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# Steady Plane Poiseuille flow of viscous incompressible Fluid between Two Porous Parallel Plates through Porous Medium with Magnetic Field

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**Abstract:** In this paper we have investigated the steady plane poiseuille flow of viscous incompressible fluid between two porous parallel plates through porous medium with magnetic field. We have studied the velocity, average velocity, shearing stress, skin frictions a, the volumetric flow, drag coefficients and stream lines.
**Keywords:** Steady poiseuille flow, viscous parallel plates, incompressible fluid, porous medium and magnetic field..

**NOMENCLATURE:**
u =velocity component along x–axis
v = velocity component along y–axis
t = the time
$\rho$ = the density of fluid
P = the fluid pressure
K= the thermal conductivity of the fluid
$\mu$ = Coefficient of viscosity
$\upsilon$ = Kinematic viscosity
Q= the volumetric flow

**INTRODUCTION:**

We have investigated steady plane poiseuille flow of viscous incompressible fluid between two porous parallel plates through porous medium with magnetic field. Attempts have been made by several researchers i.e. M. Aydin & R.T. Fenner [1] boundary element analysis of driven cavity flow for low and moderate Reynolds numbers M and moderate Reynolds numbers N. Bahloul & Boutana and P. Vasseur [2] Double-diffusive and soret-induced convection in a shallow horizontal porous layer. V. et al. Barbu [3] exact controllability magneto-hydrodynamic equations. D. Barkley & L. S. Tuckerman [4] Stability analysis of perturbed plane Couette flow. D. Barkley & L. S. Tuckerman [5] Turbulent-laminar patterns in plane Couette flow. D. Barkley & L. S. Tuckerman [6] Mean flow of turbulent-laminar patterns in plane Couette flow. D.Barkley, L. S. Tuckerman [7] Turbulent-laminar patterns in plane Couette flow. E. Barragy & G. F. Carey [8] Stream function-vorticity driven cavity solutions using p finite elements. G. K.Batchelor [9] a proposal concerning laminar wakes behind bluff bodies' at large Reynolds number. G. K. Batchelor [10] on steady laminar flow with closed streamlines at large Reynolds number. C. Baytas and I. Pop [11] free convection in oblique enclosures filled with a porous medium. R.M. Beam & R.F. warming [12] an implicit factored scheme for the compressible Navier-stokes equations. Beant Singh & Chanpreet Singh [12] Analysis of Vortex Motion in Porous Media Journal of Electronics Cooling and Thermal Control. In this paper we have investigated the velocity, average velocity, shearing stress, skin frictions a, the volumetric flow, drag coefficients and stream lines.

**FORMULATION OF PROBLEM:**

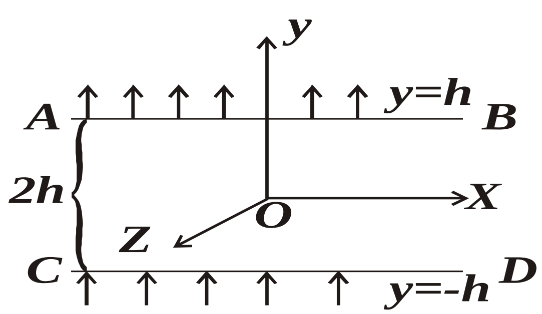


**Fig. 1**

Let us consider two infinite porous plates AB & CD separated by a distance *2h*. The fluid enters in y-direction. The velocity component along x-axis is a function of y only. The motion of incompressible fluid is in two dimension and is steady then

$$u = u\ (y), \quad w = 0\ \&\ \frac{\partial}{\partial t} = 0$$

The equation of continuity for incompressible fluid

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \text{ and put w = 0 }, \quad \frac{\partial u}{\partial x} = 0 \quad \& \quad \Rightarrow \quad \frac{\partial v}{\partial y} = 0$$

***v*** is independent of ***y*** but motion along **y**-axis. So we can say **v** is constant velocity i.e. $v = v_0$

The fluid enters the flow region through one plate at the same constant velocity $v_0$

