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Research Article

Relations Among Polygonal Numbers Through The Integer Solutions Of $z^2=6x^2+y^2$

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Abstract: The ternary quadratic equation given by $Z^2 = 6X^2 + Y^2$ is considered. Employing its non-zero integral solutions, relations among few special polygonal numbers are determined.
Keywords: Pell equations, Ternary quadratic equation.

INTRODUCTION:

In [1-3], different patterns of m-gonal numbers are presented. In [4] explicit formulas for the rank of Triangular numbers which are simultaneously equal to Pentagonal, Decagonal and Dodecagonal numbers in turn are presented. In [5,6] the relations among the pairs of special m-gonal numbers generated through the solutions of the binary quadratic equations are determined.

In this communication, we consider the ternary quadratic equation given by $Z^2 = 6X^2 + Y^2$ and obtain the relations among the pairs of special m-gonal numbers generated through its solutions.

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NOTATIONS:

$T_{m,n}$ **:** Polygonal number of rank n with m sides

METHOD OF ANALYSIS:

Consider the Diophantine equation

$$Z^2 = 6X^2 + Y^2 \qquad (1)$$

whose general solutions are

$$\left.\begin{aligned} X &= 6rs \\ Y &= 6r^2 - s^2 \\ Z &= 6r^2 + s^2 \end{aligned}\right\} \qquad (2)$$

where r and s are non-zero positive integers.

CHOICE (1):

The choices

$$2M + 1 = 6r^2 + s^2 \text{ , } 4N - 1 = 6r^2 - s^2 \qquad (3)$$

in (1) leads to the relation that

$$"4T_{3,M} - 4T_{6,N} = 3\alpha^2"$$

From (3), the values of ranks of the Triangular numbers and Hexagonal numbers are respectively given by

$$M = \frac{6r^2 + s^2 - 1}{2}, \; N = \frac{6r^2 - s^2 - 1}{4}$$

For integer values of M and N, choose $r = 2k, s = 2k - 1$

TABLE: 1- Examples

k	M	N	$4(T_{3,M} - T_{6,N})$
1	12	6	$3(4^2)$
2	52	22	$3(24^2)$
3	120	48	$3(60^2)$
4	216	84	$3(112^2)$

CHOICE (2):

The choices

$$2M + 1 = 6r^2 + s^2 \;,\; 6N - 1 = 6r^2 - s^2 \quad (4)$$

in (1) leads to the relation that

$$"4T_{3,M} - 12T_{5,N} = 3\alpha^2"$$

From (4), the values of ranks of the Triangular numbers and Pentagonal numbers are respectively given by

$$M = \frac{6r^2 + s^2 - 1}{2}, \; N = \frac{6r^2 - s^2 + 1}{6}$$

CASE: 1

choose $r = 4k - 3, s = 6k - 5$. The corresponding integer values of M, N are

$$M = 66k^2 - 102k + 39, \; N = 10k^2 - 14k + 5$$

TABLE: 2- Examples

k	M	N	$4T_{3,M} - 12T_{5,N}$
1	3	1	$3(2^2)$
2	99	17	$3(70^2)$
3	327	53	$3(234^2)$
4	687	109	$3(494^2)$

CASE: 2

choose $r = 4k - 1, s = 6k - 1$. The corresponding integer values of M, N are

$$M = 66k^2 - 30k + 3, \; N = 10k^2 - 6k + 1$$

TABLE: 3- Examples

k	M	N	$4T_{3,M} - 12T_{5,N}$
1	39	5	$3(30^2)$
2	207	29	$3(154^2)$
3	507	73	$3(374^2)$
4	939	137	$3(690^2)$

CHOICE (3):

The choices

$$2M + 1 = 6r^2 + s^2 \;,\; 3N - 1 = 6r^2 - s^2 \quad (5)$$

in (1) leads to the relation that

$$“8T_{3,M} - 3T_{8,N} = 6\alpha^2"$$

From (5), the values of ranks of the Triangular numbers and Octagonal numbers are respectively given by

$$M = \frac{6r^2 + s^2 - 1}{2},\ N = \frac{6r^2 - s^2 + 1}{3}$$

CASE: 1

choose $r = 6k - 2, s = 6k - 1$ .The corresponding integer values of M, N are

$$M = 126k^2 - 78k + 12\ \ N = 60k^2 - 44k + 8$$

TABLE: 4-Examples

k	M	N	$8T_{3,M} - 3T_{8,N}$
1	60	24	$6(40^2)$
2	360	160	$6(220^2)$
3	912	792	$6(544^2)$
4	1716	416	$6(1012^2)$

CASE: 2

choose $r = 6k, s = 6k + 1$. The corresponding integer values of M, N are

$$M = 126k^2 + 6k,\ N = 60k^2 - 4k$$

TABLE: 5

k	M	N	$8T_{3,M} - 3T_{8,N}$
1	132	56	$6(84^2)$
2	516	232	$6(312^2)$
3	1152	528	$6(684^2)$
4	2040	944	$6(1200^2)$

CHOICE (4):

The choices

$$6M - 1 = 6r^2 + s^2\ ,\ 2N + 1 = 6r^2 - s^2 \tag{6}$$

in (1) leads to the relation that

$$"8T_{3,M} - 24T_{5,N} = 6\alpha^2"$$

From (6), the values of ranks of the Triangular numbers and Pentagonal numbers are respectively given by

$$M = \frac{6r^2 + s^2 - 1}{2},\ N = \frac{6r^2 - s^2 + 1}{6}$$

CASE: 1

choose $r = 6k - 2, s = 6k - 1$. The corresponding integer values of M, N are

$$M = 126k^2 - 78k + 12,\ N = 30k^2 - 22k + 4$$

TABLE: 6- Examples

k	M	N	$8T_{3,M} - 24T_{5,N}$
1	60	12	$6(40^2)$
2	360	80	$6(220^2)$
3	912	208	$6(544^2)$
4	1716	396	$6(1012^2)$

CASE: 2

choose $r = 6k, s = 6k + 1$. The corresponding integer values of M, N are

$$M = 126k^2 + 6k,\ N = 60k^2 - 4k$$

TABLE: 7- Examples

k	M	N	$8T_{3,M} - 24T_{8,N}$
1	132	28	$6(84^2)$
2	516	116	$6(312^2)$
3	1152	264	$6(684^2)$
4	2040	472	$6(1200^2)$

CHOICE (5):

The choices

$$5M - 2 = 6r^2 + s^2\ ,\ N = 6r^2 - s^2 \qquad (7)$$

in (1) leads to the relation that

$$"5T_{12,M} - T_{4,N} = 6\alpha^2 - 4"$$

From (7), the values of ranks of the Dodecagonal numbers and Square numbers are respectively given by

$$M = \frac{6r^2 + s^2 + 2}{5},\ N = 6r^2 - s^2$$

For integer values of M and N, choose $r = 5k - 3,\ s = 5k - 3$

TABLE: 8- Examples

k	M	N	$5T_{12,M} - T_{4,N} + 4$
1	6	20	$6(8^2)$
2	69	245	$6(98^2)$
3	202	720	$6(288^2)$
4	405	1445	$6(578^2)$

CONCLUSION:

To conclude, we may search for other relations to (1) by using special polygonal numbers.

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