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Research Article

On The Higher Degree Equation with Six Unknowns $x^6-y^6-2z^3=5^{2n}T^{2m}(w^2-p^2)$

MA Gopalan[1*], S Vidhyalakshmi[2], E Premalatha[3]

[1,2] Professor, Department of Mathematics, SIGC, Trichy-620002, Tamilnadu, India;

[3] Assistant Professor, Department of Mathematics, National college, Trichy-620001, Tamilnadu, India

***Corresponding author**
Dr. M. A. Gopalan
Email: mayilgopalan@gmail.com

Abstract: We presents non zero solutions of the $(2m+2)^{th}$ degree non-homogeneous Diophantine equation in six unknowns represented by $x^6-y^6-2z^3=5^{2n}T^{2m}(w^2-p^2)$ in which $m,n \in z^+$. In particular, different patterns of non-zero integral solutions of the above equation along with a few interesting properties among the solutions are exhibited.

Keywords: Higher degree equation with six unknowns, Integral solutions

INTRODUCTION

Diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity as can be seen from [1,2].The problem of finding all integer solutions of a Diophantine equation with three or more variables and degree at least three, in general presents a good deal of difficulties. There is vast general theory of homogeneous quadratic equations with three variables [3-7].Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research [8-11]. A lot is known about equations in two variables in higher degrees. For equations with more than three variables and degree at least three very little is known. It is worth to note that undesirability appears in equations, even perhaps at degree four with fairly small co-efficients. It seems that much work has not been done in solving higher order Diophantine equations. In [12-20] a few higher order equations are considered for integral solutions. In this communication a $(2m+2)^{th}$ degree non-homogeneous equation with six variables represented by $x^6-y^6-2z^3=5^{2n}T^{2m}(w^2-p^2)$ is considered and in particular a few interesting relations among the solutions are presented.

NOTATIONS USED

- $t_{m,n}$ - Polygonal number of rank n with size m.
- P_n^m - Pyramidal number of rank n with size m.
- gn_a - Gnomonic number of rank a
- so_n - Stella octangular number of rank n
- pr_n - Pronic number of rank n
- Pt_n - Pantatope number of rank n
- $f_{m,s}^n$ - m-dimensional figurate number of rank n with s sides.

METHOD OF ANALYSIS

The diophantine equation representing the Higher degree equation with six unknowns under consideration is

$$x^6-y^6-2z^3=5^{2n}T^{2m}(w^2-p^2) \tag{1}$$

Introduction of the transformation

$$x = u + v,\ y = u - v,\ z = 2uv,\ w = uv + 3 \quad p = uv - 3 \qquad (2)$$

in (1) leads to $u^2 + v^2 = 5^n T^m$ (3)

Now, we solve (3) through different methods and thus obtain different patterns of solutions to (1)

PATTERN -I

Assume $T = T(a,b) = a^2 + b^2$ (4)

where a and b are non zero distinct integers

Write 5 as $5 = (2+i)(2-i)$ (5)

Using (4) & (5) in (3) and applying the method of factorization, define

$$u + iv = (2+i)^n (a+ib)^m = (\alpha_1 + i\beta_1)(\gamma + i\delta), say$$

Equating the real and imaginary parts, we have

$$u = \alpha_1\gamma - \beta_1\delta$$

$$v = \alpha_1\delta + \beta_1\gamma$$

Hence in view of (2), the corresponding solutions of (1) are given by

$$x = \alpha_1\gamma - \beta_1\delta + \alpha_1\delta + \beta_1\gamma$$

$$y = \alpha_1\gamma - \beta_1\delta - \alpha_1\delta - \beta_1\gamma$$

$$z = 2(\alpha_1\gamma - \beta_1\delta)(\alpha_1\delta + \beta_1\gamma)$$

$$w = (\alpha_1\gamma - \beta_1\delta)(\alpha_1\delta + \beta_1\gamma) + 3$$

$$p = (\alpha_1\gamma - \beta_1\delta)(\alpha_1\delta + \beta_1\gamma) - 3$$

where

$$\alpha_1 = \frac{1}{2}[(2+i)^n + (2-i)^n]$$

$$\beta_1 = \frac{1}{2i}[(2+i)^n - (2-i)^n]$$

Illustration-I

Let n=2, m=3

Thus the corresponding non-zero distinct integral solutions of (1) are

$$x = x(a,b) = 7a^3 - 3a^2b - 21ab^2 + b^3$$

$$y = y(a,b) = -a^3 - 21a^2b + 3ab^2 + 7b^3$$

$$z = z(a,b) = 2(3a^3 - 12a^2b - 9ab^2 + 4b^3)(4a^3 + 9a^2b - 12ab^2 - 3b^3)$$

$$w = w(a,b) = (3a^3 - 12a^2b - 9ab^2 + 4b^3)(4a^3 + 9a^2b - 12ab^2 - 3b^3) + 3$$

$$p = p(a,b) = (3a^3 - 12a^2b - 9ab^2 + 4b^3)(4a^3 + 9a^2b - 12ab^2 - 3b^3) - 3$$

$$T = T(a,b) = a^2 + b^2$$

A few interesting properties observed are as follows:

1. $[\frac{z}{2}(a,b), w(a,b) - 3, p(a,b) + 3]$ forms a Pythagorean triple.
2. $x(1, b(b+1)) + 7y(1, b(b+1)) = 25SO_{b(b+1)} - 250t_{3,a}$
3. $x^2(a,b) - y^2(a,b) = 2z(a,b) + w(a,b) - p(a,b) + 6$
4. $-\{x(a,1) + 7y(a,1) + T(a,1) - t_{4,a} - 51\}$ is a Nasty number.
5. $20\{7x(a,1) - y(a,1) - 75(gn_a - 1)\}$ is a cubical integer

PATTERN-II:

Instead of (5), write 5 as $5=(1+2i)(1-2i)$ (6)

Following the procedure similar to Pattern-I and performing a few calculations, the corresponding non-zero distinct integral solutions of (1) are found to be

$$
\begin{aligned}
x &= \alpha_2\gamma-\beta_2\delta+\alpha_2\delta+\beta_2\gamma \\
y &= \alpha_2\gamma-\beta_2\delta-\alpha_2\delta-\beta_2\gamma \\
z &= 2(\alpha_2\gamma-\beta_2\delta)(\alpha_2\delta+\beta_2\gamma) \\
w &= (\alpha_2\gamma-\beta_2\delta)(\alpha_2\delta+\beta_2\gamma)+3 \\
p &= (\alpha_2\gamma-\beta_2\delta)(\alpha_2\delta+\beta_2\gamma)-3
\end{aligned}
$$

where

$$\alpha_2=\frac{1}{2}[(1+2i)^n+(1-2i)^n]$$

$$\beta_2=\frac{1}{2i}[(1+2i)^n-(1-2i)^n]$$

Illustration-II

Let n=2, m=5

The corresponding non-zero distinct integral solutions of (1) are

$$x=x(a,b)=-13a^5+130a^3b^2-65ab^4-45a^4b+90a^2b^3-9b^5$$

$$y=y(a,b)=-9a^5+90a^3b^2-45ab^4+65a^4b-130a^2b^3+13b^5$$

$$
\begin{aligned}
z=z(a,b)=2(&-11a^5+110a^3b^2-55ab^4+10a^4b-20a^2b^3+2b^5)\\
&(-2a^5+20a^3b^2-10ab^4-55a^4b+110a^2b^3-11b^5)
\end{aligned}
$$

$$
\begin{aligned}
w=w(a,b)=&(-11a^5+110a^3b^2-55ab^4+10a^4b-20a^2b^3+2b^5)*\\
&(-2a^5+20a^3b^2-10ab^4-55a^4b+110a^2b^3-11b^5)+3
\end{aligned}
$$

$$
\begin{aligned}
p=p(a,b)=&(-11a^5+110a^3b^2-55ab^4+10a^4b-20a^2b^3+2b^5)*\\
&(-2a^5+20a^3b^2-10ab^4-55a^4b+110a^2b^3-11b^5)-3
\end{aligned}
$$

$$T=T(a,b)=a^2+b^2$$

Properties:

1. $w(A,B)+p(A,B)-z(A,B)=0$
2. $y(a,1)-x(a,1)=120f^a_{5,6}-180f^a_{3,5}+85\Pr_{a^2-1}-5t_{4,4a}+7gn_a+15$
3. $9x(a,1)-13y(a,1)+1250(12f^a_{4,4}-8f^a_{3,6}-6t_{3,a})\equiv 0(\bmod 250)$
4. $9x(1,b)-13y(1,b)-1250SO_a\equiv 0(\bmod 250)$
5. $x^2(a,b)-y^2(a,b)=z(a,b)+w(a,b)+p(a,b)$
6. $8\{13y(a,1)-9x(a,1)+t_{4,50A}-250\}$ is a biquadratic integer.