Also Navier-Stoke's equations for incompressible fluid in the absence of body force when flow is steady

$$v_0 \frac{du}{dy} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon \frac{d^2u}{dy^2} + \frac{\upsilon u}{K} + \frac{\sigma B_0^2\, \upsilon u}{\mu} \quad \text{......} (1)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \text{...........} (2)$$

**SOLUTION OF THE PROBLEM:**

Equation (2) Shows that the pressure does not depend on $y$ hence $p$ is a function of $x$ only and so (1) reduces to

$$\frac{dp}{dx} = \rho\left[\upsilon \frac{d^2u}{dy^2} - v_0 \frac{du}{dy} + \frac{\upsilon u}{K} + \frac{\sigma B_o^2\, \upsilon u}{\mu}\right] \quad \text{Where } \frac{dp}{dx} = \text{Constant} = -\text{P}$$

$$\Rightarrow \frac{d^2u}{dy^2} - \frac{v_0}{\upsilon}\frac{du}{dy} + \frac{u}{K} + \frac{\sigma B_o^2 u}{\mu} = -\frac{P}{\mu} \quad \Rightarrow \quad \left(D^2 - \frac{v_0}{\upsilon}D + \frac{1}{K} + \frac{\sigma B_o^2}{\mu}\right)u = -\frac{P}{\mu}$$

$$\text{A.E} \quad m^2 - \frac{v_0}{\upsilon}m + \frac{1}{K} + \frac{\sigma B_o^2}{\mu} = 0 \Rightarrow m = \frac{\frac{v_0}{\upsilon} \pm \sqrt{\left(\frac{v_0}{\upsilon}\right)^2 - 4\left(\frac{1}{K} + \frac{\sigma B_0^2}{\mu}\right)}}{2} = \frac{v_0}{2\upsilon} \pm \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B_0^2}{\mu}\right)}$$

$$C.F. = e^{\frac{v_0}{2\upsilon}y}\left[c_1 Cosh\, Ay + c_2 Sinh\ Ay\right] \quad P.I = -\frac{P}{B\mu}$$

$$Let \quad A = \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B_0^2}{\mu}\right)} \ and\ B = \frac{1}{K} + \frac{\sigma B_0^2}{\mu}$$

$$u(y) = e^{\frac{v_0}{2\upsilon}y}\left[c_1 Cosh\ Ay + c_2 Sinh\, Ay\right] - \frac{P}{B\mu}$$

**Using boundary conditions:** $u = 0\ at\ y = -h\ and\ u = U\ at\ y = h$

$$e^{-\frac{v_0}{2\upsilon}h}\left[c_1 Cosh\ Ah - c_2 Sinh\ Ah\right] - \frac{P}{B\mu} = 0 \text{ .......} (3)$$

$$U = e^{\frac{v_0}{2\upsilon}h}\left[c_1 Cosh\ Ah + c_2 Sinh\ Ah\right] - \frac{P}{B\mu} \text{ .......} (4)$$

$$or \quad \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}h} = c_1 Cosh\, Ah - c_2 Sinh\, Ah \quad , \quad \left(U + \frac{P}{B\mu}\right)e^{-\frac{v_0}{2\upsilon}h} = c_1 Cosh\, Ah + c_2 Sinh\, Ah$$

$$c_1 = \frac{1}{2Cosh\,Ah}\left[\left(U+\frac{P}{B\mu}\right)e^{-\frac{v_0}{2\upsilon}h} + \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}h}\right] \quad \& \quad c_2 = \frac{1}{2Sinh\,Ah}\left[\left(U+\frac{P}{B\mu}\right)e^{-\frac{v_0}{2\upsilon}h} - \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}h}\right]$$

$$u(y) = \frac{e^{\frac{v_0}{2\upsilon}y}Cosh\,Ay}{2Cosh\,Ah}\left\{\left(U+\frac{P}{B\mu}\right)e^{-\frac{v_0}{2\upsilon}h} + \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}h}\right\}$$

