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Quantum Statistics and Zeta Functions

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Abstract **Original Research Article**

The present paper aims to bridge the border between real physical-constant-based real fields and dimensionless complex fields. Unified fields have coupling constants differing by hundreds of orders of magnitude which are catchable by dimensionless fields. The border between a Fatou set and a Julia of quadratic root finding within a Newton iteration yields complex curvatures, masses and a generation of Minkowski spacetime. A scan algorithm in the vicinity of nontrivial zeros of zeta functions yields stable laps with fluctuations of Legendre modules at the critical strip.
Keyword: zeta function, quadratic Newton root finding, Minkowski spacetime, Green's function singularity.

1. INTRODUCTION

The present paper aims to bridge the border between real physical-constant-based real fields and dimensionless complex fields. Unified fields have coupling constants differing by hundreds of orders of magnitude which are catchable by dimensionless fields. This approach links fields to entropy and information maxima. In [1] the Green's function has been related to substitutions of binary invariants. This algebraic-binary invariant description of bifurcation adds equivalent elliptic curves as an infinite source of k-components. Binary invariants must overwhelm complex half-differentials [2] [3]. The first nontrivial case is complex Newtonian root finding $\mathrm{N_q}(z)$ for quadratic q(z) [4]. An angle in a planar triangle at $k\rightarrow\infty$ decides whether $\mathrm{N_q}(z)$ tends to z_1 or z_2. This statistical generation rate is used to define a thermal $\delta(\varphi)$-function source for unified fields [5]. $\delta(\varphi)$ is related to rotations about $\pm\pi$ in interval [0,1] which gives a cos representation of $\delta(\varphi)$ [5]. A root finding algorithm for entire transcendent function $\phi^{(\infty)}(z)\in\mathbb{C}$ by setting $\mathrm{q}(z)\rightarrow\phi^{(\infty)}\overline{\phi}^{(\infty)}$ searches simple zeros in abelian bases $\widehat{\mathrm{B}}$ of binary γ-eigenstates of periodic continued fractions (CF). Conformal steps $\phi_{k+1}=z_{k+1}=\gamma\circ z_k$ and $\phi_{k+2}(\phi_{k+1}(\phi_k(z_k)))$ is as well variables z_k and functions ϕ_k which create a source term in real interval [0,1]. A threefold map allows to define a plus sign [2]. This transformation of half-differentials is capable to be linked with spinors in 3+1 dimensions [2] [3]. The present algebraic approach connects a difference $z_{k+1}-z_k$ with the (inverse) Green's function in 3·4 complex dimensions. Section 2 describes the combined map of invariants. Section 3 describes a source term due to permutations of polynomial roots. Section 4 introduces details of scanning processing. Section 5 calculates metrical invariants and Section 6 shifts the origin of Minkowski spacetime $\mathcal{M}$ to a one-dimensional border between the Julia set $\mathcal{J}(N_q)$ and the Fatou set $\mathcal{F}(N_q)$ of q(z). Section 6 defines $\mathcal{M}$ by (15) for a two-step conformal mapping. Section 5 claims a number field regulator- Lagrangian relation. Section 3 explains Bethe-Salpeter equation and Dyson equation by periodic CF in γ [6] [1] [7].

2. THE COMBINED MAP

The algorithm proves a window in Newton root finding

$$z_{k+1}\leftarrow \mathrm{N_q}(z_k)=z_k-\frac{q(z)}{\partial_z q(z)}\Big|_{z=z_k}\simeq \mathrm{F}^{(n)}(w,z_k)=\gamma^{(n)}(w)\circ z_k \qquad (1)$$

in combination with binary invariant polynomials of a number field $\mathbb{K}$ for a unique factorization domain (UFD). The windows create entropy sources by equivalent permutations for an infinite number of Poncelet polygons. A spinor is defined by the standard mapping between cubic and quartic roots for elliptic curves in $\mathbb{K}$ which is a cyclic permutation. First the division in (1) is discussed in conjunction with a UFD. A convergence of quadratic Newtonian root finding $\mathrm{N_q}(z)$ to one fixed point z_1, z_2 [4]

$$q(z) = (z - z_1)(z - z_2) = e^{\int_0^z \frac{dz}{F^{(3)}(w,z)-z}} \qquad (2)$$

requires that $\partial_z N_q(z) < 2$. A regular quadratic map $\gamma^{(3)}(w)$ which transforms cubic roots e_i is seen as Feigenbaum renormalization [8]. Convergence is discussed in dependence on the angle $\mathrm{Imln}\frac{z_1-z_2}{z_0-z_3}$ of a planar triangle [4]

$$T(\Delta z, z_1, z_2) = T\big(\Delta z = z_0, z_1, z_2, z_3 = ½(z_1+z_2)\big) = T(z_q).$$

Subsequent rational Hermite-Tschirnhausen transformations of a polynomial $\phi^{(n)}(z) = \sum_{i=0,\dots,n} a_{n-i}\, z^i$ are of degree n-1

$$F^{(n)}(w,z) = \phi^{(n)}(w)/(w-z) - ⅓\, \partial_w \phi^{(n)}(w) = \gamma^{(n)}(w) \,?\, z \qquad (3)$$

Case n=3 creates transvectants of $\phi^{(3)}(z)$ with $z \wedge w$ in a cubic number field $w, z \in \mathbb{K}[\Delta]$. A special case of (3) is a permutation of cubic roots $e_{1,2,3}$ and quartic roots $x_{i,j,k,l}$ $(i,j,k,l = 1,2,3,4)$ for n=3 [9]

$$z - e_i = \gamma^{(3)} \circ x = (x_i x_k)(x_i x_l) M_{j,i}(x) \qquad (4)$$

with Moebius map $M_{1,2}(z) = \frac{z-z_1}{z-z_2}$ where $\phi^{(3)}(x_i) = \prod_{\beta=jkl}(x_i x_\beta)$ (see (4) of §5 [9]). With $z - e_i \to \frac{1}{z_{k+1}-z_k} \simeq (z_{k+1}-z_k)'(z_{k+1}-z_k)''$ conjugates of cubic roots z_k enter (4) for a given discriminant Δ. Homogeneous coordinates $x_{1,2}$ with $(x_i x_j) = \frac{1}{2i}\psi_i \wedge \overline{\psi}_j$ depend on a four-component complex $\psi_i = x_{i1} + i x_{i2}$ on Gaussian plane. Commonly used is a 1:2 relation $e_i - e_j \simeq (x_i x_j)(x_k x_l)$. The linear relation (15) between branching points $e_{1,2,3}$ and $x_{i,j,k,l}$

$$\phi^{(4)}(x) = \prod_i (x - x_i) \to \phi^{(8)}(x) = \det(\gamma_i \gamma_j \psi_i \overline{\psi}_j - x) \qquad (5)$$

permutes between degree 4 and 8. Section 3 justifies an inverse Green' s function (4) $G^{-1} \simeq z - e_i \simeq \overline{\psi}_j \psi_i$. with source $G \simeq \overline{\psi}_i \overline{\psi}_j \Gamma_{ijkl} \psi_k \psi_l$ of a rational vertex Γ_{ijkl}. Due to γ- invariance of (1) q(z) is enveloped by a Bezout matrix $B(\phi,\varphi) = \big(\phi(z)\varphi(w) - \phi(w)\varphi(z)\big)/(z-w)$

