

Projecting the Growth Population of a Community Using Second Order Differential Equation

(A Case study of Damaturu Local Government Area)

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Abstract

Original Research Article

Population is a central issue affecting both physical and social environment. As the population of a community increases, there is greater inevitable pressure of demands on the environment. When the rate of population growth increases, the pressure on the natural sources, particularly basics needs like food and water increases. For effective allocation of limited available resources in any given community, it is important to know the population of that community. This will help the government in its community development planning. This project examined two major models (Malthusian model and Logistic model) for projecting population growth of a given community, and determined the model that best fit Damaturu local Government area of Yobe State within the period of 2009-2018. The logistic model considered being best model for population projection; the analysis shows that Njiwaji/Gwange ward has the highest growth rate of 0.0758 and the percentage increase within the range of 5.47% - 6.70% for the period under study. While, Bindigari/Pawari ward has the lowest growth rate of 0.09834 with of each ward in the study area, with the percentage increase within the range of 7.09% - 8.76% for the period under study. Therefore, logistic model should be employed for continuous projections of the population for proper appropriation of limited available resources.

Keywords: Differential Equation, Population, Projection, Damaturu, Growth.

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INTRODUCTION

In modern times, during the seventeenth century men become very interested in the study of human population purely from scientific point of view. The first person who undertook the said study was an Englishman called John Graunt (1620-1674) [1]. His work was truly standard; he introduced the first life table for the study of population of London in some details. Many then followed his footsteps in the study of human population, notable among them was Rev. Thomas Malthus. Keyfitz and Flieyer [2] were the first to analyze the human population in their work on world population growth and aging.

Information on the size, distribution and characteristics of a country's population is essential for describing and assessing its economic, social and demographic circumstances and for developing sound policies and programmes (in such fields as education and literacy employment and manpower, family planning, housing, maternal and child health, rural

development, transportation and high way planning, urbanization and welfare) aimed at fostering the welfare of a country and its population [3].

Information technology has radically extended possible use of population data beyond the traditional models. As population data have become more and more pervasive in our lives, so are the calls for increasing their scope, completeness, accuracy and validity, and for improving their national value and international comparability.

The study of human population is very important, as it tends to determine factors for distribution of available limited resources to communities. There seems to be no unified mathematical model for population figures or data developed with the aid of describing the statistical properties of data related to such population figures or data. Due to ever-increasing population growth naturally, it has become necessary to introduce the most common quantitative approach to

population dynamics, taking note of the different theoretical foundations and assumptions to such population.

Exponential growth occurs when there are no barriers to growth such as limited resources, and population grows slowly when it is small but as the population gets larger growth speeds up. Exponential grows fast in the beginning and slows at the end.

Logistic model is a population growth model that starts with minimum number of individual and reach is a maximum, depending on the earning capacity of the habitat. As the population approaches the carrying capacity, resources become scarce, competition for food and shelter increases between the individuals population of a population. Logistic is a different kind of behavior, which is more realistic to population not in ideal condition.

PROBLEM STATEMENT

As Damaturu population trends have shown ever increasing growth rates, while government policy has been to keep growth rates in check, the need to understand and model population size and growth becomes more and more pressing. Jajere *et al.*, [4] bemoaned a close to 50% expansion will be doubled if the present growth pattern continues by 2030; Damaturu will engulf many close by villages.

OBJECTIVES OF THE STUDY

The objectives of the study are to:

- Explore Damaturu population trends over a ten-year period (2006-2017).
- Explore differences in the results of the same period, using different modules; and
- Show how efficient is the logistic differential equation and Malthusian model.

SIGNIFICANCE OF THE STUDY

Governments have viewed population data, especially size and growth with serious concern, thought they differ in the method of curtailment. At the center of each of each method is the future of projection and estimation.

The study is significant in many ways: its perspective is to evaluate social and economic statistics relating to population size and growth over the study period (2006-2017). In this regard, the study aimed at recommending mathematical models, which governments could use for future population projection. In addition, to an extent, it would help the state and local governments in allocation of social amenities, which include education, housing and healthcare for its citizens (elderly and children).

Organizations relying solely on census information data to provide social services would use the models herein for their processes. Developed towns

or cities also need census data when planning technical and economic assistance to developing towns such as Damaturu. Finally, other researchers would use this study as their secondary source of literature.

SCOPE OF THE STUDY

Generally, the study covers the demographics data of the world's population with emphasis on the differences in characteristics between developed and develop and developing countries. However, specifically the study focuses on Damaturu demographics trends between 2009 and 2018. We will consider basic components of demographic analysis, giving detailed qualitative examination of population size and growth during the said study period.

