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Exact and Simple Solution to "Halving a Cube" Problem with Straightedge and Compass in Euclidean Geometry

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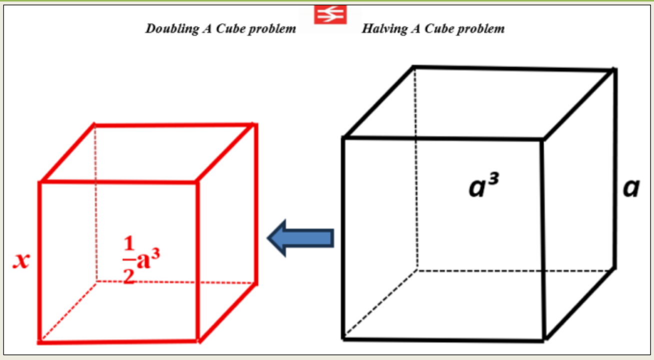
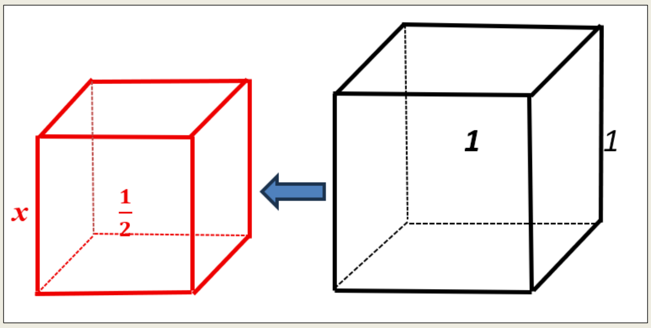
Abstract **Original Research Article**



If a is equal to a unit:



PART I

In a previous independent study published in the International Journal of Pure and Mathematical Sciences, an exact and rigorous solution to the classical Greek problem of Doubling the Cube was presented, achieved using only a straightedge and compass [30]. Building upon the logical framework established in that work, the recent study investigated a related and previously unexplored problem: *Halving A Cube.* This proposed solution is grounded in classical geometry and elements of algebraic geometry. The methodology employs geometric constructions to position a given cube with volume a^3, and side a, inside a cube with volume na^3 in a concentric configuration. From this arrangement, the side length x of the resulting cube is derived analytically using algebraic methods. This length is then interpreted geometrically, allowing for its exact construction using a straightedge and compass. The relationship between this article and problems of doubling and halving a cube is a *INDUCTIVE GENERALISATION,* highlighting their structural symmetry and methodological parallels.

Keywords: Classical geometry, straightedge and compass constructions, halving a cube, algebraic geometry, cube duplication/halving, geometric construction, divide a cube into a half.

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INTRODUCTION

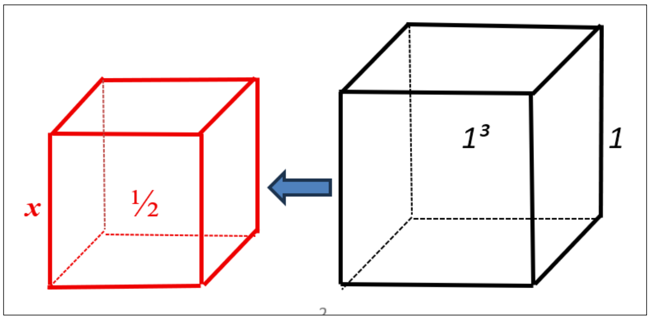
Before 2022, there are no exact solutions for all three of the ancient Greek mathematics challenged problems: "Squaring the Circle", "Trisecting an Angle", and "Doubling the Cube" using only a compass and a straightedge. It was proven by French mathematician Pierre Wantzel in 1837 that it is impossible to solve these problems using only a compass and a straightedge, except for certain specific cases [4]. Approximate solutions for these problems do exist. However, in 2022 and 2023, I did solve the "Trisecting An Angle" problem & "Squaring The Circle" problem with exact precision and accurate solutions using only the geometry method with straightedge and compass. Then, the "Doubling A Cube" problemwas also solved exactly. And then, these exact solutions were published [25,26,28,29,30,37,39,40].