$$q(z) \simeq B\big(\phi^{(3)}(z), 1\big). \qquad (6)$$

Invariances $F^{(3)}(\gamma\circ w, \gamma\circ z)$ are completed by $\gamma\circ F^{(3)}(w,z)$ of $F^{(3)}(w,z)$ via

$$z = B\big(F^{(3)}(z), 1\big) = \frac{F^{(3)}(w,z) - F^{(3)}(w,w)}{z-w} = 4w_0 z + ½ w_1 \qquad (7)$$

leading to sequential steps $q(z) \simeq B\Big(\phi^{(3)}\Big(B\big(F^{(3)}(z),1\big)\Big), 1\Big)$. (7) is a linear z- shift in $\mathbb{K}[\Delta]$. Newton identities allow to represent Bezout matrices as

$$B\big(\phi^{(2n)}, 1\big) \simeq \sum_{i=1,\dots,n} \psi_i \overline{\psi}_i = \sum_{i=1,\dots,2n} x_i^2 \qquad (8)$$

which leads to a normalization $\overline{\psi}_i \psi_i = 1$ in case of $\phi^{(8)}$. The discriminant-like Bezoutian (8) normalizes which is capable to define a mass. Next it is proven that the regulator index defines a Lagrangian [10]. The claim is that invariant one-dimensional complex root finding (1) with an information uncertainty bit $z_1 \vee z_2$ is a base for covariant coordinates. The complexity of the root finding algorithm (1) is much lower than that of a Lattes map u→2u [11]. However, the present algorithm (1) operates on reduced genus 3 to generalized split genus 1 hypersurfaces reducing the computational complexity [11]. The purpose of this paper is to propose a subroutine which scans the linear vicinity of simple nontrivial zeros of an entire transcendent $\phi^{(\infty)}(z)$ by means of a cubic polynomial $\phi^{(3)}(z)$. An open system would contain infinite cyclotomic roots $\zeta^{(\infty)}$ in a Riemann surface $\mathbb{R}_L$. Here closed systems are triangulated $\mathbb{R}_L$ of volume $\mathrm{Vol}(\mathcal{M})$ in $\mathcal{M}$. Invariants depend on cyclotomic roots $\zeta^{(m)}$ of congruences mod m=12,6,4,3 and 2. $\zeta^{(m)}$ are capable to satisfy the optimality condition $z=e^z$ and sequentially $\zeta^{(m')} = e^{e^{\zeta^{(m)}}}$ due to the Kronecker-Weber theorem (KWT). Root finding for a complex function e.g. $q(z) = \zeta(z)\overline{\zeta}(\overline{z})$ should converge near simple zeros because it would be quadratic in z. The introduced complex Lagrangian is underconstrained and provides more degrees of freedom than the physical system constraints. Generally, binary invariant root finding consists in four real functions Reζ,Imζ,Reγ(ζ)∘ζ,Imγ(ζ)∘ζ in terms of four real variables Rez,Imz,Reγ(z)∘z,Imγ(z)∘z in a quadrupolar configuration $\zeta(z), \zeta(\overline{z}), \overline{\zeta}(z), \overline{\zeta}(\overline{z})$ of complex conjugates in the vicinity of z_{nt}. Substitutions $\gamma \circ \xi$ and $\gamma \circ z$ of zeta function $\zeta(z)$ and gamma function $\Gamma(z)$

$$\xi(z) = \binom{z}{2} \pi^{-\frac{z}{2}} \Gamma(\tfrac{z}{2}) \zeta(z) = ½ \prod_n (1 - \frac{z}{z_{nt}}) = -\frac{\partial j_\xi(z)}{\partial z} \qquad (9)$$

can be represented by CF which implies a dependence on the cubic normal field $\mathbb{K}[\Delta]$. A CF of $\gamma \circ \zeta(z)$ should be connected with a product $\zeta(z)\prod_\chi L(z,\chi)$ with L-functions of character χ which needs further clarification. Orbits around nontrivial zeros z_{nt} of $\xi(z)$ are linked to masses m_n in the scattering amplitude in [6]: Masse are explainable by ± rotations of roots in interval [0,1] in Section 3 [12]. A Feigenbaum constant- fine structure constant relation for a possible explanation of the world requires a quadratic relation for the charged current [13] [14]. Sections 2-6 claim that the quadratic Newton iteration N_q

$$N_q(z) \sim \zeta(z)\zeta(z') \sim T(z,z') \sim [\lambda(z),\lambda(z')] \sim (\delta z)^2 \sim \delta z \qquad (10)$$

tends to conformal stress-energy T(z) and Legendre module λ(z) correlation. Further on, the concept of half-integer differentials [3] is extendable to a quadratic mapping $(\delta z)^2 \sim \delta z$ for a Julia set $\mathcal{J}(N_q)$ with Feigenbaum constant $\delta_F = 1 - \delta ln\delta c$ for k-components of c. A relation of differentials $d^2/dz^2 \sim d/dz$ implies a correspondence of the heat equation and the wave equation $(dz)^2 \sim dz$ as averages over times which causes a δ-function source [5]. Optimal modular units $f(\omega) = e^{\frac{-i\pi\omega}{24}}\prod_{n=1}^{\infty}\left(1+e^{i\pi\omega(2n-1)}\right)$ can be approximated by the first term $\zeta^{(12)}e^{\frac{-i\pi\omega}{24}}$ for $\omega = ½\left(1+i\sqrt{\Delta}\right)$ with very high precision for class number one $h_\Delta=1$ fields $\mathbb{K}[\Delta]$ [15].

3. FERMIONS AND RATIONAL ELLIPTIC CURVE POINT ADDITION/PERMUTATION

$N_q(z)$ is investigated on an underdetermined Riemann surface $\mathbb{R}_L$ where strings on torus 1 and 2 bifurcate into string 1' and string 2' on a genus 3 surface. A root of a genus 3 quartic polynomial can be shifted to ∞ which shifts 1' and 2' to ∞ cutting out a single torus of variable period. An inverse process would be a single addition step on an elliptic curve within a variable quartic polynomial ϕ_4 which is reducible to ϕ_3. Substitutions γ contain additions by Poncelet polygons in a spatial cone C(x,y,z) [16] [17]. For n=3 subsequent fractional substitutions γ are transvectants of $\phi^{(3)}(z)$ with $z \wedge w$ [18]. Each second step in k,k+1,k+2 yields an invariant polynomial [9]

$$\phi^{(3)}(z) \to z^3 - g_2 z - g_3 \qquad (11)$$

Accordingly, the iterated variable z transmits into an invariant f(ω) as a rational parameter for the cone C(x,y,z). Roots of (9) are $e_{1,2,3} = f_1^8(\omega), f_2^8(\omega), -f^8(\omega)$ with $e_1+e_2+e_3=0$ [9] [18]. $h_\Delta=1$ implies invariants $g_2 \simeq \gamma_2 \in \mathbb{Z}$ and $g_3 \simeq \gamma_3 \in \mathbb{Q}(\sqrt{\Delta})$. For $a_0=4, a_1=0, a_2=g_2, a_3=g_3$

$$z_{k+1} \leftarrow F^{(3)}(w,z_k) = w_0\left(4z_k^2+\tfrac{2}{3}g_2\right)+w_1 z_k. \qquad (12)$$