We shall put forward two mathematical models exponential model and logistic growth model in relation to population size and growth from 2009 to 2018. This aspect first explored basic definitions of the models followed by a further examination of the relationship between population growth rate and the population models. We shall conclude with a recommendation of application of theory and practice in future prediction and projection of population.

LITERATURE REVIEW

Islam [5] justified the use of mathematical models "for the estimation of population projection and estimation. In fact, mathematical model is essentially an endeavor to find the relationship and their dynamic behavior among the various elements in the Demography". In the present study, the structures of the models are deterministic in that the functional relationship between different variables that take definite values would be explored.

Merritt [6], posited out that "the process of translating a real-life problem into a mathematical form (model) can give a better representation and solution of certain problems, with or without some difficulties. Now the question is what mathematical model ore models could provide the right information about population data and population growth for effective and efficient development agenda? In addition, what demographic factors are more prevalent in Damaturu population growth between the periods of 2007 and 2016?

Meyer *et al.*, [7] demonstrated that simple mathematical models could account for growth-increase or decline-in human population. The number of people that the environment can support, called the carrying capacity, gives an interesting background to the population research survey and evaluation of the data.

Jajere *et al.*, [4] examined the geographical determinants of urban growth of Damaturu town over the past years. They discovered that the growth of the town from 1991 to 1999 was influenced by such factors

as the location of the administrative offices and housing states at the fringe of the town, classification of the urban land based on functions. In addition, they reported that the growth of Damaturu town from 1999 to 2005 was influenced by population growth and increased agricultural production. However, the geographical factors responsible for the growth of the town within 2005 to 2009 include the construction of housing units along major roads (ring road) that bypass the town.

Constructing a model entails the application of method developed for the study of dynamic systems to data provided poorly by demography and anthropology. Physicists poorly know these data and one of the purposes of this research is to introduce them to a new set of problems in a number of cases, it will be possible to identify concepts and recognize well-known ideas in a new setting. It should be kept in mind that both the data and model itself are but crude image of the real would. In these studies, one of the main difficulties is that of taking ideas and Methods from one field and transferring them to another. This process is perhaps best developed in mathematics when models are suggested, and in theoretical physics, although it is difficult to say when a model can reach the status of a theory, serve not as a description of events, but lead to greater insight into the nature of the phenomena treated.

UNLIMITED GROWTH

The first mathematical model exponential model is based on the work by Rev .Thomas Malthus (1766-1834). In the principle of population essay that he published in 1798, Malthus explained simple terms, his theories of human population growth and the connection between over-population and misery. One of the fundamental concepts that he brought up is that of unlimited population growth.

The mathematical model based on these ideas is that the population size for one generation depends on the size of the previous generation, and it is a multiple. This mathematically expressed by the following equation:

$$P_{t-1} = r \times P_t \text{-----} 2.1$$

Where:

- t**- Is the time (which could be minute, year etc, depending on the species under consideration),
- P_t**-Is the population size at time, t. The unit of time could be hours, days, year, etc.
- P_{t-1}**-Is the population size at the next time. In addition, it could be the next hour, next day, next year etc.
- r** -Refers to the Malthusian factor, is the multiple that determines the growth rate.

This type of equation is called a differential equation because it allows you to find out the value of P at different discrete time interval, say at year 3,4,5,10

etc. You could not use this to find out the size of P when t=3.57, because t=3.57 does not represent a discrete time. The value of r in the previous equation has a strong impact of how fast population will grow; the mathematical name for this type of growth is exponential growth.

This growth equation can be used in cases where there is truly this type of growth. For example, when a new specie arrive to an island there is plenty of food, perfect condition for reproduction and no predators, one can certainly observe this (at most perfect) type of growth; although not forever. That is why other mathematical models were developed.

NATURAL EQUILIBRIUM

Robert H *et al.*, [8], started their work in the analysis and population data through the analysis of a lot experimental data, they discovered that population size remain steady, even though the exact species carried. They were able to develop a very intricate mathematical model for this equilibrium that was published in 1962 (not presented here). Their conclusion after their work with many different species and many experiments was that: nature was this great tendency of balancing things out and reaching a very harmonious equilibrium. If nature were left alone, equilibrium would exist and population would remain close to them.

LIMITED GROWTH

The problem with equation (2.1) model is that the population continues to grow unlimited over time. A major contribution came from Pierre Francois Verhulst, a scientist interested in population growth. He showed in 1846 that the population growth not only depends on the population size but it is from its upper limit.