These classical challenge problems above have been extremely important in the development of geometry. Three such problems stimulated so much interest among later geometers that they have come to be known as the "classical problems": Trisecting An Angle, Squaring A Circle and, Doubling A Cube (i.e., constructing a cube of which volume is twice that of a given cube). This problem is described in detail as follows: *Given a cube of side a & volume a^3 then use a straightedge & a compass to construct a cube of volume $2a^3$.* The challenge of "doubling the cube" refers to constructing a cube with twice the volume of a given cube using only a straightedge and compass. In classical Euclidean geometry, it has been proven that doubling the cube using only these two tools is not possible [4]. This impossibility is known as the Doubling A Cube Problem, and it is rooted in the fact that the cube root of 2 (which is necessary for doubling the cube) is not constructible using only a straightedge and compass. The construction requires finding a length equal to the cube root of 2, which is a transcendental number. Various attempts have been made throughout history to solve the problem, but they involve more advanced mathematical techniques beyond classical constructions [17-22]. These methods typically involve algebraic or geometric concepts that go beyond the scope of the traditional straightedge and compass constructions. Until 2022, there was no exact precision and accurate solution for the challenge of doubling the cube using only a straightedge and compass, based on classical Euclidean geometry. It is fair to say that although the problem of squaring the circle was to become the most famous in more modern times, certainly the problem of "Doubling A Cube" was the more famous in the time of the ancient Greeks. The "Doubling A Cube" challenge asks for a method to construct a cube which has double volume of a given cube. That means if the given cube volume is 1 unit then we have to construct a cube with side $\sqrt[3]{2}$ from this given unit cube, using only a compass and a straightedge. Cube duplication is believed to be impossible under the stated restrictions of Euclidean geometry, because the Delian constant is classified as an irrational number, which was stated to be geometrically irreducible [2,4,17,18]. Contrary to the impossibility consideration, the solution for this ancient problem is theorem, in which an elegant approach is presented, as a refute to the cube duplication impossibility statement. Geogebra software as one of the interactive geometry software is used to illustrate the accuracy of the obtained results, at higher accuracies which cannot be perceived using the idealized platonic straightedge and compass construction. Despite the efforts of many mathematicians, the problem remained unsolved for about three thousand years, and it became one of the most famous and intriguing unsolved problems in the history of mathematics. Today, the "doubling a cube" problem is considered to be a classic example of a difficult mathematical problem that was finally solved through the application of mathematical methods and techniques that were not available to the ancient Greeks. It is still studied in mathematics courses as a historical and challenging problem, and its solution continues to inspire and influence mathematicians and students alike. The French mathematician Pierre Wantzel, 1837, proved that it is impossible to double a cube using only a straightedge and compass [4]. This proof by Wantzel's raised the issue that the problem cannot be solved with the traditional methods of ancient Greek geometry. However, if we do not limit the use of only a straightedge and compass, we can use more modern mathematical tools to solve the problem. For example, we can use functions on the number line to solve equations and calculate the dimensions of the necessary cubes to double a given cube. But this calculation method is not considered within the scope of ancient Greek geometry.

In this article paper, a groundwork proof for solving the problem of HALVING the volume of a given cube, to a certain accuracy/precision, is presented. My obtained/earned results indicated that, algebraic irrationalities should be extended to plane geometric constructions, since subject to application, the desired degree of precision could be possible for compass straightedge construction. Through the presented discussion, it can be concluded that the Wantzel's statement of impossibility is not geometrically valid, since it does not give the geometrical relationship between the quadratic and the cubic extensions used/employed in the proof of cube duplication impossibility, with respect to the formal framework of classical geometric constructions [4]. The impossibility proof simply justifies a statement and not a concept. The focus of my Core Theorem in PART II below is to convert the problem from the complex 3D consideration as presented in the impossibility proof, into a simpler 2D problem, and its solution found following the formal Greek's rules of geometry. *For centuries, the problem of doubling a cube has been a subject to pseudo mathematical approaches, which do not reach the set limits of accuracy. It can therefore be affirmed that, by following the revealed approach, it is geometrically* *certainty to solve the coefficient $\sqrt[3]{2}$, which is the magnitude of the given cube.*

