Scholars Journal of Engineering and Technology (SJET) **ISSN 2321-435X**
Sch. J. Eng. Tech., 2013; 1(4):232-237

(An International Publisher for Academic and Scientific Resources)
www.saspublisher.com

Research Article

Ranking of Stochastic DEA with using an integrated method using Data envelopment analysis and Fuzzy preference relations

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Abstract: An integrated method using data envelopment analysis and fuzzy preference relation model is one of the models in data envelopment analysis widely used by DEA people and practitioners. However, in many real applications, data is often imprecise. A successful method to address uncertainty in data is replacing deterministic data by random variables, leading to stochastic DEA. Therefore, in this paper, An integrated method using data envelopment analysis and fuzzy preference relations model is developed in stochastic data envelopment analysis, and its deterministic equivalent which is a nonlinear program is derived. Moreover, it is shown that the deterministic equivalent of the stochastic model can be converted to a quadratic program. Finally, a numerical example for illustration purpose is presented.
Keywords: Data envelopment analysis, Performance evaluation, Preference relation, Stochastic DEA, Ranking.

Introduction

Performance evaluation is of great importance for effective decision-making. The foundation of efficiency evaluation is faithfully to identify the corresponding production possibility set.

Decision and decide one of the most sensitive tasks managers and organizations are aware there is always the decision of an issue for managers is challenging. While effective managers' important addition, by seeking opportunities that are provided with the main objectives of the organization to achieve competitive advantages and increase take effective steps.

Note that we know data envelopment analysis methodology has many advantages, such as no requirement for a priori weights or explicit specification of functional relations among the multiple inputs and outputs. However, there is a weakness in conventional DEA models; in fact, original DEA models do not allow stochastic variations in input and output such as data entry errors. As a result, DEA efficiency measurement may be sensitive to such variations. A DMU, which is measured as efficient relative to other DMUs, may turn inefficient if such random variations are considered. Stochastic input and output variations into DEA have been studied by, for example, Cooper, Deng, Huang, and Li [3], Land, Lovell, and Thore[9], and Olesen and Petersen [12], Morita and Seiford[10], Khodabakhshi and Asgharian [7], Khodabakhshi[6,8]. Although original DEA models such as CCR or BCC models have been extended in stochastic data envelopment analysis, the research on ranking stochastic DMUs solely have been done in deterministic DEA. To close this gap, in this paper, an integrated method using data envelopment analysis and fuzzy preference relations model is developed in stochastic data envelopment analysis which allows stochastic variations in output data.

The concept of chance constrained programming approach introduced by Cooper et al.[3]. More recently, Asgharian, Khodabakhshi, and Neralic[1] and Khodabahshi[6,7,8] have studied stochastic input and output variations into DEA. See also Kall[5] for discussion on linear programming programs. Moreover, deterministic equivalent of the stochastic model is obtained to solve the stochastic model. Furthermore, as an empirical example, the proposed approach is applied on data of Iranian electricity distribution units.

Therefore, in this paper one ranking method is proposed based on DEA and fuzzy preference relation. The rest of the paper is organized as follows: in section 2, Stochastic DEA. In section 3, methodology for ranking of stochastic DMUs. Methods have been widely employed in Iranian electricity distribution units are collected into the last section4. Finally, in section 5 the conclusion and some remarks put forward.

Stochastic DEA

In many important situations inputs or outputs of the DMUs are often considered to be random, so efficiency conclusions upon a deterministic DEA can be misleading because of a high sensivity of the efficiency scores to the realized levels of inputs or outputs. Stochastic DEA methods have therefore been designed to deal with the problems which are introduced by uncertainty.

We assume that there are n DMUs to be evaluated. For each DMU$_j$ (j=1, 2,…, n) , $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{mj}), \tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \dots, \tilde{y}_{sj})$ represent m, s random input and output vectors, Respectively. In this paper, the inputs are considered as deterministic variables and the outputs are considered as stochastic variables. Mathematicaly, the DEA model has the following formulation:

$$\begin{aligned} Max \quad & E\left(\sum_{r=1}^{s} u_r \tilde{y}_{rd}\right) \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{id} = 1 \qquad (1) \\ & pr\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le \beta_j\right) \ge 1-\alpha_j \qquad j = 1,2,\dots,n \\ & u_r \ge 0, \qquad r = 1,2,\dots,s \\ & v_i \ge 0 \qquad i = 1,2,\dots,m. \end{aligned}$$

The above model is designed to measure the performance (DEA efficiency) of the specific dth DMU. The symbols (u_r, v_i) reperesnt weight multipliers related to the rth output and ith input. Pr stands for a probability and the superscript "$\sim$" indicates a stochastic variable.