Let us look at the mathematics behind this. Do you remember the definition of carrying capacity introduced in the fundamentals of population section? Carrying capacity, k, is the maximum population size that a given habitat can support. It will be denoted that ask if the population is far below k, it would tend to grow rapidly, but as it approaches k, the growth will slow down. If the population size would exceed its upper limit beyond k, the growth would actually be negative! In order to model this, Verhulst modified equation (2.1) to make the population size proportional to both the previous population and the new term:

$$\frac{(k - P_t)}{k} \text{-----} (2.2)$$

This term reflects how far the population is from its maximum units so that the equation using this new term is named after Verhust as Verhust population model.

$$P_{t+1} = r \times P_t \frac{(k - P_t)}{k} \text{-----} (2.3)$$

This equation is also known as a logistic differential equation, comparing it with equation (2.1) it is non-linear in the sense that one cannot simply multiply the previous population by a factor. In this case, multiplied the population P_t on the right hand side of the equation by itself. One good thing about this equation is that it is relatively easy to solve and easy to see how it behaves by looking at a chart. The logistic growth equation is a useful model for demonstrating the effect of density-dependent mechanism in population growth subject to dynamics of population complexity.

INTERSECTING AND COMPLEX ASPECT OF THE BASIC LOGISTIC EQUATION

Robert May started his carrier as a physicist but then did his post-doctoral work in Applied mathematics. He became very interested in the mathematical explanations of what enables competing species to co-exist and the mathematics behind population growth. Most of the findings on the basic Logistic equation depend on the value of r .

AGE STRUCTURE DEPENDENCY

Up until now, we assume that the size of the population in the next time depends on one factor affecting the whole size of the current population. It does not take into account the fact that not all members of the population will reproduce depending on the individual's age. In essence, it uses a simplistic probability average. Equation (2.1) represented the function expressed below:

$$P_{t+1} = f(P_t) \text{ ----- (2.4)}$$

Equation (2.4), simply says that the population for the next time period is a function (or depends) on the present population. To take into account the age structure dependency, equation (2.4) becomes:

$$P_{t+1} = f(P_t, P_{t-1}, P_{t-2}, \dots, P_{t-m}) \text{ ----- (2.5)}$$

This means that the population is now a function of the current population, the population of the previous year. f denotes a genetic function and it is different in each equation (2.4) and (2.5) above. In addition, the function for each of the previous population could be very difficult to solve i.e. the complexity of the mathematical models.

By now the mathematical models of the population growth, when incorporating many of the issue described in the previous sections, can become quiet complex, especially when the models are map to real physical characteristics of the natural world. Just imagine a predator that switches prey from time to time. It can be very challenging but when one finds a mathematical model that works, it is rewarding! Here follows an interesting example.

Maria J Malevich, an ecologist, has studied a very colorful fishpond in the Great Barrier Reef in Australia known as Damsel fish. These damsel fish lays their eggs in the reef's bottom, the full moon cause the larvae to hatch about once a month. The larvae leave the reef and about 20 days later return as mature larvae. Milicich was working with data consisting of eggs released from nest, number of larvae that hatched and the number of matured larvae that return that to the reef.

Looking specifically at the data for the mature larvae, Milicich expect the number to be equilibrium points; however, she found that the number varied dramatically. Some month she found only a few larvae and others a thousand of them. The feeling was that there fluctuations were random and attributed to weather changes; unknown diseases; sampling error or what is referred to as environmental noise (unknown causes in the environment).

Milicich used linear models, which did not work very well so decided to model the number of matured larvae using a non-linear model, her approach was to compare linear and non-linear model to predict the number is more accurate. With her team played with hundreds and hundreds of environmental variable, in trying to improve the predictability of the non-linear model. They finally came up with a model that considered the following.

- a. The moon's phase.
- b. The water turbulence around the reef.
- c. The wind speed.

The non-linear model proved to be significantly better than the previous linear model. Not only that, but also these three physical variables made perfect ecological sense. They were able to understand the roles that the moon, water turbulence and wind have on the overall damselfish population process. When the three combine positively, the larvae reach maturity in great numbers, but when they work against each other, their numbers are very small. A predator and prey model.

Taking into account two populations, where one is the prey and the other predator, is similar to the previous case.

The difference is that the species are at different tropic levels. As there are more predators, the prey population decreases, so also as the prey increase, the predator population increases as well.