This article objectively presents a provable construction of generating a length of magnitude; as the geometrical solution for the ancient classical problem of halving the volume of a given cube arbitrarily. I follow strictly the constraint use of straightedge & compass to develop a method to solve accurately the "Halving A Cube" problem by geometry and algebraic geometry with a special technique that was developed by geometers and called "analysis". Geometers assumed the problem to have been solved and then, by investigating the properties of this solution, worked back to find an equivalent problem that could be solved, based on the givens. To obtain the formally correct solution of the original problem, then, geometers reversed the procedure: firstly, the data were used to solve the equivalent problem derived in the analysis, and, from the solution obtained, the original problem was then solved. In contrast to analysis, this reversed procedure is called "synthesis". I adopted the technique "ANALYSIS" to solve precisely the "Halving A Cube" problem with only a straightedge & compass, using only classic Euclidian Geometry.

For a Unit Cube, side = 1, then its halved cube (volume $\frac{1}{2}$) side x = $\frac{\sqrt[3]{1}}{\sqrt[3]{2}}$ and this research shows how to construct, precisely, a straight-line segment x geometrically.

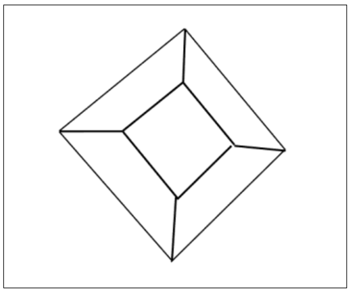


PART II

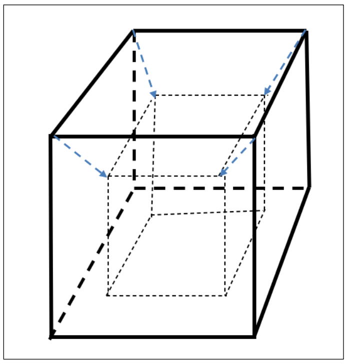
PROPOSITION

1. Definition 01: "HEAD-CUT PYRAMID"

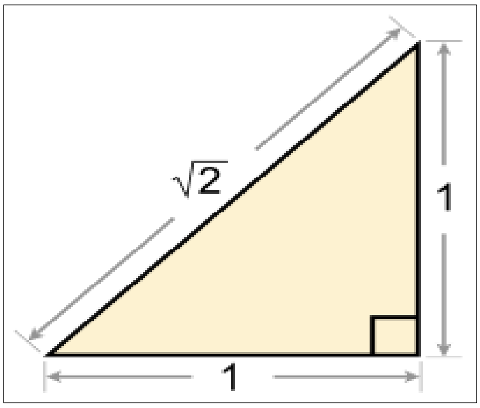
Given a regular 5-facet pyramid. A plane paralleled to the pyramid base will cut and divides the pyramid into 2 parts: one is the smaller pyramid and the other is the "Head-cut Pyramid", described in the following figure:



From the definition above, the head-cut pyramid has the top and bottom bases paralleled and squared. The top square is smaller than the bottom square. Its other 4 side faces are the 4 equal isosceles trapezoids



2. Theorem 01: Given a UNIT LENGTH, then the exact lengths of $\sqrt{2}$ and $\sqrt{3}$ are constructive in algebraic geometry with a compass and a straightedge



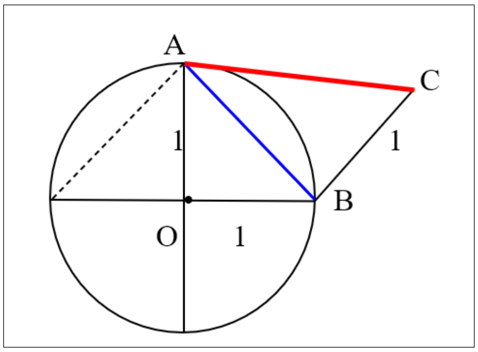


Figure 1: ***Length AB (blue) and AC (red) are the exact geometrical lengths of*** $\sqrt{\mathbf{2}}$ **and** $\sqrt{\mathbf{3}}$

PROOF:

Use the given unit length U and *a compass & a straightedge* to draw a circle (O, U=1) and 2 perpendicular diameters of the circle (Figure 01). Then, in the right-angle triangle AOB, Pythagoras' theorem gives:

