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Research Article

Small Parameter Method for a Heat Conduction Problem with a Second Order Nonlinear Term

Xin Wang
School of Civil Engineering and Architecture, Northeast Petroleum University, Daqing 163318, China

***Corresponding author**
Xin Wang
Email: vickinepu@126.com

Abstract: By the small parameter method, the approximate solution of a heat conduction problem with a second order nonlinear term is solved.
Keywords: Small parameter method, Heat conduction problem

INTRODUCTION

A general one dimensional heat conduction equation is as follows [1]

$$u_t = u_{xx} + f(x,t,u). \quad (1)$$

In some special cases, for example, if u is the density of N -order chemistry reactions, we have

$$f(x,t,u) = l u^N.$$

If $f(x,t,u) = u - u^2$, equation (1) describes the dynamics of biological group. Since equation (1) is a nonlinear differential equation, in general, it is difficult to solve it.

In the present paper, we consider a second order heat conduction problem

$$u_t = u_{xx} + l u^2, \quad (2)$$

with the initial condition

$$u(0,x) = f(x). \quad (3)$$

Our trick is to introduce a small parameter e to transform the equation (2) into

$$u_t = u_{xx} + e l u^2. \quad (4)$$

We find the solution of equation (4) as a power series of e, and in finial take $e = 1$ to give the corresponding solution of equation (2).

The small parameter method has been applied to many problems such as quantum mechanics [2], vibration theory [3] and celestial mechanics [4] and so on. This is a so powerful method that a lot of nonlinear problem can be dealt with.

SOLUTION BY SMALL PARAMETER METHOD

We deal with the traveling wave solution $u(x) = u(x - wt)$, so the equation (3) becomes

$$u' = au'' + bu^2, \quad (5)$$

where $a = 1/w, b = l/w$. Respectively, equation (4) becomes

$$u' = au'' + e bu^2. \quad (6)$$

Furthermore, we assume that the initial condition of equation (5) is

$$u(0) = A, \quad u'(0) = B. \quad (7)$$

Now, we solve the problem by small parameter method. Suppose that the solution of equation (6) can be expanded as a power series of e

$$u(x) = u_0(x) + u_1(x)e + u_2(x)e^2 + u_3(x)e^3 + \cdots, \quad (8)$$

where u_k 's are unknown functions. As $\varepsilon = 1$, the solution (8) is just the solution of equation (5). So the corresponding initial conditions are

$$u_0(0) = A, \quad u_0'(0) = B, \tag{9}$$

$$u_k(0) = 0, \quad u_k'(0) = 0. \tag{10}$$

Substituting solution (8) into equation (6) and letting the coefficients of every ε terms to be zero give the following linear equations

$$u_0' = au_0'', \tag{11}$$

$$u_1' = au_1'' + bu_0^2, \tag{12}$$

$$u_2' = au_2'' + 2bu_0u_1, \tag{13}$$

and so on.

Solving equation (11) with the initial condition (9) yields

$$u_0 = A_1e^{x/a} + B_1,$$

where

$$A_1 = \frac{a(A - B)}{a - 1}, \quad B_1 = \frac{aB - A}{a - 1}.$$

Solving equation (12) with the initial condition (10) yields

$$u_1 = A_2e^{x/a} + B_2e^{2x/a} + C_2xe^{x/a} + D_2x + E_2,$$

where

$$A_2 = 2abA_1B_1(a - 2) + b(B_1^2 - A_1), \quad B_2 = abA_1/2, \quad C_2 = 2bA_1B_1,$$

$$D_2 = -bB_1^2, \quad E_2 = 2abA_1B_1(1 + b - a) - ab(B_1^2 - A_1) - abA_1/2.$$

We can also go on solving equation (13) to give more terms. For simplicity, we omit them. According to our computation, the first order approximate traveling wave solutions of equation (3) is given as

$$u = A_1e^{x/a} + B_1 + \varepsilon(A_2e^{x/a} + B_2e^{2x/a} + C_2xe^{x/a} + D_2x + E_2).$$

By taking $\varepsilon = 1$, the first order approximate traveling wave solutions of equation (2) is given as

$$u = (A_1 + A_2)e^{x/a} + B_2e^{2x/a} + C_2xe^{x/a} + D_2x + E_2 + B_1.$$

This is an exponent function type solution.

CONCLUSION

Small parameter method is a powerful method of solving nonlinear problems. In particular, if the nonlinear term is small, the method is always gives good approximation.

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