$$+\frac{e^{\frac{v_0}{2\upsilon}y}Sinh\,Ay}{2Sinh\,Ah}\left\{\left(U+\frac{\rho}{B\mu}\right)e^{-\frac{v_0}{2\upsilon}h} - \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}h}\right\} - \frac{P}{B\mu}$$

$$u(y) = \left(U+\frac{P}{B\mu}\right)\frac{e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\,A(y+h)}{2Sinh\,Ah\;Cosh\,Ah} - \frac{P}{B\mu}\frac{e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\,A(y-h)}{2Sinh\,Ah\;Cosh\,Ah} - \frac{P}{B\mu}$$

$$u(y) = \frac{1}{Sinh\,2Ah}\left[\left(U+\frac{P}{B\mu}\right)e^{\frac{v_0}{2\upsilon}(y-h)}SinhA(y+h) - \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}(y+h)}SinhA(y-h)\right] - \frac{P}{B\mu} \quad \text{.........}(5)$$

**Plane Poiseuille flow:** In this case both plates are at rest so U = 0

$$\therefore\; u(y) = \frac{1}{Sinh\;2Ah}\left[\frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}(y-h)}\;Sinh\;A(y+h) - \frac{P}{B\mu}e^{\frac{v_0}{2\upsilon}(y+h)}\;Sinh\;A(y-h)\right] - \frac{P}{B\mu}$$

$$u(y) = \frac{P}{B\mu\,Sinh\;2Ah}\left[e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\;A(y+h) - e^{\frac{v_0}{2\upsilon}(y+h)}\;Sinh\;A(y-h) - Sinh\;2Ah\right] \quad \text{.......}(6)$$

**Shearing stress at any point**

$$\sigma_{xy} = \mu\frac{du}{dy} = \frac{P}{B\,Sinh\;2Ah}\left[\left\{\frac{v_0}{2\upsilon}e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\;A(y+h) + Ae^{\frac{v_0}{2\upsilon}(y-h)}Cosh\;A(y+h)\right\}\right.$$

$$\left. -\frac{v_0}{2\upsilon}e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\;A(y-h) - Ae^{\frac{v_0}{2\upsilon}(y+h)}Cosh\;A(y-h)\right]$$

$$\sigma_{xy} = \frac{P}{B\,Sinh2Ah}\left[\frac{v_0}{2\upsilon}\left\{e^{\frac{v_0}{2\upsilon}(y-h)}SinhA(y+h) - e^{\frac{v_0}{2\upsilon}(y+h)}SinhA(y-h)\right\}\right]$$

$$+\frac{PA}{B\,Sinh2Ah}\left\{e^{\frac{v_0}{2\upsilon}(y-h)}CoshA(y+h) - e^{\frac{v_0}{2\upsilon}(y+h)}CoshA(y-h)\right\} \text{....}(7)$$

**Skin friction at lower & upper plates**

$$\left(\sigma_{xy}\right)_{y=h} = \frac{P}{B\,Sinh\;2Ah}\left[\frac{v_0}{2\upsilon}\left\{Sinh\;2Ah\right\} + A\left\{Cosh\;2Ah - e^{\frac{v_0}{\upsilon}h}\right\}\right]$$

$$\left(\sigma_{xy}\right)_{y=h} = \frac{P}{B\,Sinh\;2Ah}\left[\frac{v_0}{2\upsilon}Sinh\;2Ah + ACosh\;2Ah - Ae^{\frac{v_0}{\upsilon}h}\right] \quad \text{.........}(8)$$

$$\left(\sigma_{xy}\right)_{y=-h} = \frac{P}{B\,Sinh\;2Ah}\left[\frac{v_0}{2\upsilon}Sinh\;2Ah + A\left\{e^{-\frac{v_0}{\upsilon}h} - Cosh\;2Ah\right\}\right]$$

$$\left(\sigma_{xy}\right)_{y=-h} = \frac{P}{B\,Sinh\,2Ah}\left[\frac{v_0}{2\upsilon}Sinh\;2Ah - ACosh\;2Ah + Ae^{-\frac{v_0}{\upsilon}h}\right] \quad \text{.........}(9)$$