Variable w defines a power integral base $w^k \to w_k$ by $\{w\} = \{w_0=1, w_1, w_2\}$. Accordingly, (12) splits into the Mandelbrot map $z_{k+1} \leftarrow 4z_k^2 + c, c = \frac{2}{3}g_2$ for base vector w_0 and a linear map $z_{k+1} \leftarrow z_k$ for base vector w_1. The additional invariance (7) shifts the cubic base vector and avoids ternary CF. The 12-component base vector $\zeta^{(12)} = \{b_0,b_1\}\otimes\{w_0=1,w_1,w_2\}\otimes\{\zeta^{(4)}\}$ is written as $\widehat{B} = \{b_0,b_1\}[\{w_0=1,w_1,w_2\}\otimes\{\zeta^{(4)}\}]$ Indices i=b,w,s denote points on three distinct circles $\mathbb{S}^1$(b), $\mathbb{S}^1$(w), $\mathbb{S}^1$(s) with congruences mod 2,3 and 4 .Shifts of the cubic base $\{w_0=1,w_1,w_2\}$ leave a cubic Diophantine index form invariant. The binary base $\{b_0,b_1\}$ depends on permutations (15) of quartic roots via DFT-4 of $\{\zeta^{(4)}\}$. Equivalent variables with $\det\gamma = \phi^{(n)} = 1$ search nearly constant

$$⅓\,\partial_z\phi^{(3)}(z) = z^2 - ⅓g_2 = (x_ix_j)(x_ix_k) - ⅓g_2 \simeq -¼\psi_i\wedge\bar{\psi}_j\psi_i\wedge\bar{\psi}_k - ⅓g_2 \simeq \bar{\psi}_i\bar{\psi}_j\Gamma_{ijkl}\psi_k\psi_l - ⅓g_2 \qquad (13)$$

The phase function $L(w,z) = \ln(z-w) = -L' - L''$ of cubic root differences $z - w \simeq (x_ix_j) = \frac{1}{2i}\psi_i\wedge\bar{\psi}_j$ depends on conjugates L', L'' where $(z-w)(z-w)'(z-w)'' = \sqrt{\Delta}$ are equivalent to cubic units. Permutations of roots e_i, x_i are rotations by ±π in phase space in real interval [0,1]. First this singularity in conjugated phase φ_q space

$$G(w,z) = \frac{1}{F(w,z)-z} = \partial L(F,z) = \frac{1}{z_{k+1}-z_k} = (z_{k+1}-z_k)'(z_{k+1}-z_k)'' \qquad (14)$$

justifies to call (14) a Green's function $G(w,z) = \partial_z L(w,z)$.

A simplest cycle $F^{(3)}\left(F^{(3)}\right) = z$ is indexed by a quadruple of shifts δ_k of steps

$$q: 1, \delta_k, \delta_k\delta_k, \delta_k\delta_k\delta_k \simeq k+3 \in k, k+1, k+2 \simeq e_i, \pm\infty, \pm i\infty \simeq x_i\ . \qquad (15)$$

For $\cos\varphi = -g_3(3/g_2)^{3/2}$ quartic roots [19]

$$x_i(\varphi_q) = \left(e_i(+\pi), e_i(-\pi), e_i(0), e_i(+i\infty)\right) \qquad (16)$$

are indexed by discrete by $\varphi_q$congruences $e_i(\varphi_q) = \sqrt{(g_2/3)}\cos\left(⅓(\varphi-\varphi_q)\right)$. A quadruple (16) contains $\varphi_q$congruences mod 3 and mod 4 in $\left(\frac{1}{3}\varphi, \frac{1}{3}(\varphi\pm\pi), \pm i\infty\right) + k\pi$ where k=0,1,2 form a 12-component congruence. A three-component transformation of a base âw would contain non-periodic ternary CF whereas only two-component shifts are

Abelian and periodic. Next it is shown that discrete changes by $\zeta^{(12)}$ of dz≃dφ and $\partial_z \simeq \partial_\varphi$ yield an infinite string circulation $\mathcal{L}(\widehat{B})$ on the Riemann surface $\mathbb{R}_L$. Its cross-section of radius $\sqrt{(g_2/3)}$ of circles $\mathbb{S}^1[\sqrt{(g_2/3)}]$ is constant for detγ=1 with constant g_2, g_3. $\mathbb{R}_L$ is differentiable infinitely sheeted complex planes for discrete sequences of roots by angle rotations φ_q. First a sum over eigenfunctions yields a Dirac δ-function [5]

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dp\cos\left(p(\varphi_q - \varphi_{q'})\right) \simeq \delta(\varphi_q - \varphi_{q'}). \quad (17)$$

A bell-shaped or double-well shaped maximum is expected for $\cos\varphi \simeq 1 - ½\varphi^2$. The heat singularity (17) transmits to $\mathcal{M}$. Doubly-periodic $z = \rho e^{i\varphi} = e^{i\varphi + \Gamma}$ can be formulated by complex φ in homogeneous roots $x_i = (x_{i1}, x_{i2})$. Shifting the origin to $\varphi - \varphi_q \to \varphi_q$ permutations of two components in (16) yield the one-dimensional Laplace equation

$$\left(\frac{\partial}{\partial \varphi_q^2} + \frac{1}{9}\right)x = \sqrt{(g_2/3)}\,\delta(\varphi_q - \varphi_{q'}) \quad (18)$$

with one-dimensional (one-periodic) tent-map Green's function

$$G^{(\pm)} = S(\varphi_q)\theta(\pi - \varphi_q) + (1 \pm S(\varphi_q))\theta(\varphi_q - \pi) \quad (19)$$

with Heavyside function θ(z). Ordered congruences m=12,6,4,3 for the mod 2 field $\varphi_q\ \zeta^{(m)}$ are capable to satisfy z=e^z and sequentially $\zeta^{(m')} = e^{e^{\zeta^{(m)}}}$. Therefore, the form (19) is equivalent to the form $G^{(\pm)} \simeq \overline{\psi}_j\psi_i$ of the Green's function in terms of quartic/cubic roots (16) which is equivalent to $G^{(\pm)} \simeq \overline{\psi}_q\psi_q \simeq \overline{\varphi}_q\varphi_q$ to a quadruple of shifts (15) of the Weber invariant f(ω). The sum of shifts $\Pi\begin{vmatrix}\zeta^{(4)} & \mathcal{L}\\ 0 & 1\end{vmatrix}\varphi_q$ yields a CF representation for the permutated basis $(\varphi_{q1}, \varphi_{q2}) \to (\varphi_{q2}, \varphi_{q1})$

$$S(\varphi_q) = \sum d\varphi_q = \Pi\begin{vmatrix}\mathcal{L} & \zeta^{(4)}\\ 1 & 0\end{vmatrix}\varphi_q \to \zeta(\mathcal{L}, \varphi_q) \quad (20)$$