The equation to model this behavior looks like this:

$$p_{t+1} = r_1 \times \frac{p_t(k_1 - q_t)}{k_1} - f_1 \times p_t \times q_t \text{ ----- (2.6)}$$

$$q_{t+1} = -r_2 \times \frac{q_t(k_2 - q_t)}{k_2} + f_2 \times p_t \times q_t \text{ ----- (2.7)}$$

An American Alfred Lotka (in 1925) and an Italian Vito Volterra (in 1925) discovered the equation and known as the Lotka- Volterra equations. You will notice that the prey population p increases, just as before, based on the term:

$$r_1 \times \frac{p_t(k - p_t)}{k} \text{----- (2.8)}$$

However, is destroyed dependent on the term?

$$-f \times p_t \times q_t \text{-----(2.9)}$$

Similarity the predator population, q , decreases based on term:

$$r_2 \times \frac{q_t(k_2 - q_t)}{k_2} \text{----- (2.10)}$$

However, increase based on the term:

$$f_2 \times p_t \times q_t \text{----- (2.11)}$$

It is interesting to note that the last two terms on both equations (2.6) and (2.7) are dependent on both the prey and predator populations. The Lotka-Volterra equations have evolved and there are many different versions of equations based on the concepts introduced 75 years ago.

In conclusion, several different concepts and models were described in the previous paragraphs, but each was separately described. A real and accurate population model would need to consider most of them at the same times. Imagine many interdependent species, and it is easy to see how the mathematical models can get quiet complicated.

General growth rate equation:

A general formula that model the growth rate, is written as shown below from the work on the previous section

$$P_n = P_0 \left(1 + \frac{f}{2}\right)^n \text{----- (2.12)}$$

Where P_0 represents the initial population, n is the number of years, f is the birth rate, and P_n is the population in year n .

The core principles of Malthus are:

1. Food is necessary for human existence. Human population tends to grow faster than the power of the earth to produce sustainable resources. Moreover, that the effects of these two unequal powers must be kept equal. Since human tend not to limit their population size voluntarily, population reduction tends to accomplished through the “positive” checks by nature such as famines, diseases, poetry and war.
2. Depending on the species that one is trying to model, the model requires an equation that can find out the sizes of the population at any point in time, called a differential equation.
3. It is called exponential growth because there is another equation, based on the constant, e , elevated to the exponent rt , which produces the same curve shape:

$$P_t = P_0 e^{rt}$$

4. The theories of learning is applicable to Verhulst’s equation. Learning depends on the amount of information learned before the learning first increase, but after sometimes, the leaner becomes saturated, so that more effort is required to acquire more knowledge.
5. The steady values where the population final stable values, the population sizes will below then settle in the long- run, are referred to as attractors because regardless of where the population starts, above or below the final stable value, the population sizes will be attracted of this value.

MATHEMATICAL MODELING METHODOLOGY

The mathematical modeling methodology enable to transform real world problem to mathematical models base on certain assumption about the real problem concerned, and to solve and interpret the mathematical solution.

This work, predicts the population of Damaturu local government area over a period of ten (10) years using the most efficient mathematical models. Below are the steps involved in mathematical modeling.

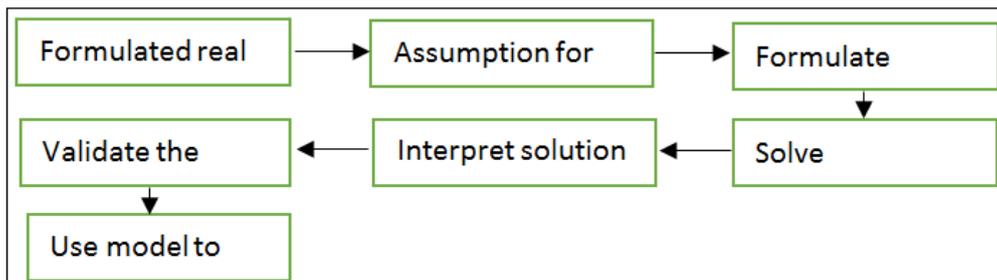


FIGURE 3.1: Steps Involved in Formulating a Mathematical Model

ASSUMPTION OF THE MODELS

The assumptions for the two models to be considered in this research work are listed below.

1. For the Malthusian (exponential) models to be realistic, Thomas Malthus made the following assumptions.
 - For all intervals, the birth and death rate are consider same.
 - The environment inhabited by the population under study, should considered close in term of migration.
 - All the individual of the population considers reproduction.
 - That the birth and death are proportional to the population size and the time interval.

2. For the logistic models to be realistic, the following assumption must be consider.
 - For all intervals, the birth rate and death rate are consider the same.
 - The environment inhabitant by the population under study is considered to be closed in term of migration.
 - The population is only concerned with the people living in that environment.
 - The carrying capacity may very over time.

The population of the community at time (t) is less than the carrying (k).

THE MODEL EQUATION

The equation of the models compared is show below:

MALTHUSIAN (EXPONENTIAL) MODEL

From the assumption mentioned above; it is assume that both the birth and deaths are propositional to the population sizes and the time interval.

