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Research Article

Sufficient conditions for meromorphic close-to-convexity of order α

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Abstract: The purpose of the present paper is to consider some sufficient conditions for close-to-convexity of order alpha of meromorphic functions in the punctured unit disc.
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INTRODUCTION

Let Σ denotes the class of functions f of the form

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_nz^n \tag{1}$$

which are analytic in $D=\{z\in C:0<|z|<1\}$. A function $f\in\Sigma$ is said to be meromorphic starlike of order α if it satisfies

$$-Re\{\frac{zf'(z)}{f(z)}\}>\alpha \quad (z\in D) \tag{2}$$

for some real $\alpha(0\le\alpha<1)$ .We denote by $\Sigma^*(\alpha)$ the class of all meromorphic starlike functions of order α. A function $f(z)\in\Sigma$ is called meromorphic close-to-convex of order α if it satisfies

$$-Re\{z^2f'(z)\}>\alpha \quad (z\in D) \tag{3}$$

We denote by $MC(\alpha)$ the class of all meromorphic close-to-convex functions of order α ..

Several authors [6,7,8] have studied meromorphic close-to-convex functions of order α . We shall unify these functions in Definition 1.1.

Definition 1.1. *Let* $g(z)$ *be a meromorphic starlike function in* Σ. *If* $f(z)\in\Sigma$ *satisfies the following inequality*

$$-Re\{\frac{zf'(z)}{g(z)}\}>\alpha, z\in D \tag{4}$$

for some $\alpha(0\le\alpha<1)$. *The function* f *can also be called a meromorphic close-to-convex function,and wo denote it by* $MC^*(\alpha)$.

THE MAIN RESULTS

Lemma 2.1. *(see [4])Let* $p(z)=1+\sum_{n=1}^{\infty}a_nz^n$ *be analytic in the unit disc* U *and* α *be a positive real number. Then suppose that there exists a point* $z_0\in U$ *such that*

$$Re\{p(z)\}>\alpha \; for\; |z|<|z_0| \tag{5}$$

and

$$Re\{p(z_0)\} = \alpha, p(z_0) \neq \alpha. \tag{6}$$

Then we have

$$Re\{\frac{z_0 p'(z_0)}{p(z_0)}\} \leq \{ -\tfrac{\alpha}{2(1-\alpha)} \text{ when } 0 < \alpha < \tfrac{1}{2} - \tfrac{1-\alpha}{2\alpha} \text{ when } \tfrac{1}{2} \leq \alpha < 1. \tag{7}$$

Lemma 2.2. *Let* $p(z) = 1 + \sum_{n=1}^{\infty} a_n z^n$ *be analytic in the unit disc* U *and suppose that there exists a point* $z_0 \in U$ *such that*

$$Re\{p(z)\} > \alpha \; for \; |z| < |z_0| \tag{8}$$

and

$$Re\{p(z_0)\} = \alpha, p(z_0) \neq \alpha. \tag{9}$$

for some real $\alpha(\alpha < 0)$. *Then we have*

$$Re\{\frac{z_0 p'(z_0)}{p(z_0)}\} > -\frac{\alpha}{2(1-\alpha)} > 0. \tag{10}$$

Theorem 2.1. *Let* $f(z) \in \Sigma$, *and suppose that there exists a meromorphic starlike function* $g(z)$ *such that*

(i)*for the case* $0 < \alpha < \frac{1}{2}$

$$1 + Re\{\frac{zf''(z)}{f'(z)}\} < Re\{\frac{zg'(z)}{g(z)}\} + \frac{\alpha}{2(1-\alpha)}, z \in D \tag{11}$$

$$-\frac{zf'(z)}{g(z)} \neq \alpha \tag{12}$$

(ii)*for the case* $\frac{1}{2} \leq \alpha < 1$

$$1 + Re\{\frac{zf''(z)}{f'(z)}\} < Re\{\frac{zg'(z)}{g(z)}\} + \frac{1-\alpha}{2\alpha}, z \in D \tag{13}$$

$$-\frac{zf'(z)}{g(z)} \neq \alpha \tag{14}$$

Then we have $f(z) \in MC^*(\alpha)$.

