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Research Article

Special family of Diophantine Triples

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Abstract: In this paper, we present two special Diophantine triples in which the sum of any two is a perfect square.
Keywords: Integral solutions, non-extendable diophantine triples

INTRODUCTION:

A set of positive integers $(a_1, a_2, ... a_m\}$ is said to have the property D(n), $n \in Z - \{0\}$, if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuple with property D(n) or a P_n set of size m were studied by Diophantus [1].For an extensive review of various article one may refer [2-15]. Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n and also for any linear polynomials in n.

In this communication we present two special Diophantine triples such that in each case the sum of any two is a perfect square

METHOD ANALYSIS

Construction of Diophantine triples:

Let $a = r^2 + s^2, b = 2rs \;\; (r > s > 0)$ be any two integers such that $(a+b)$ is a perfect square. We search for a distinct integer c such that

$$a + c = \alpha^2 \qquad \textbf{(1)}$$

$$b + c = \beta^2 \qquad (2)$$

$(1) - (2) \Rightarrow$

$$\alpha^2 - \beta^2 = (r - s)^2 \qquad (3)$$

Choice: I

Choose r and s in (3) such that

$$r - s = P^2 - Q^2$$

and thus

$$\alpha = P^2 + Q^2 \text{ and } \beta = 2PQ$$

Substituting the value of α in (1) or value of β in (2),we get,

$$c = (P^2 + Q^2)^2 - a \quad \text{or} \quad c = (2PQ)^2 - b$$

The above process is illustrated below in Table .I

Table: I

r	s	P	Q	a	b	c
15	7	3	1	274	210	-174
22	10	4	2	584	440	-184
24	11	7	6	697	528	6528
26	11	8	7	797	572	11972
32	15	9	8	1249	960	19776
42	21	5	2	2205	1764	-1364
42	21	11	10	2205	1764	46636
4n	2n-1	n+1	n	$20n^2-4n+1$	$8n(2n-1)$	$4n^4+8n^3-12n^2+8n$
4	1	2	1	17	8	8

Note that all the triples in the table are all non-extendable triples except the triple (17,8,8) which can be extended to a quadruple (17,8,8,8).

Choice: II

Choose r and s so that

$$r-s=2PQ \quad (P \succ Q)$$

Then,

$$\alpha = P^2+Q^2, \beta = P^2-Q^2 \tag{4}$$

Substituting the values of α in (1) or β in (2),we get,

$$c=\alpha^2-a(r,s) \text{ (or) } c=\beta^2-b(r,s)$$

The above process is illustrated below in Table.II

Table: II

r	s	P	Q	a	b	c	Remark
17	1	4	2	290	34	110	Non-extendable triple
20	4	4	2	416	160	-16	Non-extendable triple
15	3	3	2	234	90	-65	Non-extendable triple
26	2	4	3	680	104	-55	Non-extendable triple
29	5	4	3	866	290	-241	Non-extendable triple
25	1	4	3	626	50	-1	Non-extendable triple
23	3	5	2	538	138	303	Non-extendable triple
42	21	11	10	2205	1764	46636	Non-extendable trplea

CONCLUSION

In this paper, we have presented some non-extendable Diophantine triples. One may search for extendable Diophantine triples consisting of special numbers.

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