$AB^2 = 1^2 + 1^2 = 2$

Length AB ≡ $\sqrt{\mathbf{2}}$ *(Figure 01)*

And then, use *the straightedge & the compass* to construct the right-angle triangle ABC, of which BC = Unit = 1. Similarly to the above:

$AC^2 = AB^2 + 1^2 = (\sqrt{\mathbf{2}})^2 + 1^2 = 3$

Length AC ≡ $\sqrt{\mathbf{3}}$ *(Figure 01 above).*

Note: *To generate one more step, we draw a straight line perpendicular to AC at C then mark D, CD = 1 (unit), using the straightedge and the compass (Figure 2 below). From the right-angle triangle ACD we get:*

$AD^2 = AC^2 + 1^2 = (\sqrt{\mathbf{3}})^2 + 1^2 = 4$ ➔ $AD = \sqrt{\mathbf{4}}$

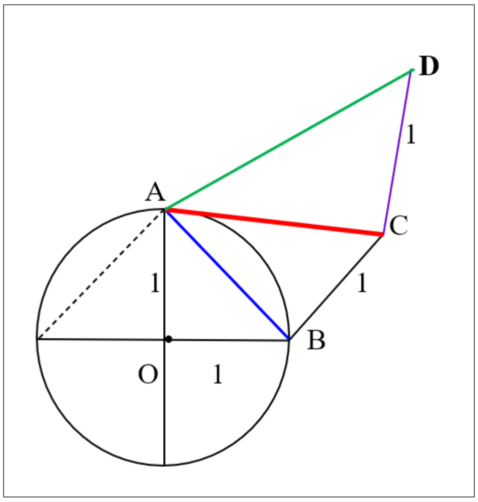
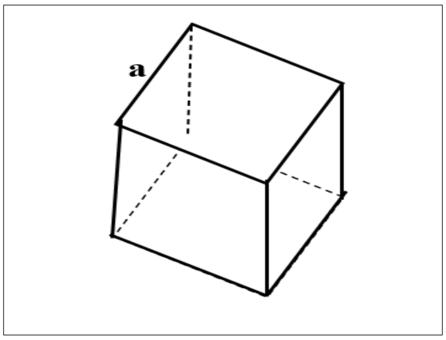


Figure 02: ***Length AD (green) is the exact length of*** $\sqrt{\mathbf{4}}$

3. Theorem 02: ***Core Theorem "Halving A Cube"***

Given a cube side a, $a \subset R$ (rational number), then side XY of a cube with volume $\frac{1}{2}a^3$ is a rational number and $XY = a(0.5 + \frac{1}{6}\sqrt{3})$ is constructive with a compass & a straightedge.



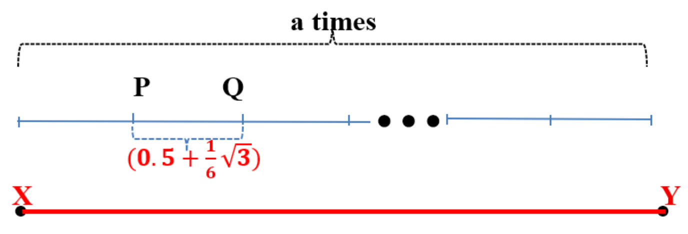


Figure 03: **Exact length XY =** $\mathbf{a(0.5+\frac{1}{6}\sqrt{3})}$.

PROOF:

Let the objective cube with volume $\frac{1}{2}a^3$ locate inside the given cube with volume a^3 and side a, concentrically as described in Figure 04 below.

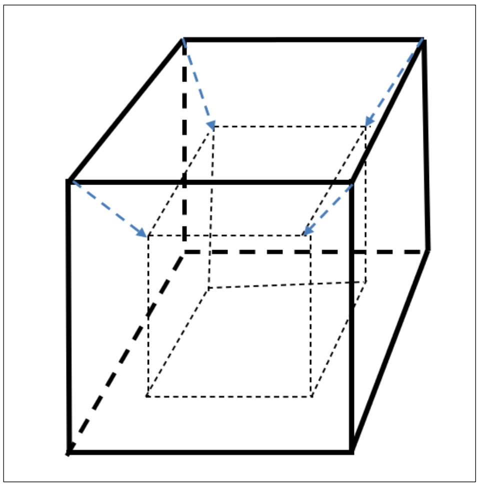
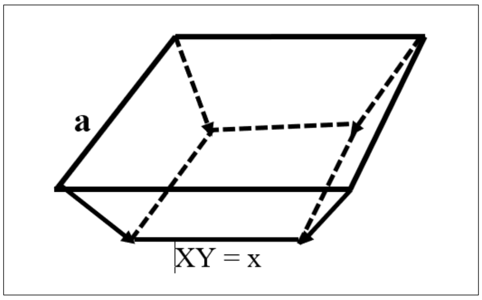