For fourth roots $\zeta^{(4)} = \pm i, \pm 1$ the product is the sum of angles in a tubular channel. If the varies $\zeta^{(4)}$ at each step one recovers the geometric zeta function form $\zeta(\mathcal{L}, \varphi) = (3^\varphi - 2)^{-1}$ for the Cantor string $\mathcal{L} = 2^j 3^{-(1+j)\varphi}$. Each step in a period-3 cycle π+π+π=3π permutes the sign $(3^\varphi - 2)^{-1} \to (-3^\varphi - 2)^{-1} \to (3^\varphi - 2)^{-1}$. It is claimed that a period-3 cycle recovers the boson-fermion vertex by the product $G^{(+)}(t,t')G^{(-)}(t,t')G^{(-)}(t,t')$. The summed angle $S(\varphi_q)$ is the occupation number in the Green's function (19) with time $t \simeq \varphi_q$. The φ_q -dependent simplest cycle $F^{(3)}(F^{(3)}) = z$ assigns cubic roots z to ranges $d\varphi_q$ in the real interval [0,1] thereby defining >,<,= relations. The real angle φ_q implies a time-causality where non-equivalent γ yield complex φ which act as a damping in (19). On Poincare plane $\mathbb{H}^2$ metric $g^{ij} = y^2\delta_{ij} = \frac{1}{4i}(z - \bar{z})^2\delta_{ij}$ and Laplacian read

$$\Delta_h = y^2\partial_z\partial_{\bar{z}} = y^2\Delta_{xy} = y^2(\partial_x^2 + \partial_y^2) = e^{-2\Gamma}(\partial_\Gamma^2 + \partial_\varphi^2) = \frac{-1}{\sqrt{|g|}}\partial_i\sqrt{|g|}g^{ij}\partial_j \quad (21)$$

The φ_q singularity in [0,1] is on horizontal lines $\mu_c(\mathrm{Im}z - m_n) = 0$ on complex plane, i.e. y=const. This links the mass m_n to Sharkovskii periods and ±π rotations in [0,1] [12]. Zeros of L(z, χ)ξ(z) with L-functions yield [20]

$$\Delta_{xy}\left(L(z,\chi)\xi(z)\right) + \mu_s L(z,\chi)\xi(z) = \mu_c(\mathrm{Im}z - m_n)$$

with Lagrange parameter μ_s and μ_c. (18) leads to

$$\Delta_{xy}\,\xi = m_n\delta(z - z_q) \quad (22)$$

The solution with Laplacian (21) is a non-analytic two-dimensional Green's function

$$G^{(2)}(w,z) = m_n \ln(z - z_q)(\bar{z} - \bar{z}_q) = m_n(L(z_q, z) + L(\bar{z}_q, \bar{z})) \quad (23)$$

In base $\widehat{B}$ discrete values of the phase φ_q are assumed to satisfy ∂L(w,z)≃ L(w,z). The angle of the planar triangle $T(z_q) = T(\Delta z = z_0, z_1, z_2, z_3)$ for q(z) changes over to discussing a net rate process where S(t) contains a Fermi level. First the Green's function (19) $G(w,z) \simeq \partial L(w,z) \simeq L(w,z) \simeq G^{(\pm)}(t,t')$ enters the z_1, z_2 decision algorithm with $\int G^{(2)}(w,z) \simeq G^{(\pm)}(t,t')$ [4]. The problem of half-differentials is attached by replacing the differential dz along the string circulation $\mathcal{L}(\widehat{B})$ by a sum over $\Sigma = \Gamma G^{(-)} + \Gamma\Gamma G^{(-)}G^{(-)}G^{(-)}$ where vertex $\Gamma = \Gamma[G^{(+)}]$ describes tangents on a bell and on twin peaks or asymmetric peaks. For k→∞ the exponent of the quadratic form q(z) tends over to a thermodynamic potential Ω in with quantum statistics [21]. One gets

$$q(z) = e^{\int_0^z \frac{dz}{z_{k+1} - z}} = e^{-\int_0^z G dz} = e^{-\int_0^{\Omega_k} d\Omega} \simeq G_0^{-1}(z)G_0^{-1}(z) \quad (24)$$

Integrating (24) a quadratic $z_{k+1}[z_k]$ confirms a quadratic law in $G_0^{-1} = z_k - z_1$ which explains a zero of $q(z) \simeq e^{\int \Sigma G}$. The discriminant root of q(z) $\sqrt{\Delta} = z_1 - z_2$ enter the self-energy as an energy gap or mass. Constant values of discriminant and regulator are capable to exhibit oscillations. First permuting iterates of $f(\omega)\zeta^{(12)}$ transform like $\Delta \to$

$\Delta(\phi^{(3)2})$ [9]. Oscillations occur for fixed discriminant Δ, regulator R_Δ and h_Δ=1 for units $\varepsilon = (\zeta^{(2)}\rho)^2, (\zeta^{(2)}\rho)^{-1}e^{i\varphi/3}, (\zeta^{(2)}\rho^{-1})e^{-i\varphi/3}$ by mod 2 and mod 3 fields $\zeta^{(2)}, \zeta^{(3)} \in \zeta^{(12)}$. One gets the discriminant

$$\sqrt{\Delta(\rho,\varphi)} = 2i\left((\zeta^{(2)}\zeta^{(3)}\rho)^3 + (\zeta^{(2)}\zeta^{(3)}\rho)^{-3} - 2cos(\varphi/3)\right)sin(\varphi/3) \qquad (25)$$

as well the regulator index

$$R_\Delta = l = \log \varepsilon = 2\zeta^{(2)}\log\rho \qquad (26)$$

(25) and (26) are subjected a minimum of the quadratic form $Q(l) = \mu_1 l^2 + \mu_2 l + \mu_3 N(e^{2l})\zeta(\mathcal{L}, l)$ with geometric zeta function form $\zeta(\mathcal{L}, l)$ [22]. Degrees of freedom of Q(l) are a modulo 2 field $\zeta^{(2)} = \zeta^{(2)}(\rho,\varphi)$, power integral bases $\{b_0, b_1\}$,$\{w_0 = 1, w_1, w_2\}$ and $\zeta^{(4)}$ which is a discrete Fourier transform (DFT) radix-4. It is noted that shifts in the cubic base $\{w_1, w_2\}$ with invariant Δ index form include the following CF process

$$g(z) = \frac{1}{z_{k+1} - z_k} = ln'q(z) \to \begin{vmatrix} \mathcal{L} & \zeta^{(4)} \\ 0 & 1 \end{vmatrix} g(z) \qquad (27)$$

The r.h.s. of (27) yields the Cantor set geometric zeta function $\zeta(\mathcal{L}, l)$ for geometric string $\mathcal{L} = 2^j 3^{-(1+j)l}$ for different roots $\zeta^{(4)} = \pm 1, \pm i$ with renormalized function $g(z) = \zeta(\mathcal{L}, l)$ and infinite periods. For vertex part $g(z) = \Gamma, \zeta^{(4)} = 1, \mathcal{L} = GG$ and $\zeta^{(4)} = 1, \mathcal{L} = \Gamma D = \Sigma, D \simeq \zeta(\mathcal{L}, l)$ periodic CF in (27) yield a Dyson equation with Green's functions a Bethe-Salpeter equation because multiplication is convolution by Fourier components $\zeta^{(12)}$ within discrete Fourier transformations (DFT) with $\zeta^{(m)}$. Bases $\{w_1, w_2\}$ enables a fast optimal solution of the optimizable quadratic form $Q(l)$ using the identity $\begin{vmatrix} 0 & \Sigma \\ 0 & 0 \end{vmatrix} = log \begin{vmatrix} 1 & \Sigma \\ 0 & 1 \end{vmatrix}$.

