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Exact and Simple Solution to the "N-Multiplying a Cube" Problem Using Straightedge and Compass Only in Euclidean Geometry

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Abstract **Original Research Article**

The problem of "n-multiplying a cube" consists of constructing, by means of a straightedge and compass within the framework of Euclidean geometry, a cube whose volume is exactly n times that of a given cube, where n is a positive integer. This work presents a generalisation of the author's previously published solutions to the classical problems of "Doubling a Cube" and "Tripling a Cube." While the classical Greek problems squaring the circle, trisecting an angle, and doubling the cube have been studied extensively throughout the history of mathematics, the more general problem of constructing a cube with volume exactly n times that of a given cube has not appeared in the traditional canon of Euclidean construction problems. Motivated by recent developments concerning the exact construction for doubling a cube and tripling a cube, this paper formulates and investigates a systematic method for the general case. In this article, we present an exact construction-based approach for enlarging a cube by an arbitrary integer factor n, using only the classical Euclidean tools of straightedge and compass. The method is derived from elementary geometric principles and extends naturally from the constructions established in the author's earlier works on cube duplication and triplication. Although the underlying principles are elementary, the complete development of the method requires a nontrivial sequence of geometric constructions. The results establish a novel framework for constructing a cube whose volume is precisely n times that of a given cube. This framework, referred to as the "n-multiplying a cube" method, provides a systematic and reproducible procedure for exact Euclidean construction without recourse to transcendental quantities or numerical approximation. This work contributes a new perspective to classical geometric construction theory and proposes an extension of constructability within Euclidean geometry through a unified method for volume integered enlargement of cubes. The present study therefore generalises the author's previously published solutions to the "Doubling a Cube" and "Tripling a Cube" problems into a broader constructive framework for arbitrary positive integers.

Keywords: multiple a cube, cube multiplication, make a cube n times larger, cube enlargement, n-multiplying cube, multiply a cube n times, enlarging cubes.

I. INTRODUCTION

For more than three millennia, the three classical problems of ancient Greek geometry - "Squaring A Circle," "Doubling A Cube," and "Trisecting An Angle" - have challenged the ingenuity of mathematicians. Formulated under the strict requirement that only a straightedge and compass be employed, these problems eventually came to be regarded as impossible within the standard framework of Euclidean construction. During the nineteenth century, this conclusion was reinforced by the foundational impossibility theorems of Pierre Wantzel and Ferdinand von Lindemann. In 1837, Wantzel, L., demonstrated, through algebraic field theory, the impossibility of arbitrary angle trisection and cube duplication by classical straightedge-and-compass methods, while Lindemann's 1882 proof of the transcendence of π established the impossibility of the classical quadrature of the circle [2-4]. Although these impossibility results are mathematically rigorous within their respective algebraic and arithmetic frameworks, they arise from methods fundamentally different from the constructive spirit of classical Greek geometry. Ancient Euclidean geometry was geometric rather than algebraic in character, relying exclusively on direct construction and deduction instead of transcendental or field-theoretic arguments. This distinction motivates a renewed examination of the classical problems strictly within the context of Euclidean geometric construction [5-8].

Recent related investigations include the paper *"Exact Solution to the Squaring the Circle Problem"* [29-30], which presents a construction claimed to produce, by Euclidean means alone, a square equal in area to a given circle. A complementary study, *"Circling the Square,"* was also published in 2024, together with an extension to the new geometric problem referred to as *"Circling the Regular Hexagon"* [31,39,40]. The latter problem concerns the construction of a circle concentric with a given regular hexagon and having the same area. The proposed method introduces a regular dodecagon whose twelve vertices lie simultaneously on a common circle and on the extensions of the sides of the hexagon. By constructing this dodecagon with straightedge and compass, a circle equal in area to the hexagon is obtained. To facilitate these constructions, a specialized geometric instrument, termed the *Regular Dodecagon Ruler*, was introduced [40].

Mathematics develops from accepted premises through deductive reasoning. Distinct premises generate distinct mathematical systems. For example, the Euclidean parallel postulate that through a point external to a line there exists exactly one parallel line to the given line gives rise to Euclidean geometry. Altering this assumption leads to non-Euclidean systems: Riemannian geometry assumes no parallel lines through such a point, whereas Lobachevskian geometry permits infinitely many [27,28]. Scientific theories and mathematical frameworks evolve continuously, and new investigations frequently emerge from reinterpretations of earlier assumptions and methods.

The classical Greek problems of squaring a circle, trisecting an angle, and doubling a cube remain formally impossible under the standard algebraic interpretation established by nineteenth-century mathematics [6,10,11]. Nevertheless, contemporary studies have proposed alternative constructive viewpoints and geometric reinterpretations intended to revisit these ancient challenges from within a purely Euclidean perspective [29,30,32-34]. Among these problems, the quadrature of circle and related constructions involving π have remained particularly influential in the history of mathematics, attracting both professional and amateur mathematicians for centuries.

Beyond its technical objectives, the present research also contributes to broader philosophical discussions concerning mathematical truth and the dependence of "possibility" or "impossibility" upon the underlying framework of reasoning. Although impossibility theorems are definitive within the logical systems from which they are derived, the constructions presented here suggest that alternative geometric approaches - consistent with the original constructive spirit of Euclidean geometry - yields exact solutions previously regarded as unattainable. This perspective is compatible with Karl Popper's philosophy of science, according to which scientific knowledge remains provisional and open to revision in light of new evidence [1].