Birth = aNdt
 Death = bNdt

Where b and a are constant

Thus the increase, say dN, in the total population, in the time interval dt is given by

$Dn = aNdt - bNdt = rNdt$
 Where $r = a - b$

Hence,

$dN = rNdt$ ----- (3.1)

Dividing equation (3.1) by dt and taking the limit as dt - 0, lead to the differential equation

$\frac{dn}{dt} = rN$ ----- (3.2)

Multiplying equation (3.2) by $\frac{dt}{n}$, we obtain.

$\frac{dN}{N} = rdt$ ----- (3.3)

And integrating both side we have

$og N = rt + A$ ----- (3.4)

And if at t=0, N=N₀, we have $log N_0 = A$, thus

$N_t = N_0 e^{rt}$ ----- (3.5)

Where,

- r=growth rate.
- N_t=the number of people at the arbitrary time,
- t.
- N₀ = total number of people at the initial time.
- t = Time of growth.

THE LOGISTIC MODEL

$\frac{dN}{dt}, rN (\frac{K-N}{K})$ ----- (3.6)

Where,

- r= b(t) – d(t)
- N_t = the number of people at the arbitrary time,
- t.
- N₀ = total number of people at the initial time.
- t=growth rate.
- k = carrying capacity.
- b(t) = birthrate at time t
- d(t) = death rate
- N_t = the number of people at the arbitrary time,
- t.

SOLUTION OF THE EQUATION 3.6

$\frac{dN}{dt} = rN \frac{(K-N)}{K}$ ----- (3.7)

We used integrating factor method to solve the above equation by first transforming it into Bernoulli's equation form of

$\frac{dy}{dx} + p(x) y = Q(x) y^n$ ----- (3.8)

We therefore have

$\frac{dN}{dt} = rN - \frac{rN^2}{k}$ ----- (3.9)

The equation can also be reduced into its linear form as

$N^2 \frac{dN}{dt} + rN^{-1} = \frac{r}{k}$ ----- (3.10)
 Let $z = N^{1-n}$

Where n=2

$z = N^{1-2}$
 $z = N^{-1}$ ----- (3.11)

By implicit differentiation on equation (3.11)

$\frac{dz}{dN} = - N^{-2} \frac{dN}{dt}$ ----- (3.12)

Making the $\frac{dN}{dt}$ the subject of the formula, we have

$\frac{dz}{dN} = - N^2 \frac{dz}{dN}$ ----- (3.13)

Substituting the form of $\frac{dN}{dt}$ of equation (3.11) and N^{-1} of equation (3.9) into

$$\frac{dz}{dN} + rz = \frac{r}{k} \text{----- (3.14)}$$

This is a linear equation

To get the integration factor the formula $I.F = e^{\int P dx}$ is used as follows

$$I.F = e^{\int r dt}$$

$$I.F = e^{rt} \text{----- (3.15)}$$

Multiplying equation (3.14) by the integrating factor, it result to

$$e^{rt} \frac{dz}{dN} + e^{rt} rz = \frac{r}{k} e^{rt} \text{----- (3.16)}$$

Or

$$d \left(\frac{e^{rt} z}{dN} \right) + = \frac{r}{k} e^{rt} \text{----- (3.17)}$$

Integrating both sides of equation (3.17) to obtain

$$e^{rt} z = \frac{r}{k} \left(\frac{e^{rt}}{r} + c \right) \text{----- (3.18)}$$

Opening the bracket and dividing the equation by e^{rt} , we have

$$z = \frac{ert+rc}{ertk} \text{----- (3.19)}$$

Substituting the form z of equation (3.11) into equation (3.19), we have

$$N^{-1} = \frac{ert+rc}{ertk}$$

Taking their inverse, we have

$$N = \frac{ertk}{ert+rc} \text{----- (3.20)}$$

Making C the subject of the formula as $t = 0$

$$N_0 = \frac{erok}{ero+rc} \text{----- (3.21)}$$

$$N_0 = \frac{k}{1+rc} \text{----- (3.22)}$$

Multiplying through by $1 + rc$, we have

$$N_0 + N_0 rc = k \text{----- (3.33)}$$

$$N_0 rc = k - N_0 \text{----- (3.34)}$$

$$C = \frac{k - N_0}{N_0 r} \text{----- (3.35)}$$

Substituting the form of c into equation (3.20)

$$N_{(t)} = \frac{dy}{ert + \left\{ \frac{k - N_0}{N_0} \right\}} \text{----- (3.26)}$$

Thus;

$$N_{(t)} = \frac{K}{1 + \left\{ \frac{K - 1}{N} \right\} e^{-rt}} \text{----- (3.27)}$$

METHOD OF DATA COLLECTION

The methods of data collection use are primary and secondary data. The primary data will be collection from the national population commission in the local government and in the National population commission Yobe state office while the secondary data will be form any relevant material of the project.