4. SCAN ALGORITHM

The aim is root finding of

$$q(z) \to \phi^{(\infty)}\bar{\phi}^{(\infty)} \to \xi(z)\bar{\xi}(\bar{z}) \qquad (28)$$

which is solvable for quadratic q(z) near simple zero z_{nt} of $\xi(z)$. The question is to translate binary information of string of decisions z_1, z_2 into the real world. The presented algorithm has an infinite number of degrees of freedom. The dependence on the number field $\mathbb{K}$ enters writing variable $z \simeq \zeta^{(12)}e^{\frac{-i\pi\omega}{24}}$ via the Kronecker product of bases $\widehat{B} = \{b_0, b_1\}[\{w_0 = 1, w_1, w_2\} \otimes \{\zeta^{(4)}\}]$. Different complex planes already arose from a numerical representation of the Mandelbrot zoom in space. $\zeta^{(12)}$ is algorithmically accessible by a solvable quartic in order to scan an infinite base of $\phi^{(\infty)}$. Binary invariances $z_{k+1} \leftarrow \gamma^{(3)}(w) \circ z_{k+1}$ and $z_k \leftarrow \gamma^{(3)}(w) \circ z_k$ create an envelope $\phi^{(6)}(\lambda)$ if substitutions are equivalent $\det\gamma^{(3)} = 1$. An envelope of (1) can be written in terms of the elliptic invariant $j(\omega) = j(\omega')$ or in terms of $\phi^{(6)}(\lambda)$ with Legendre module $\lambda(f(\omega))$ or in terms of $SL(2, \mathbb{Z})$ substitutions of λ. $\phi^{(6)}(\lambda)$ writes $\phi^{(3)}(f^8(\omega)) = f^{24}(\omega) - g_2 f^8(\omega) - g_3$ in terms of $z_k \to f^8(\omega)$. Two-component bases $\hat{b}$ and the conjugate $\hat{\hat{b}}$ yield an Abelian four-component base $b_4 = (\hat{b}, \hat{\hat{b}})$. Setting $b_4 = b_{4s} = \psi_{bws}$ one gets an oscillation around z_{nt} by three distinct circles centered by $z_q[\mathbb{S}^1(z_b), \mathbb{S}^1(z_w), \mathbb{S}^1(z_s \in \zeta^{(12)})]$. Without loss of generality, one gets a two or three 'particle' motion around a given point. With a large background mass a double peaked configuration is used for a matrix representation of ξ(z) for specified b_4. Because a substitution γ is equivalent to matrix multiplication $\hat{\gamma}\hat{b}$ one can use $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}^n = \begin{vmatrix} 1 & n \\ 0 & 1 \end{vmatrix}$, $e^{\begin{vmatrix} 0 & n \\ 0 & 0 \end{vmatrix}} = \begin{vmatrix} 1 & n \\ 0 & 1 \end{vmatrix}$ and $n^{-z} \to b_4 e^{M(nz)} b_4$ with $M(zn) = \begin{vmatrix} 0 & -zn \\ 0 & 0 \end{vmatrix}$. The zeta function $\zeta(z) = \sum_n n^{-z}$ yields the product

$$\xi(z)\bar{\xi}(\bar{z}) = b_4^+ \widehat{H}^+ \sum_{n,n'} e^{\widehat{H}M\widehat{H}^+} \widehat{H} b_4 \qquad (29)$$

with four-component $M = \begin{vmatrix} M(zn) & 0 \\ 0 & M(zn') \end{vmatrix}$ acting on up to an undetermined rotation by a Hermitian matrix $\widehat{H}$. A renormalized (iterated) behavior $\xi(z[\varphi_q])$ should traverse a bell-shaped or double well shaped configuration $z[\varphi_q \in \zeta^{(12)}]$. Rational $\varphi_q \simeq \psi_i$ in homogeneous coordinates on Gaussian plane describe a string circulation in cylindrical coordinates ρ and φ with in φ-units of π in interval [0,1] [8]. A required number of pixels for φ_q to form a smooth maximum is expected above $2^{2^{10}}$. Iterates $z_{k+1} - z_k \simeq G^{-1} \simeq z - e_i$ are proportional to differences of cubic roots $e_i - e_j \simeq x_i \simeq f(\omega)$ for optimal iscrete φ_q with $\varphi_q \simeq e^{\varphi_q}$. The algorithm aims to prove that renormalized $\Delta z[\varphi_q]$ is quadratic or quartic in φ_q with $\Delta z \simeq \varphi_q \simeq G^{-1} \simeq \Sigma$. A bell-shaped maximum corresponds to the direct term and a double-well shaped maximum to the exchange term of the scattering amplitude. The algorithm associates a fast $\mathcal{F}(N_q)$ decision by a short φ_{b_q}-string where the message carries very little information. A slow decision on the border between $\mathcal{J}(N_q) - \mathcal{F}(N_q)$ with a long φ_{b_q}- message string carries much information. The underlying idea is that a maximal topological entropy $h_t[\zeta^{(12)}]=\#\zeta^{(12)}$ is given by a high number of generators $\zeta^{(12)}$ in the set of three different circles (coordinate systems) among maximally five circles. The underdetermined Newton algorithm (1) stores least squares of a thermodynamic potential $\Omega_k = \int dz\, G(z)$ along a surface dz triangulated by $T(z_q)$. This elliptic approximation is capable to satisfy a Gaussian differential equation for $z(\lambda)$ due to γ-invariance of $\phi^{(6)}(\lambda)$. The cyclotomic approximation $b_4[\zeta^{(12)}]$ of the KWT allows to set $\partial_{\ln\lambda} \simeq \lambda^2$ which yields an oscillatory behavior $z(\lambda)$. Expanding $b_4[\zeta^{(12)}]$ in terms of Hermite polynomials the n-sum in (29) can be performed. Accordingly, for still to be found four centers z_q only nearest neighbors in (29) contribute to periodic CF. Root finding (1) converges if [4] $\partial_z N_q(z)| = \left|\frac{2}{q(\partial_z ln q)^2}\right| < 2$ where $\partial_z N_q(z) = \partial_z F^{(3)}(w,z) = 2w_0 a_0 z + w_1 a_0 + w_0 a_1 = 8w_0 z + 4w_1$ is only linear in $\mathbb{K}[\Delta]$. The roots of

$$F^{(3)}(w,z) = w_0F_1 + w_1F_0 = w_0a_0z^2 + (w_1a_0 + w_0a_1)z + ⅓w_1a_1 + ⅔w_0a_2 \quad (30)$$

are

$$F^{(3)} = \frac{1}{2w_0a_0}\left(w_1a_0 + w_0a_1 \pm \sqrt{(w_1a_0 + w_0a_1)^2 - ⅓w_0a_0(⅓w_1a_1 + ⅔w_0a_2)}\right) \quad (31)$$

For invariants $a_0 = 4, a_1 = 0, a_2 = g_2$ one has

$$F^{(3)} = \frac{w_1}{2w_0}\left(1 \pm \sqrt{1 - \frac{w_0^2}{72w_1^2}g_2}\right) \quad (32)$$