**The average velocity distribution in poiseuille flow:**

$$u_{av} = \frac{1}{2h}\int_{-h}^{h} u(y)\,dy$$

$$= \frac{P}{2B\mu h\, Sinh\, 2Ah}\int_{-h}^{h}\left[e^{\frac{v_0}{2\upsilon}(y-h)} Sinh\, A(y+h) - e^{\frac{v_0}{2\upsilon}(y+h)} Sinh\, A(y-h) - Sinh\, 2Ah\right] dy$$

$$Now\ Let \quad I_1 = \int_{-h}^{h} e^{\frac{v_0}{2\upsilon}(y-h)} Sinh\, A(y+h)\, dy$$

$$= \frac{1}{2}\int_{-h}^{h}\left\{e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)} - e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}\right\} dy = \frac{1}{2}\left[\frac{e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)}}{\left(\frac{v_0}{2\upsilon}+A\right)} - \frac{e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right]_{-h}^{h}$$

$$= \frac{1}{2}\left[\frac{e^{2Ah} - e^{-\frac{v_0}{\upsilon}h}}{\left(\frac{v_0}{2\upsilon}+A\right)} - \frac{e^{-2Ah} - e^{-\frac{v_0}{\upsilon}h}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right] = \frac{1}{2B}\left[\left(\frac{v_0}{2\upsilon}-A\right)\left(e^{2Ah} - e^{-\frac{v_0}{\upsilon}h}\right) - \left(\frac{v_0}{2\upsilon}+A\right)\left(e^{-2Ah} - e^{-\frac{v_0}{\upsilon}h}\right)\right]$$

$$= \frac{1}{2B}\left[\frac{v_0}{2\upsilon}\left\{e^{2Ah} - e^{-\frac{v_0}{\upsilon}h} - e^{-2Ah} + e^{-\frac{v_0}{\upsilon}h}\right\} - A\left\{e^{2Ah} - e^{-\frac{v_0}{\upsilon}h} + e^{-2Ah} - e^{-\frac{v_0}{\upsilon}h}\right\}\right]$$

$$I_1 = \frac{1}{B}\left[\frac{v_0}{2\upsilon} Sinh\, 2Ah - ACosh\, 2Ah + Ae^{-\frac{v_0}{\upsilon}h}\right]$$

$$I_2 = \int_{-h}^{h} e^{\frac{v_0}{2\upsilon}(y+h)} Sinh\, A(y-h)\, dy = \frac{1}{B}\left[\frac{v_0}{2\upsilon} Sinh\, 2Ah + ACosh\, 2Ah - Ae^{\frac{v_0}{\upsilon}h}\right]$$

$$I_3 = \int_{-h}^{h} Sinh\, 2Ah\, dy = 2h\, Sinh\, 2Ah$$

$$\therefore u_{av} = \frac{P}{2B\mu\, Sinh\, 2Ah}\left[I_1 - I_2 - I_3\right]$$

$$= \frac{P}{2B^2\mu\, h\, Sinh\, 2Ah}\left[\frac{v_0}{2\upsilon} Sinh\, 2Ah - A\, Cosh\, 2Ah + A\, e^{-\frac{v_0}{\upsilon}h} - \frac{v_0}{2\upsilon} Sinh\, 2Ah - A\, Cosh\, 2Ah + A\, e^{\frac{v_0}{\upsilon}h} - 2hB\, Sinh\, 2Ah\right]$$

$$u_{av} = \frac{P}{B^2\mu h\, Sinh\, 2Ah}\left[A\left(Cosh\frac{v_0}{\upsilon}h - Cosh\, 2Ah\right) - BhSinh\, 2Ah\right] \quad .......(10)$$

