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Research Article

Construction of the Diophantine Triple Involving Polygonal Numbers

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Abstract: We searched for three distinct special polygonal numbers a, b, c such that the product of any two of them added with an integer is a perfect square.
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INTRODUCTION

Let n be an integer.A set of positive integers $\{a_1, a_2, \ldots, a_m\}$ is said to have the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$;such a set is called a Diophantine m-tuple or a P_n set of size m. The problem of construction of such set was studied by Diophantus. He studied the problem of finding four (positive rational) numbers such that the product of any two of them increased by 1 is a perfect square. He obtained the solution as $\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}$[1].The first set of four positive integers with the above property was found by Fermat and it was$\{1,3,8,120\}$.Euler gave the solution$\{a, b, a+b+2r, 4r(r+a)(r+b)\}$ where $ab+1=r^2$ [2]. In [3], the extendibility of the Diophantine triple involving Jacobsthal numbers $(J_{2n-1}, J_{2n+1}-3, 2J_{2n}+J_{2n-1}+J_{2n+1}-3)$ with the property $D(1)$ is presented.For an extensive review of various articles one may refer [4-13].In this communication we present ten sections where in each of which we find the Diophantine triples from special polygonal numbers namely octagonal, decagonal, hexadecagonal, octadecagonal, centered heptagonal, centered tetra decagonal, centered dnonagonal, centered decagonal, centered dodecagonal numbers respectively. A few interesting relations among the numbers in each of the above Diophantine triples are presented.

METHOD OF ANALYSIS

Section A

(i) Let $=3n^2-2n$, $b=3n^2+4n+1$ be octagonal numbers of rank n and $n+1$ respectively such that $ab+1$ is a perfect square, sayα^2.

Let c be any non-zero integer such that

$ac+1=\beta^2$ (1)

$bc+1=\gamma^2$ (2)

Setting $\beta=a+\alpha, \gamma=b+\alpha$ and subtracting (1) from (2), we obtain $c=a+b+2\alpha$
Similarly by choosing $\beta=a-\alpha, \gamma=b-\alpha$, we obtain $c=a+b-2\alpha$ and two values of c are given by $c=12n^2+4n-1$ and $c=3$.

We notice that $\{3n^2-2n, 3n^2+4n+1, 12n^2+4n-1\}$ and $\{3n^2-2n, 3n^2+4n+1, 3\}$ are Diophantine triples with the property D (1).

Remark

Using Euler's solution [2], the Diophantine quadrapules for the above two choices are respectively $\{3n^2-2n, 3n^2+4n+1, 12n^2+4n-1, 4(3n^2+n-1)(6n^2-n-1)(6n^2+5n)\}$ and $\{3n^2-2n, 3n^2+4n+1, 3, 4(3n^2+n-1)(3n-1)(3n+2)\}$ with the property$D(1)$.

(ii) Let$=3n^2-8n+5$, $b=3n^2+4n+1$ be octagonal numbers of rank $n-1$ and $n+1$ respectively such that $ab+4$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac+4=\beta^2$ (3)

$bc + 4 = \gamma^2$ (4)

Applying the procedure as mentioned in case (i),the values of c satisfying (3) and (4) we have $c = 12n^2 - 8n$ and $c = 12$.

Thus we observe that $\{3n^2 - 8n + 5, 3n^2 + 4n + 1, 12n^2 - 8n\}$ and $\{3n^2 - 8n + 5, 3n^2 + 4n + 1, 12\}$ are Diophantine triples with the property $D(4)$.

Section B

Let $= 5n^2 - 4n$, $b = 5n^2 + 6n + 1$ be decagonal numbers of rank n and $n + 1$ respectively such that $ab + 4$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac + 4 = \beta^2$ (5)

$bc + 4 = \gamma^2$ (6)

Applying the procedure as mentioned in section A, the values of c satisfying (5) and (6) we have $c = 20n^2 + 4n - 3$ and $c = 5$.

Thus we observe that $\{5n^2 - 4n, 5n^2 + 6n + 1, 20n^2 + 4n - 3\}$ and $\{5n^2 - 4n, 5n^2 + 6n + 1, 5\}$ are Diophantine triples with the property $D(4)$.

Section C

(i) Let $a = 7n^2 - 6n$, $b = 7n^2 - 20n + 13$ be hexadecagonal numbers of rank n and $n - 1$ respectively such that $ab + 9$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac + 9 = \beta^2$ (7)

$bc + 9 = \gamma^2$ (8)

Applying the procedure as mentioned in section A,the values of c satisfying (7) and (8) are obtained as $c = 28n^2 - 52n + 19$ and $c = 7$.

Thus we observe that $\{7n^2 - 6n, 7n^2 - 20n + 13, 28n^2 - 52n + 19\}$ and $\{7n^2 - 6n, 7n^2 - 20n + 13, 7\}$ are Diophantine triples with the property $D(9)$.

(ii) Let $a = 7n^2 - 6n$, $b = 7n^2 + 8n + 1$ be hexadecagonal numbers of rank n and $n + 1$ respectively such that $ab + 9$ is a perfect square, then we have $c = 28n^2 + 4n - 5$ and $c = 7$.