Figure 4: ***Concentric image of the given cube and the halved cube***



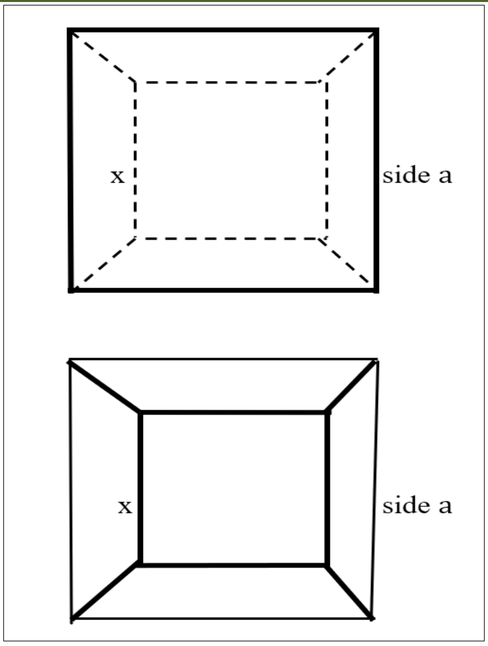


Figure 5: One of the 6 head-cut pyramid from Figure 4 above

Let unknown x be the side of the cube with volume $\frac{1}{2}a^3$, then $x = a(\sqrt[3]{\frac{1}{2}})$, and any head-cut pyramid gets volume (Figure 05):

$(\frac{a^2+x^2}{2})(\frac{a-x}{2}) = \frac{1}{12}a^3$
$(\frac{a^3-a^2x+ax^2-x^3}{4}) = \frac{1}{12}a^3$
$12a^3 - 12a^2x + 12ax^2 - 12x^3 = 4a^3$
Because $x^3 = \frac{1}{2}a^3$ (by assumption), we get
$12a^3 - 12a^2x + 12ax^2 - 6a^3 = 4a^3$

$12a^3 - 12a^2x + 12ax^2 - 6a^3 - 4a^3 = 0$
$-12a^2x + 12ax^2 + 2a^3 = 0$,
OR
$6x^2 - 6ax + a^2 = 0$
is a quadratic equation in term of the unknow x. Replace x in the equation by $a(\sqrt[3]{\frac{1}{2}})$ as $x = a(\sqrt[3]{\frac{1}{2}})$ to get:

$6a^2(\sqrt[3]{\frac{1}{2}})^2 - 6a^2(\sqrt[3]{\frac{1}{2}}) + a^2 = 0$

$6(\sqrt[3]{\frac{1}{2}})^2 - 6(\sqrt[3]{\frac{1}{2}}) + 1 = 0$ (1)

Let $(\sqrt[3]{\frac{1}{2}})$ be y, then (1) becomes a quadratic equation in term of y:
$6y^2 - 6y + 1 = 0$ (2)

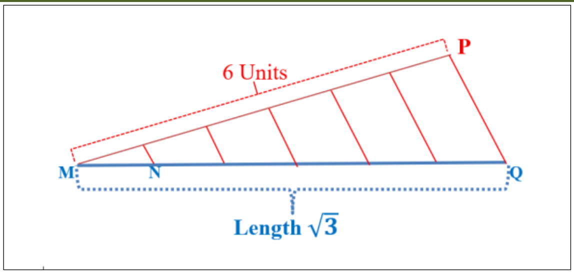
Sove this quadratic equation (2), we get
$\Delta = 36 - 24 = 12$
$\sqrt{\Delta} = \sqrt{12} = \sqrt{4(3)} = 2\sqrt{3}$