**The volumetric flow** $Q = 2h\, u_{av}$

$$= \frac{2P}{B^2\mu\, Sinh\, 2Ah}\left[A\left(Cosh\frac{v_0}{\upsilon}h - Cosh\, 2Ah\right) - BhSinh\, 2Ah\right] \quad .........(11)$$

**The Drug coefficients**: $C_f \ \&\ C_f^{'}\ at\ y = h\ \&\ y = -h$

$$C_f = \frac{\left(\sigma_{xy}\right)_{y=h}}{\frac{1}{2}\rho(u_{av})^2} = \frac{2B^2\mu^2h^2\, Sinh\, 2Ah}{\rho P}\left[\frac{\left\{\frac{v_0}{2\upsilon} Sinh\, 2Ah + ACosh\, 2Ah - Ae^{\frac{v_0}{\upsilon}h}\right\}}{\left\{A\left(Cosh\frac{v_0}{\upsilon}h - Cosh\, 2Ah\right) - BhSin\, 2Ah\right\}^2}\right] \quad .......(12)$$

$$C_f{}' = \frac{\left(\sigma_{xy}\right)_{y=-h}}{\frac{1}{2}\rho(u_{av})^2} = \frac{2B^2\mu^2h^2\ Sinh\ 2Ah}{\rho P}\left[\frac{\left\{\frac{v_0}{2\upsilon}Sin\ 2Ah - ACosh\ 2Ah + Ae^{-\frac{v_0}{\upsilon}h}\right\}}{\left[A\left(Cosh\frac{v_0}{\upsilon}h - Cos\ 2Ah\right) - BhSinh\ 2Ah\right]^2}\right] \text{.........(13)}$$

**The stream line in the plane poiseuille flow:** $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

$$\Rightarrow \frac{dx}{\frac{P}{B\mu\ Sinh\ 2Ah}\left\{e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\ A(y+h) - e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\ A(y-h) - Sinh\ 2Ah\right\}} = \frac{dy}{v_0} = \frac{dz}{o}$$

**Taking first two**

$$\frac{v_0 B\mu\ Sinh\ 2Ah}{P}\ x - \int\left\{e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\ A(y+h) - e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\ A(y-h) - Sinh\ 2Ah\right\}dy = C_1$$

$$Let\quad I_1 = \int e^{\frac{v_0}{2\upsilon}(y-h)}Sinh\ A(y+h)\ dy$$

$$I_1 = \frac{1}{2}\left[\frac{e^{\frac{v_0}{2\upsilon}(y-h)+A(y+h)}}{\left(\frac{v_0}{2\upsilon}+A\right)} - \frac{e^{\frac{v_0}{2\upsilon}(y-h)-A(y+h)}}{\left(\frac{v_0}{2\upsilon}-A\right)}\right]$$

$$= \frac{e^{\frac{v_0}{2\upsilon}(y-h)}}{2B}\left[\left(\frac{v_0}{2\upsilon}-A\right)e^{A(y+h)} - \left(\frac{v_0}{2\upsilon}+A\right)e^{-A(y+h)}\right]\quad Since\left(\frac{v_0}{2\upsilon}\right)^2 - A^2 = B$$

$$= \frac{e^{\frac{v_0}{2\upsilon}(y-h)}}{B}\left[\frac{v_0}{2\upsilon}Sinh\ A(y+h) - ACosh\ A(y+h)\right]$$

$$I_2 = \int e^{\frac{v_0}{2\upsilon}(y+h)}Sinh\ A(y-h)\ \ dy = \frac{e^{\frac{v_0}{2\upsilon}(y+h)}}{B}\left[\frac{v_0}{2\upsilon}Sinh\ A(y-h) - ACosh\ A(y-h)\right]$$

