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Research Article

On the Homogenious Cubic Equation with Four Unknowns $X^3+Y^3=7ZT^3$

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Abstract: The homogeneous cubic equation with four unknowns represented by the Diophantine equation $X^3+Y^3=7ZT^3$ is analyzed for its patterns of non-zero integral solutions. Two patterns of solutions are illustrated. A few interesting relations between the solutions and special numbers are exhibited.
Keywords: Cubic equation with four unknowns, Integral solution.

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Notations:

$T_{m,n}$: Polygonal number of rank n with size m

SO_n: Stella octangular number of rank m

$CP_{m,n}$: Centered pyramidal number rank n with size m

INTRODUCTION

The Diophantine equations offer an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-19] for cubic equation with four unknowns. This communication concerns with yet another interesting equation $X^3+Y^3=7ZT^3$ representing the homogeneous cubic equation with four unknowns for determining its infinitely many non-zero integral points. Also a few interesting relations among the solutions are presented.

METHOD OF ANALYSIS

The ternary non-homogeneous cubic Diophantine equation to be solved for its distinct non-zero integral solutions is

$$x^3+y^3=7zt^3 \quad (1)$$

Introduction of the transformations

$$\left.\begin{aligned} x&=u+v\\ y&=u-v\\ z&=2u\left(7n^2-10n+4\right)\end{aligned}\right\} \quad (2)$$

in (1) leads to

$$u^2+3v^2=\left(49n^2-70n+28\right)t^3 \quad (3)$$

Let

$$t=a^2+3b^2 \quad (4)$$

Write $49n^2-70n+28$ as

$$49n^2-70n+28=\left(-5+7n+i\sqrt{3}\right)\left(-5+7n-i\sqrt{3}\right) \quad n=1,2,3..... \quad (5)$$

Using (4) & (5) in (3) and applying the method of factorization, define

$$\left(u+i\sqrt{3}v\right)=\left(-5+7n+i\sqrt{3}\right)\left(a+i\sqrt{3}b\right)^3 \quad (6)$$

Equating real and imaginary parts, we get

$$u=-5a^3+45ab^2+7na^3-63nab^2+9b^3-9a^2b \quad (7)$$

$$v=a^3-9ab^2-21nb^3+21na^2b+15b^3-15a^2b \quad (8)$$

using (7) & (8) in (2) we have,

$$\left.\begin{aligned} x(n,a,b)&=-4a^3+36ab^2+24b^3-24a^2b+7n\left(a^3-9ab^2-3b^3+3a^2b\right)\\ y(n,a,b)&=-6a^3+54ab^2-6b^3+6a^2b+7n\left(a^3-9ab^2+3b^3-3a^2b\right)\\ z(n,a,b)&=\left(-10a^3+90ab^2+18b^3-18a^2b+14na^3-126nab^2\right)\left(49n^2-70n+28\right)\end{aligned}\right\} \quad (9)$$

Thus (4) & (9) are the corresponding non trivial integral solutions of the given equation (1)

Properties

- $x(n,a,a)-y(n,a,a)+8SO_n+8n=0$
- $x(n,a,b)-y(n,a,b)+10CP_{6,a}+18CP_{6,b}-14nCP_{6,a}=90ab^2-18a^2b-126nab^2$
- $x(n,a,b)+4y(n,a,b)+28CP_{6,a}+35nCP_{6,a}-63nCP_{6,b}=252ab^2-63na^2b+315nab^2$
- $x(n,a,a)-16SO_n+28SO_n-T_{58,n}-43=0$
- $y(n,a,1)+6CP_{6,a}-6T_{4,n}-54a+6=n\left(7CP_{6,a}-21T_{4,a}-63a\right)$

It is worth to note that $49n^2-28n+7$ is also represented as

$$49n^2-28n+7=\left(-2+7n+i\sqrt{3}\right)\left(-2+7n-i\sqrt{3}\right), i=1,2,3...... \quad (10)$$

Following the analysis presented above, the corresponding integer solutions to (1) are found to be

$$\left.\begin{aligned} x(n,a,b)&=-a^3+9ab^2+15b^3-15a^2b+7n\left(a^3-9ab^2-3b^3+3a^2b\right)\\ y(n,a,b)&=-3a^3+27ab^2+3b^3-3a^2b+7n\left(a^3-9ab^2+3b^3-3a^2b\right)\\ z(n,a,b)&=\left(-4a^3+36ab^2+14na^3-126nab^2+18b^3-18a^2b\right)\left(49n^2-28n+7\right)\end{aligned}\right\} \quad (11)$$

Properties

- $x(n,a,a)-y(n,a,a)-16SO_n-16n=0$
- $3x(n,a,b)-y(n,a,b)-42CP_{6,b}-14nCP_{6,a}-42nCP_{6,b}=84na^2b-42a^2b-126nab^2$
- $3x(n,a,1)-y(n,a,1)-42CP_{6,b}+42T_{4,a}-14nCP_{6,b}-84T_{4,a}-42CP_{6,b}+126a=0$
- $x(n,a,b)-y(n,a,b)-2CP_{6,a}-12CP_{6,b}=21na^2b-18ab^2-12a^2b$
- $z(n,1,b)=\left(-4+36T_{4,b}+18CP_{6,b}-18+14n-126T_{4,b}\right)\left(49T_{4,n}-28n+7\right)$

CONCLUSION

In this paper, we have presented two different patterns of non-zero distinct integer solutions to the cubic equation with four unknowns given by $x^3+y^3=7z\,t^2$. To conclude, one may search for other patterns of solutions and their corresponding properties.

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