The constraint $pr\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le \beta_j\right) \ge 1-\alpha_j$ is equivalent to:

$$pr\left(\sum_{r=1}^{s} u_r \tilde{y}_{rj} \le \beta_j\left(\sum_{i=1}^{m} v_i x_{ij}\right)\right) \ge 1-\alpha_j$$

For j=1,2,…,n. Minus and divide both sides of inequality inside n parentheses by fix term yield:

$$pr\left(\frac{\sum_{r=1}^{s} u_r(\tilde{y}_{rj} - \bar{y}_{rj})}{\sqrt{v_j}} \le \frac{\beta_j\left(\sum_{i=1}^{m} v_i x_{ij}\right) - \sum_{r=1}^{s} u_{r_j}\bar{y}_{rj}}{\sqrt{v_j}}\right) \ge 1-\alpha_j$$

Where $\bar{y}_{rj}$ is the expected value of $\tilde{y}_{rj}$ and v_j indicates the variance-covariance matrix of the jth DMU. Assume new variable $\tilde{z}_j$ is defined as bellows:

$$\tilde{z}_j = \frac{\sum_{r=1}^{s} u_r(\tilde{y}_{rj} - \bar{y}_{rj})}{\sqrt{v_j}} \qquad j = 1,2,\dots,n$$

Which follows the standard normal distribution with zero mean and unit variance, because we assume $\tilde{y}_j$ for j=1,2,…,n has normal distribution. Thus can write as follows:

$$pr\left(\tilde{z}_j \le \frac{\beta_j\left(\sum_{i=1}^{m} v_i x_{ij}\right) - \sum_{r=1}^{s} u_{r_j}\bar{y}_{rj}}{\sqrt{v_j}}\right) \ge 1-\alpha_j \qquad j = 1,2,\dots,n$$

Since $\tilde{z}_j$ follows the standard normal distribution, the invariability of (1) is executed as follows:

$$\left(\frac{\beta_j\left(\sum_{i=1}^{m} v_i x_{ij}\right) - \sum_{r=1}^{s} u_{r_j}\bar{y}_{rj}}{\sqrt{v_j}}\right) \ge F^{-1}(1-\alpha_j) \qquad j = 1,2,\dots,n$$

Here, F stands for a cumulative distribution function of the normal distribution and F^{-1} indicates its inverse function. So model (1) can be written as:

$$\begin{aligned} Max \quad & E\left(\sum_{r=1}^{s} u_r \tilde{y}_{rd}\right) \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{id} = 1 \qquad (2) \end{aligned}$$

$$\beta_j\left(\sum_{i=1}^{m} v_i x_{ij}\right) - \sum_{r=1}^{s} u_{r_j}\bar{y}_{rj} \geq \sqrt{v_j}F^{-1}(1-\alpha_j) \quad j = 1,2,\dots,n$$

$$u_r \geq 0, \quad r = 1,2,\dots,s$$
$$v_i \geq 0 \quad i = 1,2,\dots,m.$$

To obtain a linear programming equivalent to (2), this research assumes that a stochastic variable $\tilde{y}_{rj}$ of reach output is expressed by $\tilde{y}_{rj} = \bar{y}_{rj} + b_{rj}\varepsilon$ for r=1,2,…,s and j=1,2,…,n , where b_{rj} is its standard deviation. Also, it is assumed that a single random variable (ε) follows a normal distribution $N(0,\sigma^2)$.

Under such an assumption, and properties of variance and covariance that expressed as below:
If $x_1, x_2,\dots, x_n$ be stochastic variables and

$$y_1 = \sum_{i=1}^{n} a_i x_i \qquad , \qquad y_2 = \sum_{i=1}^{n} b_i x_i$$

Where a_i , b_i for i=1,…,n are constant, then:

$$var(y_1) = \sum_{i=1}^{n} a_i^2\ var(x_i) + 2\sum\sum_{i<j} a_i a_j\, cov(x_i, x_j)$$
$$var(y_2) = \sum_{i=1}^{n} b_i^2\ var(x_i) + 2\sum\sum_{i<j} b_i b_j\, cov(x_i, x_j)$$
$$cov(y_1, y_2) = \sum_{i=1}^{n} a_i\ b_i\ var(x_i) + 2\sum\sum_{i<j}(a_i b_j + a_j b_i)cov(x_i, x_j)$$