The investigations presented in this work were further guided by an aesthetic and philosophical principle inspired by Lao Tzu's aphorism in the *Tao Te Ching*: ***"The Great Tao is simple, very simple"*** (大道至简), [13]. Emphasizing simplicity, symmetry, and concentric geometric structure, the author develops exact geometric constructions not only for angle trisection, but also for squaring the circle and multiplying the cube, using only a straightedge and compass [32, 33, 41, 43].

This article asserts that the problem of constructing a cube whose volume is exactly n times that of a given cube can be resolved certainly within Euclidean geometry using only an unmarked straightedge and compass. The proposed construction follows the classical geometric methods of Analysis and Synthesis. In the analytic phase, the desired construction is assumed to exist and is reduced to previously solvable geometric configurations. In the synthetic phase, the solution is reconstructed systematically from the original data through explicit geometric operations [12, 22].

The resulting construction provides a complete and rigorous method for the exact n-multiplication of a cube. Although the sequence of constructions is nontrivial, the method demonstrates that the enlargement of a cube by an arbitrary positive integer factor n can, in principle, be achieved through classical Euclidean means alone, surely.

Previous investigations by the author established related constructions for doubling and tripling a cube and introduced a novel geometric instrument termed the *Head-cut Pyramid*. This device enables rapid and precise triplication of a given cube and represents a practical development within constructive geometry. The present work generalizes these earlier constructions into a broader framework applicable to arbitrary integer multiplication of cube volumes.

The remainder of this paper presents the analytic reductions, synthetic constructions, proofs of correctness, and geometric procedures underlying the proposed method of "n-multiplying a cube." Detailed diagrams and step-by-step straightedge-and-compass constructions are provided to demonstrate the exactness, consistency, and reproducibility of the method.

II. PROPOSITION

Let C_1 be a cube of side length $a > 0$, and let C_2 be a cube with volume na^3, and side length $a\sqrt[3]{n}$, $n \in N$, $N = \{4, 5, 6, \ldots\}$, and $n > 3$, constructed so that C_1 and C_2 are concentric ($C_2 = nC_1$).

Denote by $R = \frac{C_2}{C_1}$ the region contained in C_2 but exterior to C_1 (see **Figure 1**).

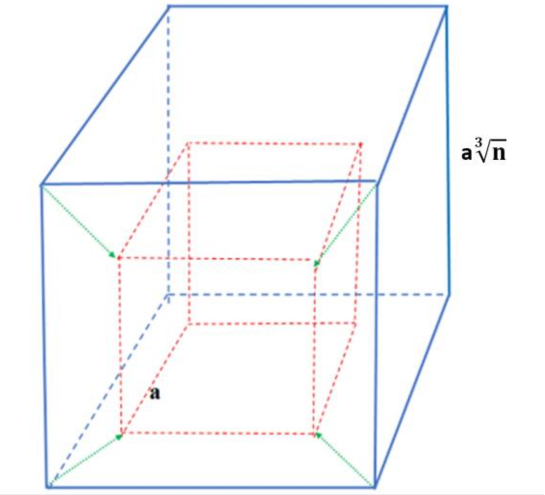


Figure 1: Two concentric cubes: the inner cube C_1 of side length a, and the outer cube C_2 of side length $a\sqrt[3]{n}$.

The region R admits a natural decomposition into finitely many congruent solids, each of which can be described in terms of truncated pyramidal geometry. This observation motivates the following definition.

II.1 Head-cut Pyramid Definition:

A *head-cut pyramid* is defined as the frustum of a regular square pyramid; that is, a solid obtained by intersecting a regular square pyramid with a plane parallel to its base and removing the portion containing the apex (see Figure 2).

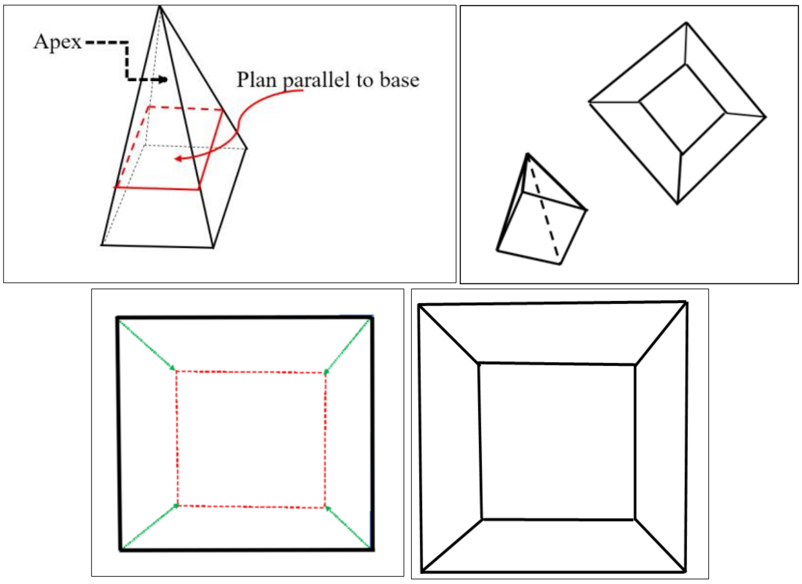


Figure 2: ***Head-cut Pyramid shapes***

II.2 Head-cut Pyramid Theorem:

Let two cubes be concentric in $\mathbb{R}^3$, with the smaller cube of volume a^3 strictly contained within the larger. Then the six head-cut pyramids (truncated pyramids), formed between corresponding faces of the two cubes, are congruent and equal.