Convergences are stored in a string $\varphi_{\{b_q=0,0,0,0\}}$of rational angles (in units of π) where a switch to 1 in the q[th] place denotes a point at z_q in a quadruple $T(z_q)$. The paper claims that the summed up φ_{b_q} is equivalent to the Green's function (19). On the border between the Julia set $\mathcal{J}(N_q)$ set and the Fatou set $\mathcal{F}(N_q)$ the length of $\{b_q\} \to \infty$ [4] [23]. Convergence to z_1 or $z_2$depends on the value of a definite angle $\varphi_T = \ln\frac{z_{k+1}-z_{k+2}}{z_k-z_{k+3}}$ in a triangle [4]

$$T\left(\Delta z = z_0, z_1, z_2, z_3 = ½(z_1 + z_2)\right) = T(z_q). \quad (33)$$

try	
$z_k[\mathbb{S}^1(z_b), \mathbb{S}^1(z_w), \mathbb{S}^1(z_s \in \zeta^{(12)})] \to \psi_i$	$\bar\psi_i\bar\psi_j\Gamma_{ijhl}\psi_h\psi_l - ⅓g_2 \simeq \zeta^{(4)} = \pm1, \pm i$
$\psi_i \wedge \bar\psi_j \to w$	$\gamma(w) = \begin{vmatrix}\zeta^{(4)} & 0\\ -1 & w\end{vmatrix}$
$\gamma(w) \circ z_k \to z_{k+1}$	$N_q(z) = \frac{z^2 - z_1z_2}{z - ½(z_1 + z_2)}$
$N_q(z_{k+1}) \to z_{k+2}$	$\lvert\partial_{z_k}N_q(z_k)\rvert < 2$
$z_k, z_{k+1}, z_{k+2} \to \{new\ triangle\ T(z_k, z_{k+1}, z_{k+2}, z_{k+3})$	
$\varphi_T = \ln\frac{z_{k+1} - z_{k+2}}{z_k - z_{k+3}} = L(z_{k+2}, z_{k+1}) - L(z_{k+3}, z_k)$	
$S(3\varphi_{\{b_q\}})\mathrel{+}= 2\pi k\theta(½\pi - \mathrm{Im}\varphi_T) + 2\pi(k+1)\theta(\mathrm{Im}\varphi_T - ½\pi)$	
catch	
$\lambda_k = e^{\varphi_T}$	$S(\varphi_{\{b_q\}}) \simeq S(\varphi_q),\ t \simeq \varphi_q$
$G(z_k) = (z_{k+1} - z_k)^{-1} = \partial_{z_k}\log q(z_k) \simeq G^{(\pm)}(t,t')$	Thermodynamic energy $\Omega = \sum G^{\pm}(t,t') = \int dz G^{\pm}(t,t')$
$G(z)^{-1} = z_{k+2} - z_{k+1} = \frac{q(z_{k+1})}{\partial_{z_{k+1}}q(z_{k+1})}$	Particle energy $G(z)^{-1} \simeq \Sigma$

Fig. 1 Statistical scan of a circulating string near zeros of $\xi(\lambda_k)$ on the border of $\mathcal{J}(N_q) - \mathcal{F}(N_q)$

A circulation of z in Fig.1 (1) indicates convergence towards $z_1, z_2 \simeq z_{nt}, \bar{z}_{nt}$ as a nontrivial zero of $\xi(z)$. A calculated condition $|\partial_{z_k} N_q(z_k)| < 2$ presupposes a UFD by $\mathbb{K}[\Delta]$. A regular (non-stochastic) behavior in Fig. 1 indicates the validity of a congruence $\Phi_3(f(\omega)) = 0$.

5. LAGRANGIAN- REGULATOR INDEX RELATION OF STRESS-ENERGY

The paper claims that thermodynamics and statistics originate from the complex plane as the infinitely sheeted Riemann surface $\mathbb{R}_L$ of the complex logarithm $L(w,z)$ for triangle string circulation $T(z_q)$ in space. The metrical line element ds^2 on Poincare plane $\mathbb{H}^2$

$$ds^2 = y^{-2}dzd\bar{z} = -d_z ln(z-\bar{z})d_{\bar{z}} ln(z-\bar{z}) \tag{34}$$

with differential d_z acts only on variable z for $w = \bar{z}$ in L(w,z). ds^2 depends on invariant properties of

$$d_z ln(\phi(z)-\phi(w))d_w ln(\phi(z)-\phi(w)) - d_z ln(z-w)d_w ln(z-w) \tag{35}$$

Steps $\phi_{k+1} = z_{k+1} = \gamma \circ z_k$ and $\phi_{k+2}(\phi_{k+1}(\phi_k(z_k)))$ are as well variables z_k and functions ϕ_k where two consecutive conformal maps $z \to \phi_{k+2} \to \phi_{k+2}(\phi_{k+1}(z))$ yield quadratic differentials in the theory of half-differentials [2]. Consecutive maps $z_k \to z_{k+1} \to z_{k+2}$ yield the exact sum

$$\begin{aligned} &d_{z_{k+1}}L_{k+2}d_{w_{k+1}}L_{k+2} - d_{z_{k+1}}L_{k+1}d_{w_{k+1}}L_{k+1} \quad + \\ &d_{z_k}L_{k+1}d_{w_k}L_{k+1} - d_{z_k}L_k d_{w_k}L_k \quad = \\ &d_{z_k}L_{k+2}d_{w_k}L_{k+2} - d_{z_k}L_k d_{w_k}L_k \end{aligned} \tag{36}$$

or in differential form

$$[w_{k+2}, z_{k+2}, w_{k+1}, z_{k+1}]dw_{k+1}dz_{k+1} + [w_{k+1}, z_{k+1}, w_k, z_k]dw_k dz_k = [w_{k+2}, z_{k+2}, w_k, z_k]dw_k dz_k \tag{37}$$

with the denotation

$$[w_{k+1}, z_{k+1}, w_k, z_k] = \frac{\partial_{w_k} w_{k+1} \partial_{z_k} z_{k+1}}{(w_{k+1}-z_{k+1})^2} - \frac{1}{(w_k - z_k)^2} \tag{38}$$

Four terms remain in (36)

$$d_{z_{k+1}}L_{k+2}d_{w_{k+1}}L_{k+2} - d_{z_{k+1}}L_{k+1}d_{w_{k+1}}L_{k+1} + d_{z_k}L_{k+1}d_{w_k}L_{k+1} - d_{z_k}L_{k+2}d_{w_k}L_{k+2} = 0 \tag{39}$$

which justifies a square root by Dirac matrices. First in the limit w→z one gets the chain rule

$$\{z_{k+2,}, z_{k+1}\}dz_{k+1}^2 + \{z_{k+1,}, z_k\}dz_k^2 = \{z_{k+2,}, z_k\}dz_k^2 \tag{40}$$

with Schwarzian derivative

$$\{F^{(3)}(w,z), z\} = 6\,\partial_z\,\partial_w \log\frac{F^{(3)}(w,z)-F^{(3)}(w,w)}{z-w}|_{w\to z} = 6\,\partial_z\,\partial_w \log B(F^{(3)}(f,z),1)|_{w\to z} = \frac{\dddot{F}}{\dot{F}} - \frac{3}{2}(\frac{\ddot{F}}{\dot{F}})^2 \tag{41}$$

where $\dot{F} = \partial_z F^{(3)}(f,z)$. The Schwarzian derivative $\{F(w,z), z\}$ enters conformal stress-energy $T(w,z)$ [24]. Limits $w \to z$ and $w \to \bar{z}$ indicate that $T(w,z)dzdw = \{F(f,z), z\}dzdw$ is not compatible with a metrical line element (29) on $\mathbb{H}^2$ .The paper solves the contradiction in terms of homogeneous coordinates $\varphi_{1,2}$ of φ_{b_q} and of φ_q for dz_q on a discretized infinite string circulation with $dz_q \simeq \begin{pmatrix} d\varphi_1 \\ d\varphi_2 \end{pmatrix}$. A sum of discrete angles φ_q in $\exp(\sigma_\mu x_\mu)\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ leaves metrics (34) invariant if the imaginary part $Imz = m_n = const.$ is constant. This defines $\mathcal{M}$ for appropriate rational values $x_\mu=(x_1,x_2,x_3,x_4)$ and Pauli matrices σ_μ. A singularity $\delta(\varphi_q - \varphi_{q'})$ in (19) would transmit to the r.h.s of (36-40). A symmetrized version of (40) is proposed to solve the problem of half-differentials as follows

$$\sum_{q,q'}\{z_{q,}, z_{q'}\}dz_q dz_{q'} = \sum_{q,q'}\delta(\varphi_q - \varphi_{q'})d\varphi_q d\varphi_{q'} \tag{42}$$