DATA PRESENTATION

In this part the comparison of Malthusian model and the logistic model projection to the actual population figure were considered. However, the model with minimum error is chosen for the projection of the population of Damaturu local government area for the period of ten years. For the comparison of the efficiency of the two models, we made use of the U.S population Logistic (Census figure) which are done from the internet (<http://www.wikipedia.com>) while, for the projection of the population of Damaturu Local Government Area, used the data obtained from the National population commission Yobe State office.

Table-4.1.1: Damaturu Local Government Area census based on Wards

S/No	WARD	1991 (Population)	2006 (Population)	LAND AREA
1	Bindigari/ Pawari	2735	9386	60.5548
2	Damakasu	2059	6321	43.5931
3	Damaturu Central	3984	10193	63.7061
4	Gambir/Maiduri	2195	5864	41.0070
5	Kalallawa/Gabai	2157	7261	47.7697
6	Kukareta/Warsala	2364	7458	49.0658
7	Maisandari/WaziriIbr. Est.	3315	8990	57.6282
8	Marfa Kalam	2292	7151	47.0461
9	Nayinawa	3538	9390	60.5806
10	Njiwaji/Gwange	3466	9747	62.8839
11	Sasawa/Kabaru	2215	6253	42.2500

Table-4.1.2: U.S Population Census Logistics

S/NO	YEAR'S	U.S POPULATION (10 ⁶)
1	1870	3.9
2	1880	5.3
3	1890	7.2
4	1900	9.6
5	1910	12.9
6	1920	17.1
7	1930	23.2
8	1940	31.4
9	1950	38.6
10	1960	50.2
11	1970	62.9
12	1980	76.0
13	1990	92.0

COMPARING THE EFFICIENCY OF THE TWO MODELS

Considering, the Malthusian mathematical model of equation (3.5) in 3.2.1 of chapter three.

$$N_{(t)} = N_0 e^{rt}$$

Making r the subject formulae of the above equation (3.5), we have

$$R = tx \ln \left(\frac{N(t)}{N_0} \right) \text{----- (3.28)}$$

Applying the above equation to get the growth rate of U.S population using the data table 4.2.2

$$N_0 = 3.9 \times 10^6$$

$$N_2 = 5.3 \times 10^6$$

$$t = 1$$

Therefore the growth rate is
r = 0.3067

Using the above calculated data on equation (3.5) to project the U.S population from 1900 – 1990.

$$N_{(t)} = N_0 e^{rt}$$

Where, t increases as the years increases.

The results are represented in the table below

Table-4.2.1: Malthusian Model Projected Figures

YEAR'S	U.S actual figure (Million)	U.S Projected figure (Million	ERROR
1900	9.6	10.0	0.4
1910	12.0	13.7	0.8
1920	17.1	18.7	1.6
1930	23.2	25.6	2.4
1940	31.4	35.0	3.6
1950	38.6	47.8	9.2
1960	50.2	65.5	15.3
1970	62.9	89.6	26.7
1980	76.0	132	56.5
1990	92.0	167.6	75.6

$$N_{(t)} = \frac{k}{1 + \left(\frac{k}{N_0 - 1}\right)e^{-rt}}$$

$$t = 1$$

$$k = 197 \times 10^6$$

Marking r the subject of the above equation (3.27), we have

Therefore the growth rate of the United State population is r = 0.3134

$$r = t \times \ln \left[\frac{N \left(\frac{k}{N_0 - 1} \right)}{k - N} \right] \text{----- (3.29)}$$

Applying the above calculated data to equation (3.27) to project the population of U.S from 1900-1990 where, t increases as the years increases. The results are represented in the table below.

$$N_0 = 3.9 \times 10^6$$

$$N_2 = 5.3 \times 10^6$$

Table-4.2.2: Logistic Model Projected Figures

YEAR'S	U.S actual population (Million)	U.S Projected population (Million	ERROR
1900	9.6	9.7	0.1
1910	12.0	13.7	0.4
1920	17.1	17.7	0.3
1930	23.2	23.6	-0.2
1940	31.4	30.2	-1.2
1950	38.6	38.1	-0.5
1960	50.2	49.9	-0.3
1970	62.9	62.4	-0.5
1980	76.0	76.5	0.5
1990	92.0	91.6	-0.4

From table 4.2.1 and table 4.2.2 we can say that the logistic mathematical model is the most efficient model, since it has the minimum error; hence the logistic mathematical model is going to be used for the projection of Damaturu local government area.