So v_j becomes
$v_j = (\sum_{r=1}^{s} u_r b_{rj}\,\sigma)^2$ (3)

Incorporating (3) in (2) provides:

$$Max \qquad E(\textstyle\sum_{r=1}^{s} u_r \tilde{y}_{rd})$$
$$s.t. \qquad \sum_{i=1}^{m} v_i x_{id} = 1 \qquad (4)$$
$$\beta_j\left(\sum_{i=1}^{m} v_i x_{ij}\right) - \sum_{r=1}^{s} u_{r_j}\bar{y}_{rj} \geq (\sum_{r=1}^{s} u_r b_{rj}\,\sigma)\ \ F^{-1}(1-\alpha_j) \quad j = 1,2,\dots,n$$
$$u_r \geq 0, \quad r = 1,2,\dots,s$$
$$v_i \geq 0 \quad i = 1,2,\dots,m.$$

Next paying attention to $\tilde{y}_{rj} = \bar{y}_{rj} + b_{rj}\varepsilon$, we reformulate the objective of (4) as follows:

$$E\left(\sum_{r=1}^{s} u_r \tilde{y}_{rd}\right) = E\left(\sum_{r=1}^{s} u_r(\bar{y}_{rd} + b_{rd}\,\varepsilon)\right)$$

$= E\ (\sum_{r=1}^{s} u_r \bar{y}_{rd})+ E\ (\sum_{r=1}^{s} u_r b_{rd}\,\varepsilon)$

$$= E\left(\sum_{r=1}^{s} u_r \bar{y}_{rd}\right)$$

Because of $E(\sum_{r=1}^{s} u_r \bar{y}_{rd}) = \sum_{r=1}^{s} u_r \bar{y}_{rd}$ and $E(\varepsilon) = 0$. consequently, (4) can be formulated as the following linear programming model that is equivalent to (1):

$$Max \qquad \textstyle\sum_{r=1}^{s} u_r \bar{y}_{rd}$$

$$s.t. \qquad \sum_{i=1}^{m} v_i x_{id} = 1 \qquad (5)$$

$$\beta_j \left(\sum_{i=1}^{m} v_i x_{ij} \right) - \sum_{r=1}^{s} u_{r_j} \bar{y}_{rj} \geq \left(\sum_{r=1}^{s} u_r b_{rj} \sigma \right) \; F^{-1}(1-\alpha_j) \qquad j = 1,2,\dots,n$$

$$u_r \geq 0, \qquad r = 1,2,\dots,s$$

$$v_i \geq 0 \qquad i = 1,2,\dots,m.$$

That, we used this model in our method to ranking DMUs.

The proposed method

In this section, the preference relation is constructed by implementing a three-stage methodology as Wu et.al were stated in [16]. Note that we argue in the introduction that a preference relation is actually constructed based on a self-rated scheme and sexton model, thus we need to establish self-rated and cross-rated problem by use of SDEA at first. Hence, of the three stages, first we yield pairwise efficiency scores using two DEA models: the CCR model and the sexton model. The resulting pairwise efficiency scores are then utilized to construct the fuzzy preference relations at the second stage. At the last stage, by use of the row wise summation technique, the priority vector for ranking DMUs is obtained.

For simplicity, we suppose *n* DMUs peer for evaluation, DMUs produces multiple stochastic outputs y_{rj} (r = 1,2, . . . ,s) by utilizing multiple inputs x_{ij} (i= 1, 2, . . . ,m).

Step 1: The multiplier CCR model for DMUs by is as follows, the inputs are considered as deterministic variables and the outputs are considered as stochastic variables.

$$Max \qquad \sum_{r=1}^{s} u_r \bar{y}_{rd}$$

$$s.t. \qquad \sum_{i=1}^{m} v_i x_{id} = 1$$

$$\beta_j \left(\sum_{i=1}^{m} v_i x_{ij} \right) - \sum_{r=1}^{s} u_{r_j} \bar{y}_{rj} \geq \left(\sum_{r=1}^{s} u_r b_{rj} \sigma \right) \; F^{-1}(1-\alpha_j) \qquad j = 1,2,\dots,n$$

$$u_r \geq 0, \qquad r = 1,2,\dots,s$$

$$v_i \geq 0 \qquad i = 1,2,\dots,m.$$

The process of obtaining this model from SDEA has been discussed in last section.

