

Short Communication

Some Indices for Hexagonal Triangle Graph

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Abstract: In this short communication, we determine the vertex PI index, second atom bond connectivity index and revised Szeged index for hexagonal triangle graph.

Keywords: vertex PI index, second atom bond connectivity index, revised Szeged index, Hexagonal Triangle Graph

INTRODUCTION

Wiener index, PI index, Szeged index and atom bond connectivity index are applied to reflect certain structural features of organic molecules Yan et al., [1] and [2], Gao and Shi [3] for more details). Let $e=uv$ be an edge of the molecular graph G . The number of vertices of G whose distance to the vertex u is smaller than the distance to the vertex v is denoted by $n_u(e)$. Analogously, $n_v(e)$ is the number of vertices of G whose distance to the vertex v is smaller than the distance to the vertex u . The vertex PI index of G is defined as

$$PI_v(G) = \sum_{e=uv} [n_u(e) + n_v(e)].$$

Let $n(e)$ be the number of vertices equidistant from both ends of $uv \in E(G)$, and m be the edge number of molecular graph G . The revised Szeged index $Sz^*(G)$ and normalized revised Szeged index $Szs^*(G)$ are defined as

$$Sz^*(G) = \sum_{e=uv} (n_u(e) + \frac{n(e)}{2})(n_v(e) + \frac{n(e)}{2}),$$

$$Szs^*(G) = \sqrt{\frac{Sz^*(G)}{m}}.$$

The second atom bond connectivity index is denoted by

$$ABC_2(G) = \sum_{uv \in E(G)} \sqrt{\frac{n_u(e) + n_v(e) - 2}{n_u(e)n_v(e)}}.$$

In our short communication, we present the vertex PI index, second atom bond connectivity index and revised Szeged index for Hexagonal triangle graph $T(n)$, containing j hexagons in the j th row, $1 \leq j \leq n$.

INDICES FOR HEXAGONAL TRIANGLE GRAPH

To determine the vertex PI index and second atom bond connectivity index of this molecular graph, we note that the graph $T(n)$ has an equilateral figure and so $|E(G)| = \frac{3}{2}(n^2 + 3n)$ and $|V(G)| = n^2 + 4n + 1$. Consider a vertical edge $e=uv$ of the j th row of the graph $T(n)$. We note that this row has exactly $j+1$ vertical edges and so $S_j = n_u(e) = j^2 + 2j$ and $T_j = n_v(e) = n^2 + 4n + 1 - j^2 - 2j$. Therefore,

$$PI_v(T(n)) = 3 \sum_{1 \leq i \leq n} [n_u(e) + n_v(e)] = 3 \sum_{1 \leq i \leq n} (1+i)[S_i + T_i]$$

$$\begin{aligned}
 &= 3 \sum_{1 \leq i \leq n} (1+i)(n^2 + 4n + 1) = 3(n^2 + 4n + 1) \left(\frac{n^2}{2} + \frac{3n}{2} \right). \\
 ABC_2(T(n)) &= 3 \sum_{1 \leq i \leq n} \sqrt{\frac{n_u(e) + n_v(e) - 2}{n_u(e)n_v(e)}} = 3 \sum_{1 \leq i \leq n} \sqrt{(1+i) \frac{S_i + T_i - 2}{S_i T_i}} \\
 &= 3 \sum_{1 \leq i \leq n} \sqrt{(1+i) \frac{n^2 + 4n - 1}{(j^2 + 2j)(n^2 + 4n + 1 - j^2 - 2j)}}. \\
 Sz^*(T(n)) &= 3 \sum_{1 \leq i \leq n} \left(n_u(e) + \frac{n(e)}{2} \right) \left(n_v(e) + \frac{n(e)}{2} \right) = 3 \sum_{1 \leq i \leq n} (1+i) \left(S_i + \frac{n(e)}{2} \right) \left(T_i + \frac{n(e)}{2} \right) \\
 &= 3 \sum_{1 \leq i \leq n} (1+i) S_i T_i = 3 \sum_{1 \leq i \leq n} [-i^5 - 5i^4 + (n^2 + 4n - 7)i^3 + (3n^2 + 12n - 1)i^2 + 2(n^2 + 4n + 1)i] \\
 &= \frac{n^6 + 12n^5 + 49n^4 + 84n^3 + 58n^2 + 12n}{4}. \\
 Sz_s^*(T(n)) &= \sqrt{\frac{Sz^*(T(n))}{|E(T(n))|}} = \sqrt{\frac{n^6 + 12n^5 + 49n^4 + 84n^3 + 58n^2 + 12n}{6(n^2 + 3n)}}.
 \end{aligned}$$

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