DAMATURU LOCAL GOVERNMENT AREA PROJECTION

To make use of logistical mathematical model, first of all the carrying capacity for the wards is obtained by multiplying the land area by 556 (which is the maximum population per square kilometer). The results are represented in the table below.

Table-4.4.1: Wards Carrying Capacity

S/NO	WARDS	LAND AREA	CARRIYING CAPACITY
1	Bindigari /Pawari	60.5548	33668
2	Damakasu	43.5931	24237
3	Damaturu Central	63.7061	35420
4	Gambir /Maiduri	41.0070	22799
5	Kalallawa /Gabai	47.7697	26559
6	Kukareta /Warsala	49.0658	27280
7	Maisandari /WaziriIbr. Est.	57.6282	32041
8	Marfa Kalam	47.0461	26158
9	Nayinawa	60.5806	33683
10	Njiwaji /Gwange	62.8839	34963
11	Sasawa /Kabar	42.2500	23491

To make use of the logistic mathematical model we secondly calculated for the growth rate of Damaturu local government area wards by marking r the subject of the formula from equation (3.27),

$$N_{(t)} = \frac{k}{1 + (\frac{k}{N_0 - 1})e^{-rt}}$$

We have equation (3.29)

$$r = t \times \ln \left[\frac{(N_{(t)} \cdot k - 1)}{k - N_{(t)}} \right]$$

Where, t varies as the year changes.

Using the data from table 4.1.1 and table 4.4.1, the population growth for the wards in Damaturu local government area is calculated. The results are represented in the table below.

Table-4.4.2: Wards Growth Rate

S/No	WARD	GROWTH RATE (r)
1	Bindigari /Pawari	0.09834
2	Damakasu	00890
3	Damaturu Central	0.0773
4	Gambir /Maiduri	0.0786
5	Kalallawa /Gaabai	0.0966
6	Kukareta /Warsala	0.0918
7	Maisandari /WaziriIbr. Est.	0.0812
8	Marfa Kalam	0.0910
9	Nayinawa	0.0795
10	Njiwaji /Gwange	0.0758
11	Sasawa /Kabar	0.0832

Using the data from table 4.1.1 and table 4.4.2 on logistic mathematical model, the projected population of Damaturu local government area from

2008 to 2017 is given in the tables below. The value of t increases as the years increases.

Table-4.4.3: Projected Population for the First Five Years

WARD	YEAR				
	2007	2008	2009	2010	2011
Bindigari /Pawari	10052	10121	10817	11539	12285
Damakasu	6737	7170	7619	8084	8564
Damaturu Central	10,754	11333	11929	12541	13168
Gambir /Maiduri	6206	6561	6928	7307	7697
Kalallawa /Gabai	7771	8302	8853	9423	10010
Kukareta /Warsala	7956	8473	9009	9563	10133
Maisandari /Waziri Ibr. Est.	9515	10058	10618	11195	11786
MarfaKalam	1625	8117	8626	9152	9693
Nayinawa	9928	10485	11059	11650	12256
Njiwaji /Gwange	10280	10830	11397	11979	12576
Sasawa /Kabar	6635	7031	7441	7864	8299

Table-4.4.4: Projected Population for the Second Five Years

WARD	YEAR				
	2012	2013	2014	2015	2016
Bindigari /Gwange	13052	13838	14640	15454	16276
Damakasu	9057	9562	10077	10601	11132
Damaturu Central	13807	14458	15119	15789	16465
Gambir /Maiduri	8098	8508	8927	9352	9788
Kalallawa /Gabai	10613	11229	11588	12489	13128
Kukareta /Warsala	10718	11315	11923	12539	13161
Maisandari /WaziriIbr. Est.	12391	13008	13635	14271	14914
MarfaKalam	10248	10815	11392	11977	12568
Nayinawa	12876	13508	14151	14803	15463
Njiwaji /Gwange	13186	13809	14442	15085	15735
Sasawa /Kabar	8746	9203	9669	10142	10622