Involutions (15) k→k+1→k+2 in (36-40) yield four neighboring triangles $T(z_q) \to T(z_{q'}) \to T(z_{q''}) \to T(z_{q'''})$ in $\mathbb{R}_L$. For optimal dL≃L with $dL \simeq dz\,\partial L \simeq dzG^{(\pm)}(z) \simeq dz\bar{\psi}_j\psi_i$ the quadratic differential form $d\varphi_q d\varphi_{q'}$ of (36) should be invertible in terms of $\gamma^\nu \partial_\mu G^{(\pm)}{}_{q,q'}$. This triangulation process should yield

$$\{z_{\mu,}, z_\nu\} = \gamma^\nu \partial_\mu G^{(\pm)}{}_{q,q'} \tag{43}$$

which is claimed to be stress-energy for a string of 4 points in triangles $T(z_q)$ propagating on $\mathbb{R}_L$ in space. Green's functions $G^{(\pm)}(z)$ in $dL \simeq dz\partial L \simeq dzG^{(\pm)}(z)$ are cubic root -dependent units ε in $z_k, z_{k+1}, z_{k+2}, z_{k+3}$. Accordingly, the thermodynamic potential Ω depends via the regulator index $R_\Delta = ln\varepsilon$ in (25-26) on a mod 2 field $\zeta^{(2)}$. Simultaneously, $R_\Delta = ln\varepsilon = detln\varepsilon_{0,r-1}$ is a circulant determinant. $dz_q \simeq \begin{pmatrix} d\varphi_1 \\ d\varphi_2 \end{pmatrix}$ implies a binary representation for $\varepsilon_{0,r-1}$. Except the zeroth component ε_0 all contributions $ln\begin{vmatrix} 1 & \varepsilon \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & \varepsilon \\ 0 & 0 \end{vmatrix}$ are nilpotent of degree 2 which shows that a cubic field exists. Simultaneous fractional substitutions $\gamma \circ \zeta(z)$ possess an analytic continuation to the entire complex plane and to $\mathbb{H}^2$ where

$$\zeta(z \to 1) - c_0 \sim \frac{c_{-1}}{z-1} + 0(z) \sim \begin{vmatrix} 0 & c_{-1} \\ 1 & -1 \end{vmatrix} z \tag{44}$$

$$\zeta(z \to z_{nt}) \sim c_1(z - z_{nt}) + 0(z^2) \sim \begin{vmatrix} 1 & -z_{nt} \\ 0 & 1 \end{vmatrix} z \qquad (45)$$

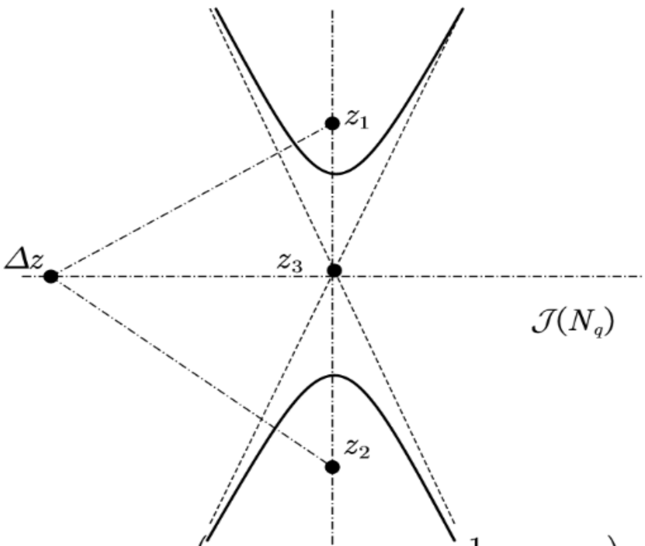
Supposing that $\gamma \circ \zeta(z)$ validates a CF representation of $\gamma \circ \zeta(z) \to \zeta(z) \prod_\chi L(z,\chi)$. Then residua $c_{-1} \to R_\Delta$ near $z_{crit} = 1$ and zeros z_{nt} are linked by a circular motion of differences of cubic roots where $\Delta z \simeq \lambda\Delta z$. Accordingly, the potential
$\delta\Omega \simeq \delta L \simeq \delta z \partial L \simeq \delta z G^{(\pm)}(z) \simeq \delta R_\Delta$

is proportional to the regulator index R_Δ. Periodic units $\varepsilon_{0,r-1}$are nilpotent of degree 2 in (25) and (26). For $q(z) = (z - z_1)(z - z_2)$ the Julia set $\mathcal{J}(N_q) = z - z_*: z_* = ½(z_1 + z_2)$ contains the involution $M_{1,2}(z_*) = -1$ as a period-3 cycle. $N_q(z)$ is conjugate to z^2 by a Moebius map $M_{1,2}(z) \circ N_q(z) \circ M_{1,2}(z)^{-1} = z^2$ which yields $M_{1,2}(z_k) = M_{1,2}(z_{k+1})^2$ [25]. This approach approximates z by cubic roots e_i. For zero-start value $z_{k+1}(z_k) = \xi(z) = 0$ $M_{1,2}(z_k) = \frac{z_k - e_2}{z_k - e_3}$ can be viewed as a Legendre module λ. Then z_{nt} corresponds to a lemniscate with $g_3 = 0, e_1 = 0, e_2 = -e_3$ and $M(z_{k+1}) = \frac{0-e_2}{0-e_3} = -1$. Subsequent steps k yield k-components of γ $M_{1,2}(z_k) = M_{1,2}(z_{k+N})^{2^N}$ which are viewed as real particles.