$$I_3 = \int Sinh\ 2Ah\ .\ dy = y.\ Sinh\ 2Ah$$

$\therefore$ **First stream line.**

$$\frac{v_0 B\mu\ Sinh\ 2Ah}{P}x - \{I_1 - I_2 - I_3\} = C_1$$

$$\Rightarrow \frac{v_0 B\mu\ Sinh\ 2Ah}{P}x - \frac{e^{\frac{v_0}{2\upsilon}(y-h)}}{B}\left\{\frac{v_0}{2\upsilon}Sinh\ A(y+h) - A\ Cosh\ A(y+h)\right\}$$

$$+\frac{e^{\frac{v_0}{2\upsilon}(y+h)}}{B}\left\{\frac{v_0}{2\upsilon}Sinh\ A(y-h) - A\ Cosh\ A(y-h)\right\} + y\ Sinh\ \ 2Ah = C_1 \text{ .......(14)}$$

**Second stream line** $z = c_2$........(15)

Clearly the curl $\bar{q} \neq \bar{0}$ $\therefore$ the fluid is Rotational

**Table for velocity:**

$$P=9,\quad \mu=.5,\quad h=.5,\quad \frac{v_0}{2\upsilon}=6,\quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{K}+\frac{\sigma B_0^2}{\mu}\right)}=2,\quad When\quad \frac{1}{K}=\frac{\sigma B_0^2}{\mu}=16$$

$$\therefore \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\frac{1}{K}}=\sqrt{20}=A$$

**Table-1 (for velocity)**

| | y | 0 | .1 | .2 | .3 | .4 | .5 | .6 |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{K}=16$ | u(y) | 1.269 | 1.627 | 1.978 | 2.195 | 1.9 | 0 | -6.654 |
| $\frac{\sigma B_0^2}{\mu}=16$ | u(y) | 1.269 | 1.627 | 1.978 | 2.195 | 1.9 | 0 | -6.654 |
| $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ | u(y) | 3.11 | 4.5 | 6.07 | 7.29 | 6.6 | 0 | -22.26 |