Proof. Let

$$p(z) = -\frac{zf'(z)}{g(z)}, \tag{15}$$

then $p(z)$ is analytic in U and $p(0) = 1$.Now using (15), we have

$$1 + \frac{zf''(z)}{f'(z)} = \frac{zg'(z)}{g(z)} - \frac{zp'(z)}{p(z)}. \tag{16}$$

(i) For the case $0 < \alpha < \frac{1}{2}$, if there exists a point $z_0 \in U$ such that

$$Re\{p(z)\} > \alpha \quad for \quad |z| < |z_0| \tag{17}$$

and

$$Re\{p(z_0)\} = \alpha, \tag{18}$$

then applying Lemma 2.1 and the hypothesis of Theorem 2.1,we have

$$p(z_0) \neq \alpha, \tag{19}$$

and

$$Re\{\frac{z_0 p'(z_0)}{p(z_0)}\} \le -\frac{\alpha}{2(1-\alpha)} \tag{20}$$

Then it follows that

$$1 + Re\{\frac{zf''(z)}{f'(z)}\} = Re\{\frac{zg'(z)}{g(z)}\} - Re\{\frac{zp'(z)}{p(z)}\} \tag{21}$$

$$\ge Re\{\frac{zg'(z)}{g(z)}\} + \frac{\alpha}{2(1-\alpha)}, \tag{22}$$

which contradicts the hypothesis of Theorem 2.1.

(ii) For the case $\frac{1}{2} \le \alpha < 1$, applying the same method as above, we also have that

$$-Re\{\frac{zf'(z)}{g(z)}\} > \alpha. \tag{23}$$

Therefore the proof of the Theorem 2.1 is completed.

Corollary 2.1. *Let* $f(z) \in \Sigma$. *Suppose that for arbitrary* α, $f(z)$ *satisfies*

$$-z^2 f'(z) \ne \alpha \tag{24}$$

and the following inequalities:

(i)*for the case* $0 < \alpha < \frac{1}{2}$

$$2 + Re\{\frac{zf''(z)}{f'(z)}\} < \frac{\alpha}{2(1-\alpha)}, z \in D \tag{25}$$

(ii)*for the case* $\frac{1}{2} \le \alpha < 1$

$$2 + Re\{\frac{zf''(z)}{f'(z)}\} < \frac{1-\alpha}{2\alpha}, z \in D \tag{26}$$

Then we have $f(z) \in MC(\alpha)$.

Proof.Let $g(z) = 1/z$ in Theorem 2.1,we can obtain Corollary 2.1.

Theorem 2.2. *Let* $f(z) \in \Sigma$ *and* α *(*$\alpha < 0$*) be a real number. Suppose that there exists a meromorphic starlike function* $g(z)$ *such that*

$$1 + Re\{\frac{zf''(z)}{f'(z)}\} \ge Re\{\frac{zg'(z)}{g(z)}\} + \frac{\alpha}{2(1-\alpha)}, z \in D \tag{27}$$

and

$$\min_{|z|\le r} Re\{-\frac{zf'(z)}{g(z)}\} = Re\{-\frac{z_0 f'(z_0)}{g(z_0)}\}_{|z_0|=r} \ne -\frac{z_0 f'(z_0)}{g(z_0)} \tag{28}$$

for arbitrary $r(0 < r < 1)$. *Then we have* $f(z) \in MC^*(\alpha)$.

Proof. Let

$$p(z) = -\frac{zf'(z)}{g(z)}, \tag{29}$$

then $p(z)$ is analytic in U and $p(0) = 1$.Now using (29), we have

$$1 + \frac{zf''(z)}{f'(z)} = \frac{zg'(z)}{g(z)} - \frac{zp'(z)}{p(z)}. \tag{30}$$

If there exists a point $z_0 \in U$ such that

$$Re\{p(z)\} > \alpha \quad for \quad |z| < |z_0| \tag{31}$$

and

$$Re\{p(z_0)\} = \alpha, \tag{32}$$

then applying Lemma 2.2 and the hypothesis of Theorem 2.2,we have

$$Re\{p(z_0)\} \neq p(z_0), \tag{33}$$

Therefore, applying Lemma 2.2,we have

$$1 + Re\{\frac{zf''(z)}{f'(z)}\} = Re\{\frac{zg'(z)}{g(z)}\} - Re\{\frac{zp'(z)}{p(z)}\} \tag{34}$$

$$< Re\{\frac{zg'(z)}{g(z)}\} + \frac{\alpha}{2(1-\alpha)}. \tag{35}$$

This contradicts the hypothesis of Theorem 2.2,and therefore we have $f(z) \in MC^*(\alpha)$.

Corollary 2.2. *Let* $f(z) \in \Sigma$*, suppose that for arbitrary* α *,* $f(z)$ *satisfies*

$$-z^2 f'(z) \neq \alpha \tag{36}$$

and the following inequalities:

$$1 + Re\{\frac{zf''(z)}{f'(z)}\} \geq \frac{3\alpha - 2}{2(1-\alpha)}, z \in D \tag{37}$$

Then we have $f(z) \in MC(\alpha)$.

Proof. Let $g(z) = 1/z$ in Theorem 2.2,we can obtain Corollary 2.2.

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