Proof:

Let C_{out} and C_{in} denote the outer and inner cubes, respectively, sharing a common centre. Denote by F_i^{out} and $F_i^{\text{in}} (i = 1, \ldots, 6)$ the corresponding parallel faces of C_{out} and C_{in}.

Join the vertices of C_{out} to the corresponding vertices of C_{in}. This partitions the region between the two cubes into six head-cut pyramids (truncated pyramids), each bounded by a pair of parallel square faces F_i^{out} and F_i^{in}, together with four lateral faces (**Figure 3**, below).

Since the cubes are concentric, the perpendicular distance between each pair F_i^{out} and F_i^{in} is constant for all i. Hence, all six head-cut pyramids (truncated pyramids) have equal heights.

Moreover, all faces of a cube are congruent squares and equal correspondingly. Therefore, for each i, the base F_i^{out} has the same area for all pyramids, and similarly the top face F_i^{in} has the same area for all pyramids.

Thus, each head-cut pyramid (truncated pyramid) has identical height and congruent parallel faces. It follows that the six head-cut pyramids (truncated pyramids) are congruent and equal.

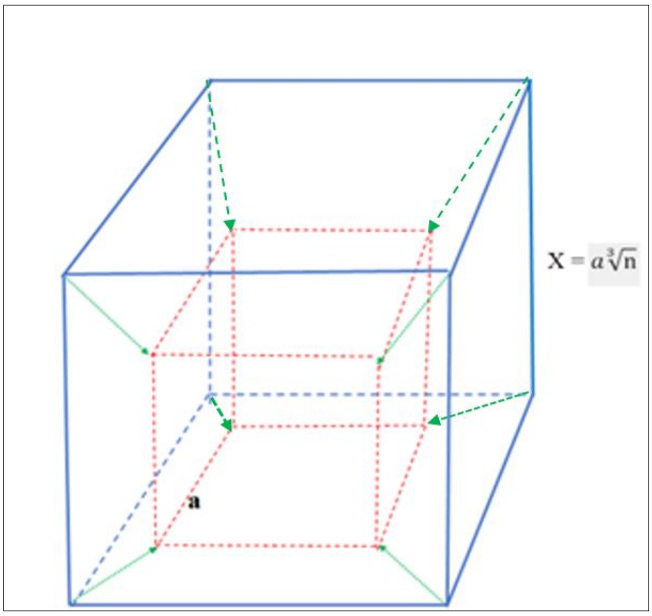


Figure 3: Six congruent and equal head-cut pyramids (truncated pyramids, see the green arrows) surrounding the cube of volume a^3

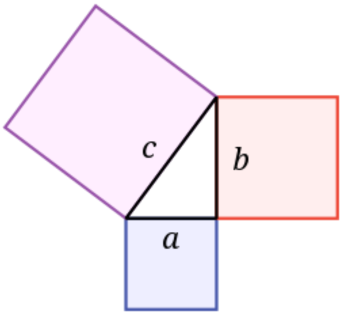
II.3 Theorem 01:

Given a UNIT LENGTH u, and a number n, n ∈ N, N = {4, 5, 6, …}, and n > 3, then

a. The exact lengths of √2, √3 and √4 are constructive in algebraic geometry with a compass and a straightedge. and,

b. The exact length ($\sqrt{12n-3}$) is also constructive in algebraic geometry with a compass and a straightedge.

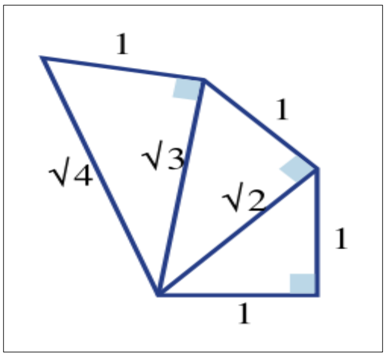
Note and reminding on the Pythagorean theorem

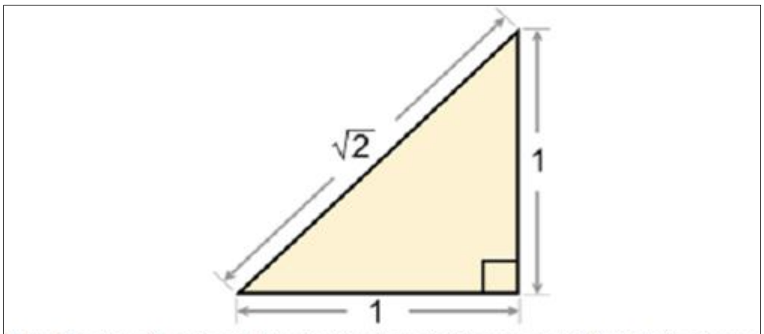


In mathematics, the Pythagorean theorem or Pythagoras's theorem is a fundamental relation in Euclidean geometry between the three sides of a right triangle. It states that the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides. The theorem can be written as an equation relating the lengths of the sides a, b and the hypotenuse c, sometimes called the Pythagorean equation:

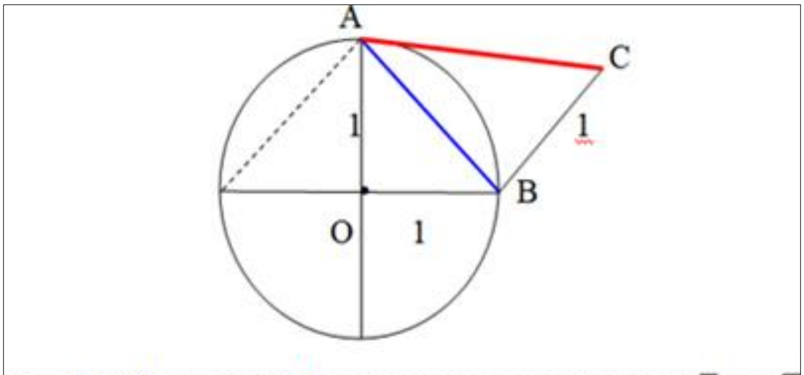
$$a^2 + b^2 = c^2.$$

Greek philosopher Pythagoras, born around 570 BC. The theorem has been proved numerous times by many different methods – possibly the most for any mathematical theorem. The proofs are diverse, including both geometric proofs and algebraic proofs, with some dating back thousands of years (see Images below).

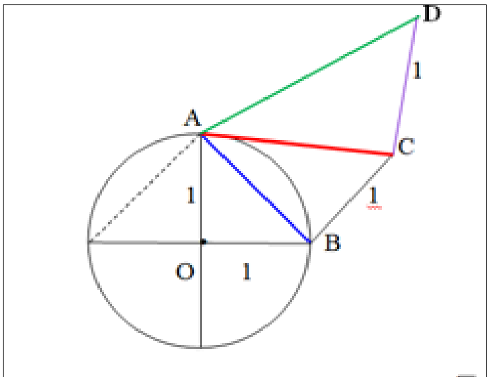




Note: *The above figure is a right-angle triangle marked 'square root of 2' at the hypotenuse.*



: *Length AB (blue) and AC (red) are the exact geometrical lengths of* $\sqrt{2}$ **and** $\sqrt{3}$



: *Length AD (green) is the exact length of* $\sqrt{4}$.

End of Note.

PROOF:

Geometric Construction of Successive Square Roots and a Derived Segment

a.- Let a unit length $u = 1$ be given. All constructions, in this article paper, are performed using only a straightedge and compass.

Construction of $\sqrt{2}$ and $\sqrt{3}$ (See images in the Note above)

- Construct a circle with centre O and radius 1. Let A and B be end points of two perpendicular diameters of the circle.
- Then triangle AOB is right-angled at O, with:

$$\mathrm{OA} = \mathrm{OB} = 1.$$

By the Pythagorean theorem:

$\mathrm{AB}^2 = \mathrm{OA}^2 + \mathrm{OB}^2 = 1^2 + 1^2 = 2$

Hence, $\mathrm{AB} = \sqrt{2}$.

➤Extend the construction by forming a right triangle ABC such that:

$$\mathrm{BC} = 1 \text{ and } \mathrm{AB} = \sqrt{2}.$$

Applying the Pythagorean theorem again:

$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 = (\sqrt{2})^2 + 1^2 = 3$

Therefore, $\mathrm{AC} = \sqrt{3}$.

Iterative Construction up to $\sqrt{n}$ (See images in the Note above)

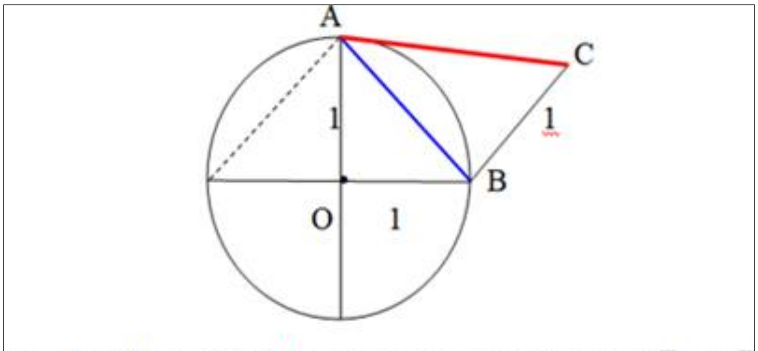
➤At point C, construct a line perpendicular to AC. On this perpendicular, mark a point D such that (see illustrated images below):

CD = 1 (See images below)

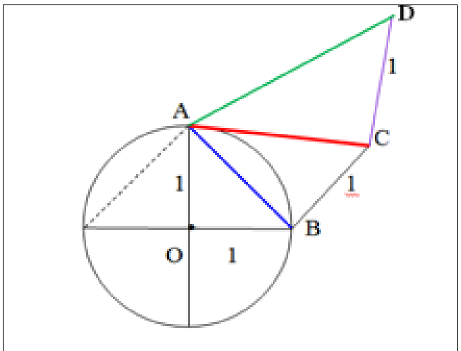
Then triangle ACD is right-angled at C, and:

$\mathrm{AD}^2 = \mathrm{AC}^2 + \mathrm{CD}^2 = (\sqrt{3})^2 + 1^2 = 4$

Hence, $\mathrm{AD} = \sqrt{4}$.