6. $\mathcal{M}$ ORIGIN

$\mathcal{M}$ is seen as the common action of (thermal) chaotic γ-cycles superimposed by converging N_q cycles (φ-series). Root finding (1) by the scan in Fig.1 is a two-conformal step algorithm by iterating N_q and independently iterating γ. The invariant form (12) is a Mandelbrot map with $c = \frac{2}{3}g_2$. Writing $D_{\mu\upsilon} = 2\Re z_k$ and $T_{\mu\upsilon} = 2\Im z_k$, $\mathcal{F} = ½\Re(c - z_{k+1}), \mathcal{G} = ½\Im(c - z_{k+1})$ the map (12) rewrites as $D^4_{\mu\nu} + 2\mathcal{F}D^2_{\mu\nu} - \mathcal{G}^2 = 0$ or $\mathcal{F} + i\mathcal{G} = D^2_{\mu\nu} + 2iD_{\mu\upsilon}I_{\mu\upsilon} - I^2_{\mu\nu}$. The invariant g_2 in $\mathcal{F} + i\mathcal{G}$ is like an energy density in quantum electrodynamics for tensor $D_{\mu\upsilon}$ [26-31]. $D_{\mu\upsilon}$ and current tensor $I_{\mu\upsilon}$ are seen as four-component invariants in $\mathcal{M}$ with normal $\mathbf{X} = \sqrt{(\mathcal{F} + i\mathcal{G})} \simeq \mathbf{E} + i\mathbf{B}$ to a complex plane z. Variable z is subjected two transformations

(i) the fractional substitution γ of z itself leads to binary invariants (11). Four-component states ψ_i of (4) are decomposable in cyclotomic bases of $\zeta^{(12)}$ which leaves (11) invariant
(ii) converging iterates (1) are discrete rotations of homogeneous angles ±π in (19) of triangle iterates $T(z_q)$ with $\Delta z \simeq z_{k+1} - z_k \simeq D_{\mu\upsilon} + iI_{\mu\upsilon} - \mathcal{F} - i\mathcal{G} \simeq dz_q \simeq \begin{pmatrix} d\varphi_1 \\ d\varphi_2 \end{pmatrix} \to \exp(\sigma_\mu x^\mu) \begin{pmatrix} d\varphi_1 \\ d\varphi_2 \end{pmatrix}$ as CF in (19)



$$T(z_q) = T\left(z_0 = \Delta z, z_1, z_2, z_3 = \frac{1}{2}(z_1 + z_2)\right)$$

Here z and Δz is one point of four points in a triangle $T(z_q)$ in Fig.2.

Fig.2 the planar triangle $T_q(z)$- configuration for deciding whether root finding (1) tends to z_1 or z_2. The Fatou set is the range of the hyperbola [4]

Both substitutions (i) and (ii) yield $T_q(z)$ shifts (index q is denoted by s)

$$\Delta z_{k,s} \leftarrow \Delta z_{k-1,s} + D_{k-1,ss'}\Delta z_{k-1,s'} \quad (46)$$

$D = D_{\mu\nu}[\gamma^\mu,\gamma^\nu]_-$ is a four-component representation of $\exp(\sigma_\mu x^\mu)$ with $D_{\mu\nu} = [x^\mu, x^\nu]_-$. Period-n cycles of $\gamma^{(3)}$ consist of n period-3 cycles $\gamma^{(3)} \circ \gamma^{(3)} \circ \gamma^{(3)} = \gamma^{(3)}$ or $F^{(3)}(F^{(3)}) = z$

$$D_{k,n} \leftarrow \frac{X}{n} D_{k,n-1} \quad (47)$$

which yields a product $\prod_n \frac{1}{n} D_{k,n} = \frac{1}{n!} D_k^n$ where $X = |\mathbf{X}| = \sqrt{D^2}$. Quadruples (15) imply $F(F) = z$, $\gamma \circ \gamma = 1$ which is equivalent to $D_{k,n}^3 \simeq X^2 D_{k,n-1}$ which is period-3. As a consequence, one gets from (46) and (47) a matrix exponential $\Delta z_{k+1} \leftarrow e^{D_k}\Delta z_k$. Even powers are diagonal matrices $D^3 = X^2 D, D^4 = X^4, D^5 = X^4 D$. One gets the unit transformation matrix $e^D \simeq \mathrm{ch}X + X^{-1}\mathrm{sh}XD$. A mod 2 field $\zeta^{(2)}$ involution is realized by squared Dirac matrices $(\gamma^\mu)^2 = 1$. It is argued that a mod 3 field $x = \zeta^{(2)} \in \zeta^{(12)}$ with $e^x \simeq \sum_{i,j=0}^{2} \frac{-1}{3} x^{(1-i)j} e^{x^j} \mathrm{mod}(x^3 - 1)$ yields the CF representation of the Cantor set (20) with a quadratic expansion $e^{i\varphi(\widehat{B})} \simeq \phi_2(\widehat{B})$. A subsequent tower of roots should require a mod 2 field in $\widehat{B}(\zeta^{(12)})$. The present picture is an infinite discrete string circulation $\mathcal{L}(\widehat{B})$ on $\mathbb{R}_L$ with a long φ_{b_q}- message in (ii) is expected on the line Δz-z_3 of involutions $M_{1,2}(z_*) = -1$. This occurs for a Julia set $\mathcal{J}(N_q)$ with $\partial_z log q(z) \to 0$ which yields $G(\Delta z - z_3) \to 0$. But $G\left(\gamma^\mu \partial_{\gamma^\mu} - m_{\mathcal{J}(N_q)}\right) = 1$ [26] which yields an infinite hypothetical (dark) background mass $m_{\mathcal{J}(N_q)} \to \infty$. $T(z_q)$ triangulations of $\mathbb{R}_L$ are disc-like and remembers a gap state with Fermi level $\Delta z - z_3$. Two independent steps (i) and (ii) should yield two irreducible susceptibilities χ_{drift} and $\chi_{diffusion}$. Drift χ_{drift} is caused by time $t \simeq S(\varphi_q)$ with highest velocity by the maximal angle of the hyperbola in Fig.2, Diffusion $\chi_{diffusion}$ is caused by theta constants $\vartheta = \eta(\omega) f^2(\omega)$ which satisfy a heat equation with unit diffusion constant. Circulations in the vicinity of z_{nt} in Fig. 1 are subsequent iterates $\Delta z \leftarrow \lambda \Delta z$ with $\mathfrak{R}\lambda[f(\omega)] = ½$ on the critical strip.

7. CONCLUSIONS

It is argued that spacetime curvature is complex, i.e. there exists simultaneously two curvatures and two masses. This claims a dark, complex environment for Minkowski spacetime $\mathcal{M}$. A quadratic root finding fits in an infinite mass on the border to a Julia set $\mathcal{J}(N_q)$. A proposed scan algorithm of zeros of entire transcendent functions is optimal because cubic and quartic polynomials have algorithmically accessible roots. As a result, the detection of zeros in quadratic, complex Newtonian root finding is non-unique in the general case. This non-uniqueness embeds covariant transformations $x^\mu = e^\mu_\nu x^\nu$ of real coordinates and vierbeins e^μ_ν and spacetime $\mathcal{M}$ by parametrizations of a planar triangle in a triangulated complex Riemann surface $\mathbb{R}_L$ of algebraic units. A point z with two possible complex solutions z_1, z_2 of the quadratic polynomial q(z) stands for two possible complex curvatures K_1, K_2 of spacetime. Alternatively, each point z is described by two complex masses m_1,m_2 where their imaginary part stands for dark matter. This chaotic dark matter state as the set of frayed paths of (1) is viewed as one origin of Minkowski spacetime $\mathcal{M}$. Whereas $\mathcal{M}$ is caused by Abelian permutations of two components of cubic/quartic roots e_i/x_i (16) the creation of fermions as mass generating laps at root finding (1) is a 1:2 relation (4) between Δe_i and Δx_i. The entropy source window permutes an infinite number of Poncelet polygons which defines spacetime $\mathcal{M}$, mass and charge.

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