Thus we observe that $\{7n^2 - 6n, 7n^2 + 8n + 1, 28n^2 + 4n - 5\}$ and $\{7n^2 - 6n, 7n^2 + 8n + 1, 5\}$ are Diophantine triples with the property $D(9)$.

Section D

Let $= 8n^2 - 23n + 15$, $b = 8n^2 + 9n + 1$ be octadecagonal numbers of rank $n - 1$ and $n + 1$ respectively such that $ab + 49$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac + 49 = \beta^2$ (9)

` $bc + 49 = \gamma^2$ (10)

Employing the procedure as mentioned above,the values of c satisfying (9) and (10) are given by $c = 32n^2 - 28n$ and $c = 32$.

Thus we observe that $\{8n^2 - 23n + 15, 8n^2 + 9n + 1, 32n^2 - 28n\}$ and $\{8n^2 - 23n + 15, 8n^2 + 9n + 1, 32\}$ are Diophantine triples with the property $D(49)$.

Section E

Let= $7n^2-7n+2$, $b=7n^2+21n+16$ be two times centered heptadecagonal numbers of rank $n-1$ and $n+1$ respectively such that $ab-7$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac-7=\beta^2$ (11)

$bc-7=\gamma^2$ (12)

Employing the procedure as mentioned above, the values of c satisfying (11) and (12) are given by $c=28n^2+28n+8$and$c=28$.

Thus we observe that $\{7n^2-7n+2,7n^2+21n+16,28\,n^2+28n+8\}$ and $\{7n^2-7n+2,7n^2+21n+$ *16,28* are Diophantine triples with the property *D(−7)*.

Section F

Let= $7n^2-7n+1$, $b=7n^2+21n+15$ be centered tetradecagonal numbers of rank $n-1$ and $n+1$ respectively such that $ab+21$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac+21=\beta^2$ (13)

$bc+21=\gamma^2$ (14)

Employing the procedure as mentioned above, the values of c satisfying (13) and (14) are given by $c=28n^2+28n+4$ and $c=28$.

Thus we observe that $\{7n^2-7n+1,7n^2+21n+15,28\}\,n^2+28n+4\}$ and $\{7n^2-7n+1,7n^2+21n+$ *15,28* are Diophantine triples with the property*D(21)*.

Section G

Let= $9n^2-9n+2$, $b=9n^2+27n+20$ be two times centered nonagonal numbers of rank $n-1$ and $n+1$ respectively such that $ab+9$ is a perfect square, sayα^2.

Let c be any non-zero integer such that

$ac+9=\beta^2$ (15)

$bc+9=\gamma^2$ (16)

Applying the procedure as mentioned above,the values of c satisfying (15) and (16) are given by $c=36n^2+36n+8$ and $c=36$.

Thus we observe that $\{9n^2-9n+2,9n^2+27n+20,36\}n^2+36n+8\}$ and $\{9n^2-9n+2,9n^2+27n+$ *20,36* are Diophantine triples with the property*D(9)*.

Section H

Let= $10n^2-10n+2$, $b=10n^2+30n+22$ be two times centered decagonal numbers of rank $n-1$ and $n+1$ respectively such that $ab+20$ is a perfect square, sayα^2.

Let c be any non-zero integer such that

$ac+20=\beta^2$ (17) $bc+20=\gamma^2$ (18)

Applying the procedure as mentioned above, the values of c satisfying (17) and (18) are given by $c=40n^2+40n+8$and$c=40$.

Thus we observe that $\{10n^2-10n+2,10n^2+30n+22,40\}n^2+40n+8\}$ and $\{10n^2-10n+2,10n^2+$ *30n+22,40* are Diophantine triples with the property*D(20)*.

Section I

Let= $5n^2-5n+1$, $b=5n^2+15n+11$ be centered decagonal numbers of rank $n-1$ and $n+1$ respectively such that $ab+5$ is a perfect square, say α^2.

Let$c$ be any non-zero integer such that

$ac + 5 = \beta^2$ (19) $bc + 5 = \gamma^2$ (20)

Applying the procedure as mentioned above, the values of c satisfying (19) and (20) are given by $c = 20n^2 + 20n + 4$ and $c = 20$.

Thus we observe that $\{5n^2 - 5n + 1, 5n^2 + 15n + 11, 20\}\, n^2 + 20n + 4\}$ and $\{5n^2 - 5n + 1, 5n^2 + 15n + 11, 20$ are Diophantine triples with the property $D(5)$.

Section J

Let$= 6n^2 - 6n + 1$, $b = 6n^2 + 18n + 13$ be centered dodecagonal numbers of rank $n - 1$ and $n + 1$ respectively such that $ab + 12$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$ac + 12 = \beta^2$ (21) $bc + 12 = \gamma^2$ (22)

Applying the procedure as mentioned above,the values of c satisfying (21) and (22) are given by $c = 24n^2 + 24n + 4$ and $c = 24$.

Thus we observe that $\{6n^2 - 6n + 1, 6n^2 + 18n + 13, 24\}\, n^2 + 24n + 4\}$ and $\{6n^2 - 6n + 1, 6n^2 + 18n + 13, 24$ are Diophantine triples with the property $D(12)$.

CONCLUSION

To conclude, one may search for other Diophantine triples using various special numbers.

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