: *Length AB (blue) and AC (red) are the exact geometrical lengths of* $\sqrt{2}$ **and** $\sqrt{3}$

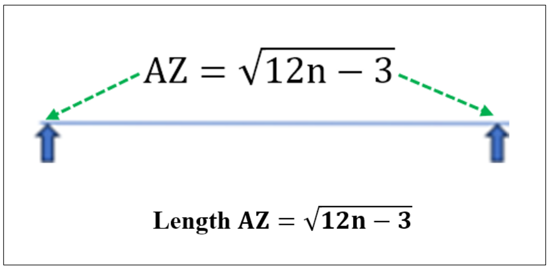


: *Length AD (green) is the exact length of* $\sqrt{4}$.

➢Repeating this procedure inductively each time erecting a perpendicular of unit length at the latest point produces a sequence of segments:

$\sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots, \sqrt{n-1}, \ldots, \sqrt{12n-3}$

b.- After (12n-6) such steps, one obtains line segment **AZ =** $\boldsymbol{\sqrt{12n-3}}$**,** using only straightedge & compass, as required.



Thus, this construction provides:

- A classical straightedge-and-compass method to generate successive square roots,
- A systematic extension up to $\sqrt{n}$, $n \in N$, $N = \{4, 5, 6, \ldots\}$, and $n > 3$.
- And a proportional construction yielding length $AZ = \sqrt{12n-3}$.

II.4 Core Theorem (n-Multiple Cube Theorem):

The n-multipled cube volume na^3, having side $a[\frac{1}{2} + \frac{1}{6}(\sqrt{12n-3})]$ from a given cube volume a^3, is constructive exactly and simply with straightedge and compass in Euclidean Geometry.

PROOF:

Let the given cube c with volume a^3 be located inside the targeted cube C with volume na^3, concentrically (Figure 3 above and Figure 4 below), then the space, surrounding cube c and limited by the 6 faces of C, has volume

$$na^3 - a^3 = (n-1)a^3 \quad (1)$$

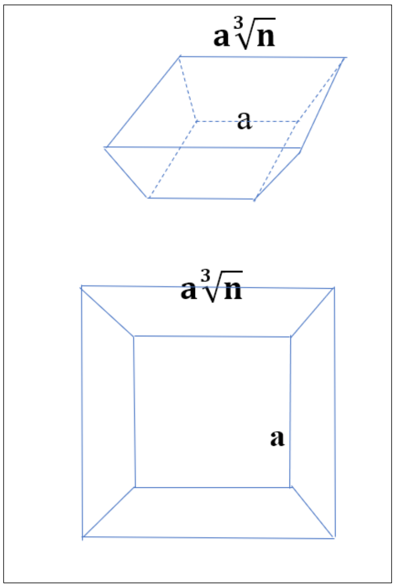


Figure 4: A head-cut pyramid from the concentric location of the given cube and the resulting cube

Because of the concentric property of these cubes, the 8 straight line segments connected the centre to the 8 vertices of the double cube go through the 8 vertices of the given cube. This causes the inner space surrounding the given cube within the double cube to be divided into 6 equal head-cut pyramids (Definition 01, above). One of the 6 head-cut pyramids is illustrated as follows:

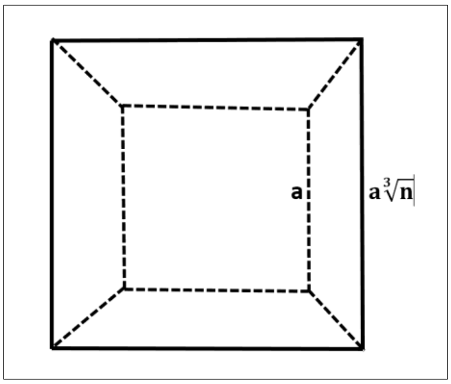


Figure 5: The 2 square bases of the head-cut pyramid are given by the concentric location/arrangement/position of the given cube & the resulting cube. (The 6 head-cut pyramids surrounding the given cube a³ can be also called 6 regular isosceles trapezoid cuboids)

Consider the volume expression (1) where its righthand side shows the volume of one of the 6 equal head-cut pyramids surrounding the given cube c (Figure 5, above). Therefore, one head-cut pyramid has volume $\frac{1}{6}(n-1)a^3$.

Let X be the length side of the resulting cube with volume na^3, then $X = a\sqrt[3]{n}$ and the height of one head-cut pyramid is $\frac{X-a}{2}$. Therefore volume $\frac{1}{6}(n-1)a^3$ in (2) of the head-cut is also calculated by algebra as follows:

$$\left(\frac{X^2+a^2}{2}\right)\left(\frac{X-a}{2}\right) = \frac{1}{6}(n-1)a^3 \qquad (2)$$

$$\frac{X^3 - aX^2+a^2X-a^3}{4} = \frac{1}{6}(n-1)a^3$$

$6(X^3 - aX^2 + a^2X - a^3) = 4(n-1)a^3$
$6X^3 - 6aX^2 + 6a^2X - 6a^3 = 4na^3 - 4a^3$
$6X^3 - 6aX^2 + 6a^2X - 6a^3 + 4a^3 - 4na^3 = 0$
$6X^3 - 6aX^2 + 6a^2X - 2a^3 - 4na^3 = 0$ (3)

Replace X by $a\sqrt[3]{n}$ to get: $6na^3 - 6a^3(\sqrt[3]{n})^2 + 6a^3(\sqrt[3]{n}) - 2a^3 - 4na^3 = 0$
$6n - 6(\sqrt[3]{n})^2 + 6(\sqrt[3]{n}) - 2 - 4n = 0$
$- 6(\sqrt[3]{n})^2 + 6(\sqrt[3]{n}) - 2 + 2n = 0$ (4)

Then (4) is a quadratic equation with unknown $(\sqrt[3]{n})$. Let $(\sqrt[3]{n})$ be unknown y, then (4) becomes
$- 6y^2 + 6y + (2n -2) = 0$ (5)

$\Delta = 36 + 24(2n-2) = 48n - 12 > 0$, due to n>3 and $\sqrt{\Delta} = \sqrt{4(12n-3)} = 2\sqrt{12n-3}$

(Note that if n =2, then the above problem was “Doubling A Cube” problem solved and published in [34]).

