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Research Article

On Special Diophantine Triples

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Abstract: This paper concerns with the study of constructing a special Diophantine triples (a,b,p) such that the product of any two elements of the set added with their sum and increased by a polynomial with integer coefficients is a Perfect square.
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INTRODUCTION

A Set of positive integers ($a_1, a_2, a_3, ...a_m$) is said to have the property D(n), $n \in z - \{0\}$, if $a_i a_j + n$ is a perfect square for all $1 \le i \lhd j \le m$ and such a set is called a Diophantine –m-tuple with property D(n). Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n [1] and also for any linear polynomial in n. Further, various authors considered the connections of the problems of diophantus, davenport and Fibonacci numbers in [2-19]. In this communication, we find special Diophantine triple (k, k+3,4k+8) in which the product of any two elements of the set added with their sum and increased by (1-k) is a perfect square.

Construction Of Special Diophantine Triple With Property D(1-K)

Let $a = k, b = k+3$ be any two non-zero distinct integers such that $ab + a + b + (1-k)$ is a perfect square. We search for a non-zero distinct integer p such that in the triple (a,b,p) the product of any two elements added with their sum and increased by (1-k) is a perfect square.

That is,

$$p(k+1)+1 = \alpha^2 \qquad (1)$$

$$p(k+4)+4 = \beta^2 \qquad (2)$$

Eliminating "p'' we get

$$(k+1)\beta^2 - (k+4)\alpha^2 = 3k \qquad (3)$$

The introduction of the linear transformations

$$\alpha = X + (k+1)T \qquad (4)$$

$$\beta = X + (k+4)T \qquad (5)$$

In (3) leads to the pell equation

$$X^2 = (k^2+3k+2)T^2 + (k+2) \qquad (6)$$

Whose initial solution is $T_0 = 1, X_0 = k+2$. Thus (4) yields $\alpha_0 = 2k+3$ and using (1), we get $p = 4k+8$

Hence (a,b,p) = (k, k+3, 4k+8) is the required special Diophantine triples with property $D(1-k)$.

The repetition of the above process leads to the generation of special Diophantine triples $(F_{m+2}^2 k + F_{m+3}^2 - 1, F_{m+3}^2 k + F_{m+4}^2 - 1, F_{m+4}^2 k + F_{m+5}^2 - 1)$ with property $D(1-k)$. Here $F_{-1} = 1, F_0 = 1, F_1 = 2, F_2 = 3, ...$

For illustration, a few examples are presented below

$$(k+3, 4k+8, 9k+24), (4k+8, 9k+24, 25k+63), (9k+24, 25k+63, 64k+168),$$
$$(25k+63, 64k+168, 169k+440), (64k+168, 169k+440, 441k+1155)$$

REMARK 1

Note that, when k=0, the triple (0, 3, 8) is the special Diophantine triple with property $D(1)$. Observe that $3 = 2^2 - 1, 8 = 3^2 - 1$, if α_0 is any non-zero integer such that $(0, 3, 8, \alpha_0)$ is the special Diophantine quadruples with property $D(1)$, then it is seen that $\alpha_0 = 24 = 5^2 - 1$.

The repetition of the above process leads to the generation of special Diophantine tuples $\{0, F_1^2 - 1, F_2^2 - 1, F_3^2 - 1, ...F_{m+2}^2 - 1, ...\}$ with property $D(1)$.

Where $F_{-1} = 1, F_0 = 1, F_{m+2} = F_m + F_{m+1}, (m = -1, 0, 1, 2, 3...)$

REMARK 2:

Replacing k by a Gaussian integer and irrational numbers respectively in each of the above triples, it is noted that each resulting triple is a Gaussian triple and irrational triple satisfying the required property.

K	Triples (a,b,p)	Property
$1 + i\sqrt{3}$	$(4+i\sqrt{3}, 12+i4\sqrt{3}, 33+i9\sqrt{3}), (12+i4\sqrt{3}, 33+i9\sqrt{3}, 88+i25\sqrt{3}),$ $(33+i9\sqrt{3}, 88+i25\sqrt{3}, 232+i64\sqrt{3}), (88+i25\sqrt{3}, 232+i64\sqrt{3}, 609+i169\sqrt{3}),$ $(232+i64\sqrt{3}, 609+i169\sqrt{3}, 1596+i441\sqrt{3})$	$D(-i\sqrt{3})$
$2 + i3$	$(5+i3, 16+i12, 42+i27), (16+i12, 42+i27, 113+i75)$ $(42+i27, 113+i75, 296+i192), (113+i75, 296+i192, 778+i507),$ $(296+i192, 778+i507, 2037+i1323)$	$D(-1-i3)$

CONCLUSION

To conclude, one may search for other choices of triples with suitable property.

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REFERENCES

1. Bashmakova IG editor; Diophantus of Alexandria. In Arithmetics and the Book of Polygonal Numbers. Nauka, Moscow, 1974.
2. Thamotherampillai N; The set of numbers {1,2,7}. Bull Calcutta Math Soc., 1980; 72: 195-197.
3. Brown E; Sets in which xy + k is always a square. Math Comp., 1985; 45: 613-620.

4. Gupta H, Singh K; On k-triad sequences. Internet J Math Sci., 1985; 5: 799-804.
5. Beardon AF, Deshpande MN; Diophantine triples. The Mathematical Gazette, 2002; 86: 253-260.
6. Deshpande MN; One interesting family of diophantine triples. Internet J Math Ed Sci Tech., 2002; 33: 253-256.
7. Deshpande MN; Families of diophantine triplets. Bulletin of the Marathawada Mathematical Society, 2003; 4: 19-21.
8. Bugeaud Y, Dujella A, Mignotte; On the family of diophantine triples $(k-1, k+1, 16k^3-4k)$. Glasgow Math J., 2007; 49: 333-344.
9. Liqun T; On the property P_{-1}. Electronic Journal of Combinatorial Number Theory, 2007; 7: #A47.
10. Fujita Y; The extensibility of diophantine pairs (k-1,k+1). J Number Theory, 2008; 128: 322-353.
11. Srividhya G; Diophantine quadruples for fibbonacci numbers with property D (1). Indian Journal of Mathematics and Mathematical Science, 2009; 5(2): 57-59.
12. Gopalan MA, Pandichelvi V; The non extendibility of the diophantine triple $(4(2m-1)^2n^2, 4(2m-1)n+1, 4(2m-1)^4n^4-8(2m-1)^3n^3)$, Impact J Sci Tech., 2011; 5(1): 25-28.
13. Fujita Y, Togbe A; Uniqueness of the extension of the $D(4k^2)$-triple $(k^2-4, k^2, 4k^2-4)$. NNTDM 17, 2011; 4: 42-49.
14. Gopalan MA, Srividhya G; Some non extendable P_{-5} sets. Diophantus J Math., 2012; 1(1): 19-22.
15. Gopalan MA, Srividhya G; Two special diophantine triples. Diophantus J Math., 2012; 1(1): 23-27.
16. Gopalan MA, Srividhya G; Diophantine quadruple for fionacci and lucas numbers with property D(4). Diophantus J Math., 2012; 1(1): 15-18.
17. Flipin A, He B, Togbe A; On a family of two parametric D(4) triples. Glas Mat Ser III, 2012; 47: 31-51.
18. Fujita Y; The unique representation $d = 4k(k^2-1)$ in D(4)-quadruples{k-2,k+2,4k,d}. Math Commun., 2006; 11: 69-81.
19. Filipin A, Fujita Y, Mignotte M; The non extendibility of some parametric families of D(-1)-triples. Q J Math., 2012; 63: 605-621.