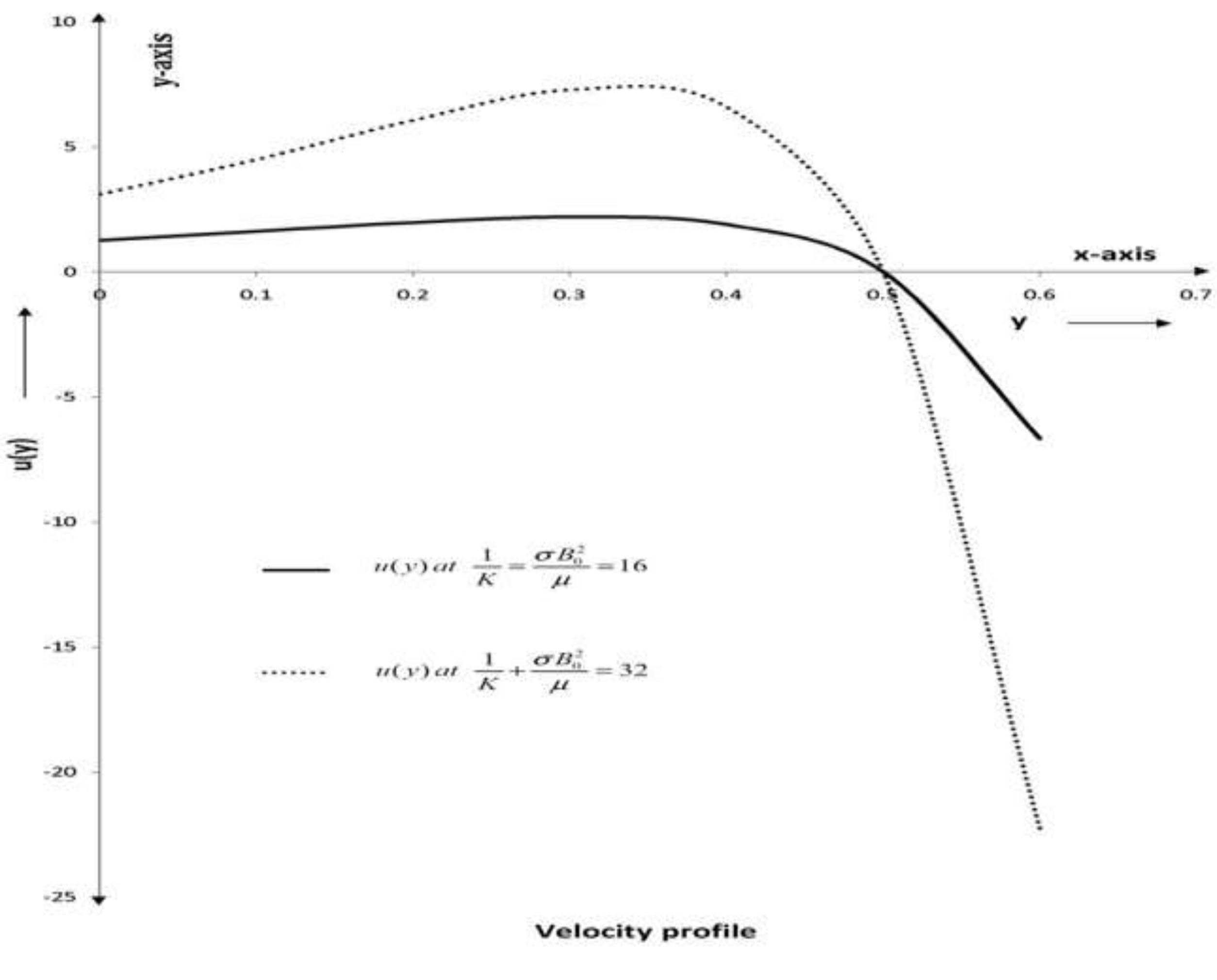


**Fig. -1**

**Table for Skin friction**

$$P=9,\quad \mu=.5,\quad h=.5,\quad \frac{v_0}{2\upsilon}=6,\quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{K}+\frac{\sigma B_0^2}{\mu}\right)}=2,\quad When\quad \frac{1}{K}=\frac{\sigma B_0^2}{\mu}=16$$

$$\therefore \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\frac{1}{K}}=\sqrt{20}=A$$

**Table-2 (for skin friction)**

| | y | 0 | .1 | .2 | .3 | .4 | .5 | .6 |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{K}=16$ | $\sigma_{xy}$ | **1.732** | **1.827** | **1.586** | **.30** | **-4.06** | **-17.297** | **-55.96** |
| $\frac{\sigma B_0^2}{\mu}=16$ | $\sigma_{xy}$ | **1.732** | **1.827** | **1.586** | **.30** | **-4.06** | **-17.297** | **-55.96** |
| $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ | $\sigma_{xy}$ | **6.215** | **7.62** | **7.695** | **3.30** | **-13.27** | **-60.3** | **-180.09** |