$y = \frac{-6 - 2\sqrt{12n-3}}{-12} = \frac{1}{2} + \frac{1}{6}(\sqrt{12n-3}) > 0$ is chosen.

$y' = \frac{-6 + 2\sqrt{12n-3}}{-12} < 0$, due to n > 2.

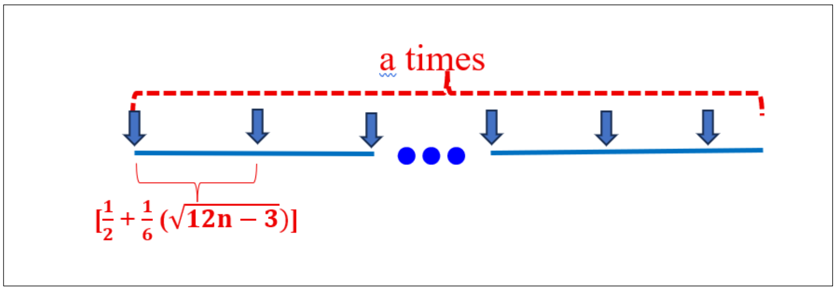
Let’s remind that now,
$y = (\sqrt[3]{n}) = \frac{1}{2} + \frac{1}{6}(\sqrt{12n-3})$, n>3 (6)

then the side of the target cube with volume na^3 is as follows:
$S = ay = a[\frac{1}{2} + \frac{1}{6}(\sqrt{12n-3})]$ (7)

Let AB be the length of the target cube above then AB is identified by straightedge & compass as follows:

- Length $(\sqrt{12n-3})$ is constructively given by Theorem 01 in section II.3 above.
- With a straightedge & a compass, it is obviously length $y = [\frac{1}{2} + \frac{1}{6}(\sqrt{12n-3})]$ is constructive, as n ∈ N, N = {4, 5, 6, …}, and n > 2.
- Then length $S = AB = a[\frac{1}{2} + \frac{1}{6}(\sqrt{12n-3})]$, a ∈ R, is constructive as follows:

Use a straightedge & a compass to construct length $y = [\frac{1}{2} + \frac{1}{6}(\sqrt{12n-3})]$ then multiply this y by a times, as a description below:



III. MATERIALS & METHODS

The materials and methods include straightedge, compass, ANALYSIS Method and SYNTHESIS Method, within the scope of Algebraic Geometry and Pure Geometry.

IV. DISCUSSION & CONCLUSION

IV.1 Discussion

Innovation Cannot Thrive When Constrained by Traditional Theories

Generating new mathematics arises from the creative synthesis of existing theories, rigorous formalism, and innovative physical or computational insights. One can extend known frameworks to produce novel theorems, apply advanced models such as four-dimensional topology or B-matrix statistics to reveal previously hidden relationships, or reinterpret classical equations like Schrödinger’s or relativistic formulations to uncover new structures and distributions. At its core, new mathematics emerges when deep theoretical insight meets cross-disciplinary vision, producing results that are both logically consistent and generative of further inquiry. This approach demonstrates that mathematics is far from closed; it continues to expand wherever curiosity, abstraction, and experimentation intersect.

In the past, those who entered the construction industry typically gained some understanding of its history. One notable figure is Joseph Monier (1823–1906), the inventor of reinforced concrete. He first presented his invention at the Paris Exhibition in 1867 and was granted the world’s first patent for reinforced concrete. Subsequently, he received additional patents for reinforced concrete pipes, tanks, beams, and other applications. The first reinforced concrete bridge was also built according to his design. However, Monier's invention was not initially recognized by construction engineers and leading experts in France and around the world. Bound by conventional theories that treated steel and concrete as separate materials, and lacking knowledge of their combined potential, they doubted the durability of the composite material. Furthermore, due to Monier’s status as a common individual rather than an academic or professional insider, his work was largely disregarded. As a result, meaningful application of his invention was delayed until the late 19th and early 20th centuries *(nearly 30 years delay!)*. Nevertheless, reinforced concrete eventually became one of the greatest innovations in human history, revolutionizing the construction industry. This breakthrough is attributed to Monier, a self-taught inventor whose ideas ultimately prevailed despite initial resistance. His success was made possible by a few individuals in the engineering community who recognized the invention’s value and either acquired the rights or continued to develop it. Today, it is widely acknowledged that without reinforced concrete, it would be impossible to construct skyscrapers, strong bridges, modern highways with overpasses and underpasses, and the vast infrastructure required for large contemporary cities. A similar example can be seen in the invention of Blockchain technology by Satoshi Nakamoto an individual whose identity remains unknown due to a deliberate choice to remain anonymous. Like Monier, Nakamoto’s work has had a profound impact on the world, despite coming from outside traditional academic or institutional frameworks. These examples demonstrate that strict adherence to established theories can hinder creativity and innovation. True progress often originates from those willing to think beyond conventional boundaries and from those who dare to explore uncharted territory. Another example of great invention is the Blockchain technology of Satoshi Nakamoto, a person who has a name but no one knows who he is.