Step2: By using Sexton [14] commute the value of E_{dj}.

$$E_{dj} = \frac{u_d^* Y_j}{v_d^* X_j} \qquad d, j = 1,2,\dots,n. \qquad (6)$$

Step3: Fuzzy preference relation was proposed by Nurmi [11]; Fan et al.[4]; Xu and Da [17]; Saaty [13]; Yager and Kacprzyk [18]; Chiclana et al [2].

Definition 1. Let $R = (r_{dj})_{n\times n}$, be a preference relation (matrix), then R is called a fuzzy preference relation, $r_{ij} \in [0,1]$; $r_{ij} + r_{ji} = 1$; $r_{ii} = 0.5$ for all $i, j \in \{1,2,\dots,n\}$.A value of 0.5 for r_{ij} or r_{ji} indicates an indifference between alternative i and j and a value of 1 for r_{ij} or a value of 0 for r_{ji} indicates that alternative i is unanimously preferred to j. Similarly, a value between 0.5 and 1 for r_{ij} or a value between 0 and 0.5 for r_{ji} stands for that alternative i is preferred to j.

Definition 2 :Tanino [15]. Let $R = (r_{dj})_{n\times n}$ be a fuzzy preference relation (matrix), then R is called a additive transitive consistency fuzzy preference relation, if $r_{ij} \in [0,1]$; $r_{ij} = r_{ik} - r_{kj} + 0.5$ for all $i, j, k \in \{1,2,\dots,n\}$.

Definition 3:Tanino [15]. Let $R = (r_{dj})_{n\times n}$ be a fuzzy preference relation (matrix), then R is called a product transitive consistency fuzzy preference relation, if $r_{ij} \in [0,1]$; $r_{ij} r_{jk} r_{ki} = r_{ji} r_{kj} r_{ik}$ for all $i, j, k \in \{1,2,\dots,n\}$.

We construct the pair wise comparison fuzzy preference relation (matrix) $R = (r_{dj})_{n\times n}$ for every pair of units d and j.

$$r_{dj} = \frac{E_{dd} + E_{jd}}{E_{dd} + E_{jd} + E_{dj} + E_{jj}}, r_{jj} = 0.5 \qquad j = 1,2,...,n. \qquad (7)$$

Step 4: A fuzzy preference relation $R = (r_{dj})_{n\times n}$ can be transformed to an additive transitive consistency fuzzy preference relation $B = (b_{dj})_{n\times n}$ by the following two formulas:

$$r_i = \sum_{j=1}^{n} r_{ij} \qquad i = 1,2,...,n \qquad (8)$$

$$b_{ij} = \frac{r_i - r_j}{2(n-1)} + 0.5 \qquad (9)$$

Step5: The consistency fuzzy preference relation B provides a ranking order of the alternatives. This is accomplished by using the row wise summation technique as follows. The ranking weight (score) w_d given to DMU_d is calculated:

$$w_d = \frac{\sum_{j=1}^{n} b_{dj}}{\sum_{d=1}^{n}\sum_{j=1}^{n} b_{dj}} \qquad (10)$$

Step 6: Rank the DMUs in the descending order of ranking scores w_d (d = 1, 2, . . . n). The most desirable DMU is the one with the highest score.

APPLICATION

As an empirical example, the proposed method is applied using some actual data of Iranian electricity distribution units. The Iranian electricity distribution units are public and act under the supervision of TAVANIR. The result are documented in Table1.

Table1. Scores by proposed model

DMU	Company	Stocha. Score, $\alpha = 0.5$	Rank
1	Azarbaijan Gharbi	0.5010	11
2	Esfahan	09573	4
3	Hamedan	0.5254	10
4	Khozestan	0.9600	3
5	Zanjan	0.4971	12
6	Fars	0.8407	5
7	Ardabil	0.6713	7
8	Markazi	0.5368	9
9	Ghazvin	0.9760	2
10	Semnan	0.9891	1
11	Hormozgan	0.5422	8
12	Yazd	0.7516	6

CONCLUSION

Stochastic models may be better suited for DEA when there is uncertainty associated with the inputs and/or outputs of DMUs or when an analyst may be wondering how much change can be incurred in the ranking of DMUs if some inputs and/or outputs change. In this paper we have discussed how to construct a preference relation using DEA, and derive the priority vector of the preference by a row wise summation technique in a multi attribute decision-making context, and then use the derived priority vector to better rate DMUs.

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