INTERPRETATION OF THE RESULT

From table 4.1.1 and table 4.1.2 it can be seen that the logistic mathematical model has the minimum error, which means that it is the best mathematical model population projection concerned. From table 4.4.2 it can be seen that Njiwaji /Gwange ward has the highest growth rate of 0.0758 which causes its population to be increasing by 5.47% to 6.7% for the ten years period of projection, where as Bindigari/Pawari ward has the most less growth rate of 0.09834 in Damaturu local government area which causes the population to increase within the range of 7.09% to 8.76% for ten years period. Nayinawa ward has a growth of 0.0795 and a percentage increase ranging between 5.73% to 7.02%, Kukareta/Warsala ward has a growth rate of 0.0918 and a percentage increase in population ranging between 6.7% to 8.3%, and Maisandari/ Waziri Ibrahim estate has a growth rate of 0.0812 and a percentage increase in population of 5.8% to 7.1%. In the analysis above, Damaturu central ward has the highest population figure among the wards, but it is the second highest growth rate of 0.0773 and makes its population to increase by 5.5%- 6.64% within ten years period. From table 4.4.3 and 4.4.4, the result obtained shows that the population of the wards

in Damaturu local government area increases at a decreasing rate which means that the population percentage increase, decreases as the years goes by (increase).

SUMMARY

The introduction was outlined the objectives and significant of the researcher work to economic activities. The reviews of literatures on mathematical model of population. The method and the steps involve in the formulation of mathematical model of population and the assumptions considered in each model were outlined.

Finally, in results and discussion, we went further by showing how efficient logistic mathematical model is in the projection of population than the Malthusian mathematical model. From the comparison, it has shown that logistic mathematical model has the minimum error compared to the actual population figure than the Malthusian mathematical model. Logistic mathematical model was used to project the population of Damaturu local government area wards, which shows how the population of each ward grows within the ten years period of projection.

CONCLUSION

Based on the analysis in chapter four we conclude that, the logistic mathematical model stands as the best mathematical model for the projection of Damaturu local government area ward population because it minimize error.

RECOMMENDATION

From the result of the previous part, the following recommendations were put forward as follows.

- Damaturu local government area should allocate resources based on the wards population that is the ward with the highest population should be giving more consideration.
- Make use of the result to plan for the prosperity of the wards and the local government in general.
- Appropriate its plans considering the influx of migrants in to its wards as a result of the preventing insurgency (Boko Haram).

APPENDIX

Table-4.1.2: Considering Malthusian Model to Project U.S Population

S/NO	YEAR'S	U.S POPULATION (X10 ⁶)
1	1870	3.9
2	1880	5.3

Considering, the Malthusian mathematical model of equation (3.5) in table 3.21 of chapter three.

$$N_{(t)} = N_0 e^{rt}$$

Making r the subject of the above equation (3.5), we have

$$r = t \times \ln \left[\frac{N(t)}{N_0} \right]$$

Applying the above equation to get the above equation to get the growth rate of U.S population using the Data in Table 4.2.2

$$N_0 = 3.9 \times 10^6$$

$$N_2 = 5.3 \times 10^6$$

$$t = 1$$

Substituting the value of the variables in to equation (3.6), we have

$$r = 1 \times \ln \frac{5.3 \times 10^6}{3.9 \times 10^6}$$

$$r = 1 \times \ln (1.3590)$$

Hence, r= 0.3067

Now, project for year 1900, the following data will be used.

$$N_0 = 3.9 \times 10^6$$

$$t = 3$$

$$r = 0.3067$$

$$N_{(t)} = N_0 e^{rt}$$

$$N_3 = 3.9 \times 10^6 \times e^{0.3067 \times 3}$$

$$N_3 = 3.9 \times 10^6 \times 2.51$$

$$N_3 = 10.0 \times 10^6$$

While considering the logistic mathematical model, equation (3.26).

$$N_t = \frac{k}{\left[1 + \frac{k}{N_0} - 1\right] e^{-rt}}$$

Making r the subject of the above equation (3.25), we have

$$r = t \times \ln \frac{N \left[\frac{k}{N_0} - 1 \right]}{k - N}$$

$$N_0 = 3.9 \times 10^6$$

$$N_2 = 5.3 \times 10^6$$

$$t = 1$$

$$k = 197 \times 10^6$$

Substituting the value of the variable above in to the equation above

$$r = 1 \times \ln \left[\frac{5.3 \times 10^6 \left(\frac{197 \times 10^6}{3.9 \times 10^6} - 1 \right)}{197 \times 10^6 - 5.3 \times 10^6} \right]$$

Therefore the growth rate, r of the US population is 0.3134.

Now, projecting for year 1900, the following data will be used.

$$N_0 = 3.9 \times 10^6$$

$$t = 3$$

$$r = 0.3134$$

$$k = 197 \times 10^6$$

Substituting them into equation (3.26), we have

$$N_3 = \frac{197 \times 10^6}{\left\{1 + \frac{197 \times 10^6}{3.9 \times 10^6} - 1\right\} e^{-0.3134 \times 3}}$$

$$N_3 = 9.7 \times 10^6$$

Therefore, the projected population for 1900 is 9.7 million people.

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