PATTERN-III:

In addition to (5) & (6) , write 5 as $5=\frac{1}{25}(11+2i)(11-2i)$

Following the procedure similar to Pattern-I, and performing a few calculations, the corresponding non-zero distinct integral solutions of (1) are given by

$$x = \alpha_3\gamma - \beta_3\delta + \alpha_3\delta + \beta_3\gamma$$
$$y = \alpha_3\gamma - \beta_3\delta - \alpha_3\delta - \beta_3\gamma$$
$$z = 2(\alpha_3\gamma - \beta_3\delta)(\alpha_3\delta + \beta_3\gamma)$$
$$w = (\alpha_3\gamma - \beta_3\delta)(\alpha_3\delta + \beta_3\gamma) + 3$$
$$p = (\alpha_3\gamma - \beta_3\delta)(\alpha_3\delta + \beta_3\gamma) - 3$$

where

$$\alpha_3 = \frac{1}{2.5^n}[(11+2i)^n + (11-2i)^n]$$
$$\beta_3 = \frac{1}{2i.5^n}[(11+2i)^n - (11-2i)^n]$$

Illustration-III

Let n=2, m=5

The corresponding non-zero distinct integral solutions of (1) are

$$x = x(A,B) = 65A^2 - 65B^2 + 90AB$$
$$y = y(A,B) = 45A^2 - 45B^2 - 130AB$$
$$z = z(A,B) = 2(55A^2 - 55B^2 - 20AB)(10A^2 - 10B^2 + 110AB)$$
$$w = w(A,B) = (55A^2 - 55B^2 - 20AB)(10A^2 - 10B^2 + 110AB) + 3$$
$$p = p(A,B) = (55A^2 - 55B^2 - 20AB)(10A^2 - 10B^2 + 110AB) - 3$$
$$T = T(A,B) = 25(A^2 + B^2)$$

Properties:

1. $9x(t_{3,A}, t_{3,A+2}) - 13y(t_{3,A}, t_{3,A+2}) = 15000Pt_A$
2. $9x(A, A(A+1)) - 13y((A, A(A+1)) + T(A, A(A+1)) - 5000P_A^3 \equiv 0(\mathrm{mod}\, 25)$
3. $x^2(A,B) - y^2(A,B) = 4[w(A,B) - 3] = 4[p(A,B) + 3]$
4. $13x(A,B) + 9y(A,B) + 2t_{4,25B} \equiv 0(\mathrm{mod}1250)$
5. $3\{9x(A,(A+1)(A+2)) - 13y(A,(A+1)(A+2)) + T(A,A) - 15000P_A^3\}$ is a nasty number:

PATTERN-IV:

Substituting m=0 in (3), we have

$$u^2 + v^2 = 5^n \tag{7}$$

Applying the method of factorization, the corresponding non-zero distinct integral solutions of (7) are given by

$$u_0 = \frac{1}{2}[(2+i)^n + (2-i)^n]$$
$$v_0 = \frac{1}{2i}[(2+i)^n - (2-i)^n] \tag{8}$$

Taking m=1 in (3),we have

$$u^2 + v^2 = 5^n T \tag{9}$$

Considering $T = T(a,b) = a^2 + b^2$ and employing the method of factorization, the corresponding non-zero distinct integral solutions of (9) are given by

$$u_1 = au_0 - bv_0$$
$$v_1 = au_0 + bv_0$$

The repetition of the above process leads to the solutions of (3) represented by

$$u_m = \frac{1}{2i}(iAu_0 + Bv_0)$$

$$v_m = \frac{1}{2i}(Bu_0 + iAv_0)$$

where

$$A = (a+ib)^m + (a-ib)^m$$

$$B = (a+ib)^m - (a-ib)^m$$

Hence, the corresponding non-zero distinct integral solutions of (1) are given by

$$x = \frac{1}{2i}\{(iAu_0 + Bv_0) + (Bu_0 + iAv_0)\}$$

$$y = \frac{1}{2i}\{(iAu_0 + Bv_0) - (Bu_0 + iAv_0)\}$$

$$z = \frac{-1}{2}\{(iAu_0 + Bv_0)(Bu_0 + iAv_0)\}$$

$$w = \frac{-1}{4}\{(iAu_0 + Bv_0)(Bu_0 + iAv_0)\} + 3$$

$$p = \frac{-1}{4}\{(iAu_0 + Bv_0)(Bu_0 + iAv_0)\} - 3$$

$$T = a^2 + b^2$$

CONCLUSION

In linear transformations (2),the variables w and p may also be represented by

$$w = 3u + v \ , \ p = 3u - v$$

Applying the procedure similar to that of patterns I to IV, other choices of integral solutions to (1) are obtained. To conclude, one may search for other patterns of solutions and their corresponding properties.

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