In the past, knowledge was often considered scientific if it could be confirmed through specific evidence or experiments. However, Karl Popper, in his book Logik der Forschung (The Logic of Scientific Discovery), published in 1934, demonstrated that a fundamental characteristic of scientific hypotheses is their ability to be proven wrong (falsifiability) [1]. Anything that cannot be refuted by evidence is temporarily regarded as true until new evidence emerges. For instance, in astronomy, the Big Bang theory is widely accepted, but in the future, anyone who discovers a flaw in this theory will be acknowledged by the entire physics community. Furthermore, no scientific theory lasts forever; rather, it is specific research and discoveries that continually build upon one another [21] & [23].

Moreover, PEOPLE MAY STAND STILL – BUT THE EARTH DOES NOT: In 1851, in the nave of the Panthéon, Mr. Léon Foucault conducted a quiet yet revolutionary experiment. He suspended from the dome a pendulum 67 meters in length. As it oscillated, observers noticed that the plane of its swing gradually rotated over time. The pendulum itself did not change direction; rather, the Earth was rotating beneath it.

With this simple but profound demonstration, Foucault provided direct, visible evidence that the Earth spins on its axis. Prior to this experiment, the heliocentric model proposed by Nicolaus Copernicus and later supported by Galileo Galilei had already established the theory of Earth’s rotation. However, these conclusions rested primarily on mathematical reasoning and astronomical observation. Foucault sought something more immediate: empirical proof accessible to all.

No complex equations were required. No debate was necessary. One needed only to stand and observe. For the first time in history, people could witness with their own eyes that they inhabited a moving planet.

Foucault did more than demonstrate a physical phenomenon; he transformed humanity’s perception of its place in the universe. He made the invisible visible. In an age of intellectual contention, he chose demonstration over argument, evidence over rhetoric.

This principle extends beyond physics. In contemporary society, opinions are abundant, and debates are constant. Many claim to possess ideas, ambitions, and potential. Yet ideas alone do not alter reality. The world does not revolve around assertions; it advances through action.

Foucault's pendulum did not persuade because he spoke about it it persuaded because it moved.

If one believes oneself capable of meaningful achievement, the path forward is not endless discussion but deliberate creation. As the ancient philosopher Laozi (Lao Tzu) expressed, "The Great Tao is simple." Simplicity, however, does not imply passivity. It calls for clarity of purpose and decisive effort: build something tangible, write with substance, develop expertise, initiate a project and substantiate claims with results [13].

When work is visible and measurable, validation becomes unnecessary. People will not ask whether you are capable; they will observe what you have created and draw their own conclusions.

We live on a planet in constant motion. The Earth continues to rotate, indifferent to hesitation. So too do opportunities evolve and pass. The essential question, then, is not whether the world moves but whether we move with it.

The lesson drawn from great figures is not merely admiration of their achievements. It is the recognition that decisive action transforms theory into experience, and potential into reality. It looks like the Newton's apple showed people the visible gravity force. The opportunity to begin remains always today, as this article transformed an invisible "n-multiplicated cube" from a given cube, into a constructive "--multiplicated cube" visibly.

The "N-multiplying A Cube" problem refers to the ancient Greek problem of constructing a cube with n volumes, $n > 3$, of a given cube, using only a straightedge and compass. The problem can be compared to a problem date back to at least the 5th century BC and was one of the three famous unsolved problems of ancient Greek mathematics, alongside the "Trisecting An Angle" and the "Squaring The Circle" problems [41] & [44]. Solution to the "Doubling The Cube" problem was published [34]. At present, this "N-multiplying A Cube" problem also refers to the ancient Greek problem of constructing a cube with double the volume of a given cube, using only a straightedge and compass.

My past research result objectively presents a provable construction of generating a length of magnitude; as the geometrical solution for the ancient classical problem of doubling the volume of a given cube, and published [34]. The "Doubling The Cube" problem, which has challenged mathematicians since the time of the ancient Greeks, is precisely solved by the ANALYTICS method to concentrically locate a given cube of volume a^3 in its double cube with volume $2a^3$, side $a\sqrt[3]{\mathbf{2}}$. In other words, I did succeed the concentric location for the given cube a^3 inside the goal cube $2a^3$ to solve exactly the problem with a straightedge and a compass; then this solution was published in [34].

The impossibility proof of doubling a cube, published by mathematician Wantzel, was based on three-dimensional cubic extensions in abstract algebra, an approach that entirely shifted the problem to solid geometry from its Greek's definition in plane geometry, and therefore the algebraic statement of impossibility has no geometrical validity [2-4].