$y = \frac{6+2\sqrt{3}}{12} = (\frac{1}{2} + \frac{1}{6}\sqrt{3}) \approx 0.5 + 0.2887 = 0.7887 > 0$

and
$y' = \frac{6-2\sqrt{3}}{12} = (\frac{1}{2} - \frac{1}{6}\sqrt{3}) \approx 0.5 - 0.2887 = 0.2113 > 0$

Consider the approximated side length of the halved cube volume $\frac{1}{2}a^3$, which is 0.7937, as $\sqrt[3]{2} \approx 1.2599$. Because the approximated side of the halved cube (0.7937) is the nearest value to $y = (\frac{1}{2} + \frac{1}{6}\sqrt{3}) \approx 0.7887$, this y - root of equation (2) - is chosen.
Then, the length $x = ya = \mathbf{a(0.5 + \frac{1}{6}\sqrt{3})} = a(\frac{1}{2} + \frac{1}{6}\sqrt{3})$ is proved constructively by straightedge & compass as follows:

- By Theorem 01 above, $\mathbf{\sqrt{3}}$ is constructive with a straightedge & compass.
- Then, $\mathbf{\frac{1}{6}\sqrt{3}}$ is also constructive as described below
  Let MQ be the identified length $\mathbf{\sqrt{3}}$ above, then draw MP that is a length of 6 given Units. Connect PQ and draw 5 other 6 line segments parallel to PQ starting from 6 unit ends on the MP (Figure 06, a.) below). These parallel line segments divide MQ = **Length** $\mathbf{\sqrt{3}}$ into 6 equal parts. Therefore, any of these parts, i.e. MN, is the required length $\mathbf{\frac{1}{6}\sqrt{3}}$. (Figure 06, a.))
- And then, MN + length 0.5 Units is the required length constructive by straightedge & compass (Figure 06, b.) below).



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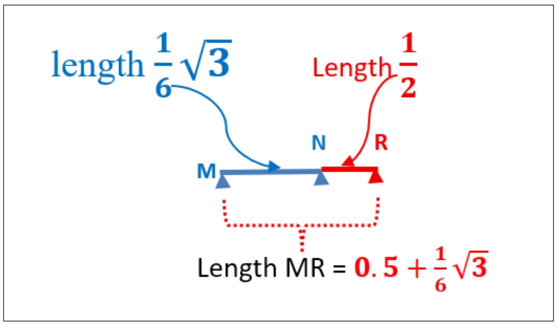

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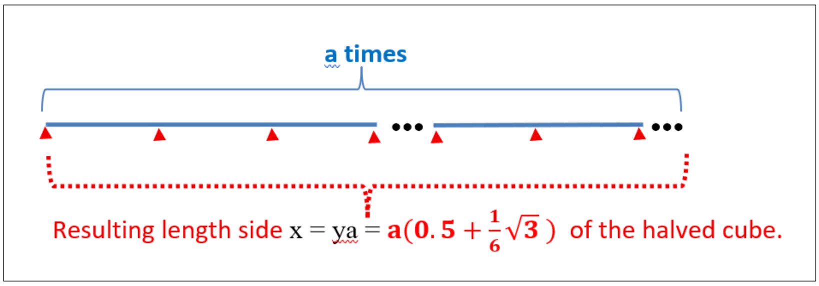

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Figure 6: Illustrating image of Theorem 02: ***Core Theorem "Halving A Cube"***

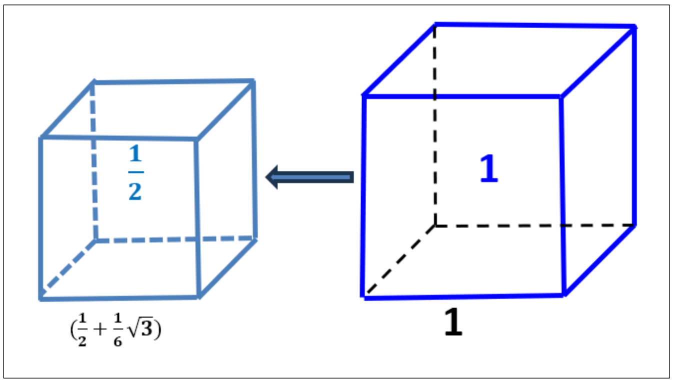
- And then, add length MR a times, $\mathbf{a} \subset \mathbf{R}$, to get the resulting length of side y for constructing the resulting cube with volume $\frac{1}{2}\mathbf{a}^3$ by straightedge & compass in the Euclidean Geometry (Figure 06, c.) above).

