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Research Article

On the Sextic Diophantine Equation with Three Unknowns $x^2-xy+y^2=7z^6$

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Abstract: Sextic Diophantine equation with three unknowns $x^2-xy+y^2=7z^6$ has been analyzed for its nonzero integral solutions. Two different patterns of solutions and their corresponding properties are obtained.
Keywords: Sextic equation with three unknowns, integral solutions.

SUBJECT CLASSIFICATION: MSC 2010 11D41

NOTATIONS:

1. $OH_n = \frac{n}{3}(2n^2+1)$ = Octahedral number
2. $Pt_n = \frac{n(n+1)(n+2)(n+3)}{24}$ = Pentatope number of rank n
3. $t_{m,n} = n\left[1+\frac{(n-1)(m-2)}{2}\right]$, polygonal number of rank n with sides m
4. $P_n^r = \frac{1}{6}n(n+1)[(r-2)n+(5-r)]$, pyramidal number of rank n with sides r
5. $SO_n = n(2n^2-1)$ = Stella Octangula number
6. $\Pr_n = n(n+1)$ Pronic number of rank n
7. gn_p = 2p – 1 , Gnomonic number of rank p
8. $HRD_n = (2n-1)(8n^2-14n+7)$, Hauy Rhombic dodecahedral number
9. $f_{m,s}^r = (rs+m-s)(r+m-2)!/m!(r-1)!$, m dimensional figurate no. of rank r whose generating polygon has s sides

INTRODUCTION

Diophantine equations, homogeneous and non-homogeneous, have aroused the interest of numerous mathematicians since antiquity as can be seen from [1, 2]. The problem of finding all integer solutions of a Diophantine equation with three or more variables and degree atleast three, in general, presents a good deal of difficulties. There is a vast general theory of homogeneous quadratic equations with three variables [1-5]. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research [6, 7]. A lot is known about equations in two variables in higher degrees. For equations with more than three variables and degree at least three, very little is known. It is worth to note that undesirability appears in equations, even perhaps at degree four and degree five with fairly small coefficients. In this context one may refer [9-12]. In [13-15], sextic equations with three unknowns are studied for their non-zero integer solutions.

In this communication, yet another interesting sextic diophantine equation with three unknowns $x^2 - xy + y^2 = 7z^6$ is considered for integral solutions. Two different patterns of solutions and their corresponding properties are obtained.

METHOD OF ANALYSIS

The problem under consideration is

$$x^2 - xy + y^2 = 7z^6 \tag{1}$$

PATTERN 1

Consider the linear transformations

$$\left.\begin{array}{l} x = u + v,\ y = u - v \\ \text{Assume } z = a^2 + 3b^2 \end{array}\right\} \tag{2}$$

Write 7 as

$$7 = (2 + i\sqrt{3})(2 - i\sqrt{3}) \tag{3}$$

Define

$$u + i\sqrt{3}v = (2 + i\sqrt{3})(a + i\sqrt{3}b)^6$$

By equating real and imaginary parts, we get

$$\left.\begin{array}{l} u = 2a^6 - 90a^4b^2 + 270a^2b^4 - 54b^6 - 18a^5b + 180a^3b^3 - 162ab^5 \\ v = a^6 + 12a^5b - 45a^4b^2 - 120a^3b^3 + 135a^2b^4 + 108ab^5 - 27b^6 \end{array}\right\} \tag{4}$$

By employing (4) in (2), the solutions of (1) are given by

$$x(a,b) = 3a^6 - 6a^5b - 135a^4b^2 + 60a^3b^3 + 405a^2b^4 - 54ab^5 - 81b^6$$

$$y(a,b) = a^6 - 30a^5b - 45a^4b^2 + 300a^3b^3 + 135a^2b^4 - 270ab^5 - 27b^6$$

$$z(a,b) = a^2 + 3b^2$$

PROPERTIES

1. Each of the following expressions represents a nasty number
   (i) $306[x(a,1) - 108(P_a^3)^2 + 12SO_a * \Pr_a + 6t_{7,a}^2 - 336P_a^5 - 54t_{6,a}]$
   (ii) $6[z(a,1) - 3]$
   (iii) $126[3y(a,1) - x(a,1) + 21(t_{6,a} * 3OH_a) + 42(\Pr_a)^2 + 482SO_a - 137gn_a - 137]$
   (iv) $2x(a,a)$
   (v) $y(a,a)$
   (vi) 6z (a, a)
   (vii) 9x(a, a) - 3y(a, a)
2. $x(1,b) + 324(P_b^5)^2 - 54SO_b * \Pr_b - 378(\Pr_b)^2 \equiv 0 \pmod 3$
3. $x(a,1) - 2160 f_{6,1}^a + 6120 f_{5,1}^a - 2880Pt_a - 3360P_a^5 + 33\Pr_a \equiv 0 \pmod 3$
4. 3 x(a, a) –y (a, a) = difference of 2 perfect squares.
5. $y(a,1) + x(a,1) - 144(P_a^3)^2 + 30SO_a * \Pr_a + 133(\Pr_a)^2 + 7HRD_a - 128t_{6,a} \equiv 1 \pmod 2$
6. $y(a,1) - 4(P_a^5)^2 + 16SO_a * \Pr_a + 14\mathrm{P}t_a - 184SO_a - 17t_{6,a} - 2t_{3,a} + 35gn^a \equiv 0 \pmod 2$
7. $x(a,1) - 108(P_a^3)^2 + 12SO_a * \Pr_a + 6t_{7,a}^2 - 54t_{6,a} - 336P_a^5 \equiv 0 \pmod 3$
8. $x(a,1) - 3y(a,1) + 21\Pr_a - 21t_{6,a} * 3OH_a - 42(\Pr_a)^2 - 482SO_a \equiv 0 \pmod 5$

PATTERN 2

It is to be noted that on the R.H.S of (1), 7 can also be written as

$$7 = \frac{(1+i3\sqrt{3})(1-i3\sqrt{3})}{4} \tag{5}$$

Following the procedure as in pattern1, the integer solutions of (1) are given by

$$x(a,b) = 2a^6 - 24a^5b - 90a^4b^2 + 240a^3b^3 + 270a^2b^4 - 216ab^5 - 54b^6$$

$$y(a,b) = -a^6 - 30a^5b + 45a^4b^2 + 300a^3b^3 - 135a^2b^4 - 270ab^5 + 27b^6$$

$$z(a,b) = a^2 + 3b^2$$

CONCLUSION

We have presented two different patterns of solutions for the Sextic diophantine equation with three unknowns $x^2 - xy + y^2 = 7z^6$. To conclude one may search for other patterns of solutions and their corresponding properties.

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