor $\mathrm{A}(\omega)\subseteq\Lambda(\omega)\bigcap B_0(\omega)$.

Proof: Any $\omega\in\Omega$, by theorem 4.5, let $B_r(\omega)=v^{r+2}\times v^r\times v^r$ be a random closed ball with a radius of $r_1(\omega)$, Definition $\Lambda(\omega)=B_r(\omega)\times\tilde{B}(\omega)$, then $\Lambda(\omega)\in D(V^0)$, because

$$v^{r+2}\times v^r\times v^r\to H_0^2(U)\times L^2(U)\times L^2(U)\quad\text{(Embed)}$$
$$B_r(\omega)\to H_0^2(U)\times L^2(U)\times L^2(U)\quad\text{(Embed)},$$

using theorem 2.2 again, $\tilde{B}(\omega)$ is compact in M^1, so $\Lambda(\omega)$ is compact in V^0. Next, prove the attractiveness of set $\Lambda(\omega)$.

For any $B(\omega)\in D(V^0)$

$$\lim_{t\to\infty}d_H(\Phi(t,\vartheta_{-t}\omega)B(\vartheta_{-t}\omega),\Lambda(\omega))=0.$$

From Theorem 4.5

$$\varphi_N(0,\omega,\varphi_0(0,\omega))=\varphi(0,\omega,\varphi_0(0,\omega))-\varphi_L(0,\omega,\varphi_0(0,\omega))\in\Lambda(\omega),$$

Combined with Theorem 4.4 there is

$$\mathrm{d}(\Phi(t,\vartheta_{-t}\omega)B(\vartheta_{-t}\omega),\Lambda(\omega))\le e^{-2\beta_1 t}(\|u_0\|_2^2+\|u_1+\varepsilon u_0\|^2+\|\theta_0\|^2+\|\eta_0\|_{\mathrm{M}^1}^2).$$

Therefore, the random dynamic system $\Phi(t,\omega)$ generated by equation (4.8) has a random attractor $\mathrm{A}(\omega)\subseteq\Lambda(\omega)\bigcap B_0(\omega)$.

To sum up, the theorem is proved.

REFERENCES

1. W.J,Ma,Q.Z.Ma,Attractors for Stochastic Strongly damped plate equations with additive noise,Electron.J.Differ.Equ.111(2013):1-12.
2. L.Aronld.random Dynamical Systems,Springer-Verlag,1998
3. A.Pazy.Semigroups of Linear Operators and Applications to Partial Differential Equations,Appl.Math.Sci.vol.44,Springer-Verlag,New York,1983.
4. M.H.Yang.J.Q.Duan,P.kloeden.Asymptotic behavior of solutions for random wave equation with nonlinear damping and white noise,*Nonlinear Anal*.12(2010):464-478
5. F. Zhou and M. Zhao, Random attractors for damped non-autonomous wave equations with memory and white noise, *Nonlinear Anal.* 120 (2015), 202–226
6. Shen Xiaoying,Ma Qiaozhen.The existences of random attractors for plate equations with memory and additive white noise,*Korean J .Math*.2016,24:461-481.
7. Hao Wu,Long-time behavior for a nonlinear plate equation with thermal memory.*J.Math.Anal.Appl.* 348 (2008) 650-670
8. Hans Crauel, Franco Flandoli ,Attractors for random dynamical systems. Probab. Theory Relat. Fields 100, 365-393 Springer-Verlag (1994)
9. Hans Crauel, Arnaud Debussche, and Franco Flandoli, Random Attractors .Journal of Dynamics and DiJferential Equations, Vol. 9, No. 2, 1997
10. Z.W. Shen, S.F. Zhou, W.X. Shen, One-dimensional random attractor and rotation number of the stochastic damped sine-Gordon equation, J. Differential Equations 248(2010): 1432- 1457.
11. Shen Xiaoying,Ma Qiaozhen.Existences of random attractors for weakly dissipative plate equations with memory and additive white noise,Comput.Math.Appl.2017,73:2258-2271.
12. S. Woinowsky-Krieger, The effect of an axial force on the vibration of hinged bars, J. Appl. Mech. 17 (1950) 35–36.
13. H.M. Berger, A new approach to the analysis of large deflections of plates, J. Appl. Mech. 22 (1955) 465–472.

14. Maurizio Grasselli,Jaime E.Munoz Rivera,Vittorino Pata,On the energy decay of the linear thermoelastic plate with memory.J.Math.Anal.Appl. 309 (2005) 1-14
15. V.Pata,A.Zucchi.Attractors for a damped hyperbolic equation with linear memory, Adv. Math. Sci.Appl.11(2001):505-529
16. M.E. Gurtin, A.C. Pipkin, A general theory of heat conduction with finite wave speeds, Arch. Ration. Mech. Anal. 31 (1968) 113–126.
17. S. Gatti, M. Grasselli, V. Pata, Exponential attractors for a conserved phase-field system with memory, Phys. D 189 (2004) 31–48.
18. C. Giorgi, M. Grasselli, V. Pata, Uniform attractors for a phase-field model with memory and quadratic nonlinearity, Indiana Univ. Math. J. 48 (1999) 1395–1445.
24.
19. V. Belleri, V. Pata, Attractors for semilinear strongly damped wave equation on R^3,Discrete Contin.Dyn.Syst.(2001) 719-735
20. Liu Y , Ueda Y . Decay estimate and asymptotic profile for a plate equation with memory[J]. Journal of Differential Equations, 2019, 268(5).
21. Liu R , Chai S , Cao X . Exact controllability of thermoelastic plate equation with memory[J]. Mathematical Methods in the Applied Sciences, 2019.
22. Al-Mahdi A M . General stability result for a viscoelastic plate equation with past history and general kernel[J]. Journal of Mathematical Analysis and Applications, 2020.
23. Baowei Feng,Long-Time Dynamics of a Plate Equation with Memory and Time Delay.New Series (2018) 49:395-418.