I don't know how many creative ideas are accidentally stifled like the so-called IMPOSSIBILY above. Many people cannot overcome this "Thinking Trap", not only binding themselves but also hindering others. "Thinking Trap" is the word used to refer to the stuckness of people who have the habit of constantly establishing a state of impossibility for their will. They always react to strange, new, unusual things; to things that they are not confident in or feel threatened by saying no to them. The human brain operates to provide information and arguments that its owner desires. Therefore, when it is determined to be impossible, one's thoughts and actions will be in the direction of trying to prove oneself right - that is, making oneself and related people see that it is impossible. In extreme cases, people even try to sabotage the work of those who are proving it is possible. How disastrous is such an extreme case that falls on people in power? But these are common cases in society [5], [6] & [8].

People with low self-esteem often fall into the trap of thinking. They not only think they can't do it, but also doubt the ability of others. If they are powerful people, they will give themselves the right to ensure safety. As a result, they will stifle creativity. Only people with confidence and a scientific attitude can overcome the trap of thinking. If we see difficulties, we should point them out and analyse them scientifically, not in-still or impose low self-esteem. Doing so will cause a decline in will, and never dare to overcome things that are bigger than ourselves.

This is evident from the fact that no two facets of a cube can share all four vertices from two different planes. However, according to this study result, the impossible imprecise classification should not be extended to geometry so that the irrationality definition was stated as "algebraic irrationality is not a constructible number of the geometry". The possibility to solve geometrically the coefficient constant $a\sqrt[3]{n}$, $n \in N$, $N = \{4, 5, 6, \ldots\}$, and $n > 3$, to an exact precision is proved. This study paper also presents a geometrically certain method under the set restrictions of Euclidean geometry (in the sense that, all presented constructions have been reduced to the Euclidean postulates of practical geometry), by the construction of the relation as depicted in the justification section in Scholar Journal of Physics, Mathematics and Statistics [34].

The above solution, using only a straightedge and compass, referred to as the “N-multiplying A Cube”, did not previously arise in classical geometric construction. It emerged only after I resolved the long-standing challenge of “Doubling The Cubbe” problem and published the solution in Sch J Phys Math Stat | 24-32, SJPMS 13/02/2025, [34].

IV.2 CONCLUSION

Most mathematicians and mathematics enthusiasts accept that the three classical Greek problems Squaring the Circle, Doubling the Cube, and Trisecting an Angle - are impossible to solve using only a straightedge and compass. This consensus largely stems from the work of Pierre Wantzel (1837), who employed field theory and algebraic methods to prove the impossibility of certain geometric constructions [2-4,6]. However, it is important to recognize that “Squaring the Circle” is fundamentally a geometric construction problem, and the algebraic approach may not fully address the nuances of Euclidean geometry. Further support for the impossibility of squaring the circle came from Ferdinand von Lindemann’s proof in 1882 that π is transcendental. From this, it is commonly concluded that since π cannot be constructed using a finite sequence of straightedge and compass steps, it is impossible to square the circle in the classical sense [2,3].

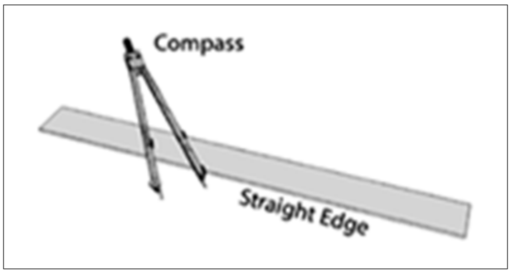
However, it is worth considering a different perspective. This article can construct an n-multiplicated cube, $n \in N$, $N = \{4, 5, 6, \dots\}$, and $n > 3$, from an arbitrary cube by placing the given cube inside and concentric with the n-multiplicated cube whose edge is unknown. To do so, the idea of shapes surrounding the given cube and adjacent to the 6 square faces of the n-multiplicated cube must be considered and verified and defined: “The Head-cut Pyramid”. I calculate the volume of a Head-cut Pyramid to get a cubic equation, of which when substituting the volume of 1/6 of the space bounded by the 12 faces of the 2 cubes, we obtain a quadratic equation with unknown being $\sqrt[3]{n}$. I solve this quadratic equation to gain $\sqrt[3]{n} = (\frac{1}{2} + \frac{1}{6}\sqrt{12n - 3})$, which is a number that can be expressed into a length using straightedge & compass. Therefore, the side of the resulting cube (the n-multiplicated cube) is identified as $a\sqrt[3]{n} = a[\frac{1}{2} + \frac{1}{6}(\sqrt{12n - 3})]$, exactly in term of length by means of straightedge & compass.

I hold a different point: I believe I have constructed a valid solution to the “n-Multiplying A Cube” problem using only a straightedge and compass, in accordance with the classical constraints. This belief strengthens my resolve as I pursue solutions to the “Multiply a cube by 2 times”, “Multiply a cube by 3 times”, “Multiply a cube by 4 times”, …, “Multiply a cube by n times” problems. The techniques of geometrical analysis and synthesis are instrumental in this effort.

It is difficult to realise why the “n-Multiplying A Cube” challenge above has not existed before this published article, meanwhile its solution within the classic Euclidean Geometry is proved exactly and simply as above.

V. AN OPEN AREA FOR RESEARCH

The Core Theorem of n-Multiple Cube Theorem (in section II.4 above), applied for n-multiplying a cube of volume a^3 into a cube with volume na^3, certainly converted from “its cubic equation to its quadratic equation successfully”, in order to have a precise geometrical length constructed by straightedge & compass. Therefore, the new problem of “converting a cubic equation to a quadratic equation, equivalently” is a possible open research area in algebraic geometry.



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