PART III

METHOD OF HALVING A CUBE

Apply all the proved Theorems in PART II above to use a straightedge and a compass to construct a halved cube of volume $\frac{1}{2}a^3$ from a given cube of side a, volume a^3.

If a = 1, then it is a special case of the "halving a cube" problem to create a cube with volume $\frac{1}{2}$ and then the side x of this cube, which is $(\frac{1}{2} + \frac{1}{6}\sqrt{3})$, is easier to construct by Theorems 01, 02, and the Core Theorem with straightedge & compass, as follows:



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PART IV

DISCUSSION AND CONCLUSION

The "Doubling The Cube" problem refers to the ancient Greek problem of constructing a cube with double the volume of a given cube, using only a straightedge and compass. The problem dates back to at least the 5th century BC and was one of the three famous unsolved problems of ancient Greek mathematics, alongside the "Trisecting An Angle" and the "Squaring The Circle" problem.

This research result objectively presents a provable construction of generating a length of magnitude, as the geometrical solution for a new "Halving A Cube" problem, similar to the ancient classical problem doubling the volume of a given cube. The "Halving A Cube" problem, is precisely solved by the ANALYTICS method to concentrically locate an unknown cube of volume $\frac{1}{2}a^3$ in its double cube with volume a^3, side a. In other words, I did succeed the concentric location for the unknown cube of volume $\frac{1}{2}a^3$ inside the given cube of volume a^3 to solve exactly the problem with a straightedge and a compass. Such positioning creates six regular head-cut pyramids that take up the space around the halved cube. These six 3D shapes are six specific cuboids, called Head-cut Pyramid. From there calculate the volume V of 1 of the 6 head-cut pyramids, and then set up the equation $V = \frac{1}{6}(\frac{1}{2}a^3)$. In that cubic equation, the cubic term will be equal to the volume of the halved cube itself which is $\frac{1}{2}a^3$, therefore the equation is reduced from a cubic equation to a quadratic equation. Solving this quadratic equation will have a root of the irrational number that is the algebraic value of the edge of the halving cube with volume $\frac{1}{2}a^3$. Then apply the theorems in Part II to use a straightedge & compass to construct the exact lengths for edges/sides of the halving cube.

I believe the problem of halving a cube is a well-known one compared to the millenary problem of "Doubling The Cube", that mathematicians stated as impossible to geometrically resolve because the its unknown side x in the expression $x^3 = 2a^3$ is classified as an irrational number. The incomprehensible proof of impossibility concerned showing that the cubic equation $x^3 = 2a^3$ is unsolved, which is not reducible; and thus geometrically unsolvable. Irrational numbers are mathematically defined as being not a finite solution from a division. However, this is not a fashionable definition, as most number divisions are open-loop operations that can never be ended. The impossibility proof of doubling a cube was based on three-dimensional cubic extensions in abstract algebra, an approach that entirely shifted the problem to solid geometry from its Greek"s definition in plane geometry, and therefore the algebraic statement of impossibility has no geometrical validity. This is evident from the fact that no two facets of a cube can share all four vertices from two different planes. However, according to this study result, the impossible imprecise classification should not be extended to geometry so that the irrationality definition was stated as "*algebraic irrationality is not a constructible number of the geometry*". The possibility to solve geometrically the coefficient constant $\sqrt[3]{2}$ to an exact precision is proved. This study paper also presents a geometrically certain method under the set restrictions of Euclidean geometry (in the sense that, all presented constructions have been reduced to the Euclidean postulates of practical geometry), by the construction of the relation as depicted in the justification section in PART II above.

Open area for research

The Core Theorem "Halving A Cube", applied for halving the cube of volume a^3 into the cube with volume $\frac{1}{2}a^3$, certainly converted from its cubic equation to its quadratic equation successfully, to have a precise geometrical length constructed by straightedge & compass. Therefore, the new problem of "*converting a cubic equation to a quadratic equation, equivalently*", is a possible open research area in algebraic geometry & pure geometry.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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