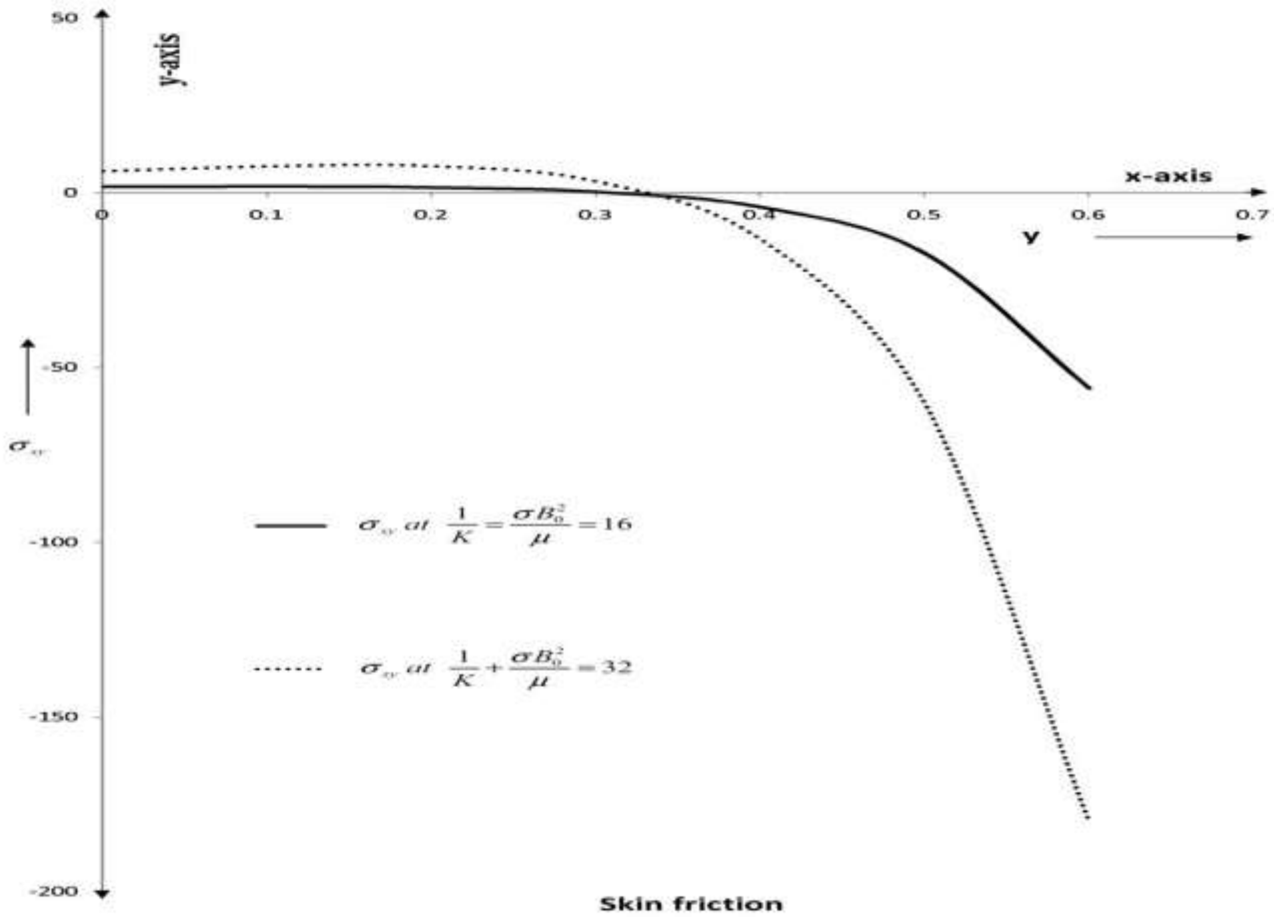


**Fig.-2**

**Table for velocity:**

$$P = 9, \quad \mu = .5, \quad h = .5, \quad \frac{v_0}{2\upsilon} = 6, \quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B_0^2}{\mu}\right)} = 2, \quad When \quad \frac{1}{K} \text{ p } \frac{\sigma B_0^2}{\mu}$$

$$\therefore \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15} = A$$

**Table-3(for velocity)**

| | y | 0 | .1 | .2 | .3 | .4 | .5 | .6 |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{K} = 11$ | u(y) | 1.05 | 1.31 | 1.56 | 1.69 | 1.45 | 0 | -5.178 |
| $\frac{\sigma B_0^2}{\mu} = 21$ | u(y) | 1.58 | 2.09 | 2.616 | 2.965 | 2.597 | 0 | -8.966 |
| $\frac{1}{K} + \frac{\sigma B_0^2}{\mu} = 32$ | u(y) | 3.11 | 4.5 | 6.07 | 7.29 | 6.6 | 0 | -22.26 |

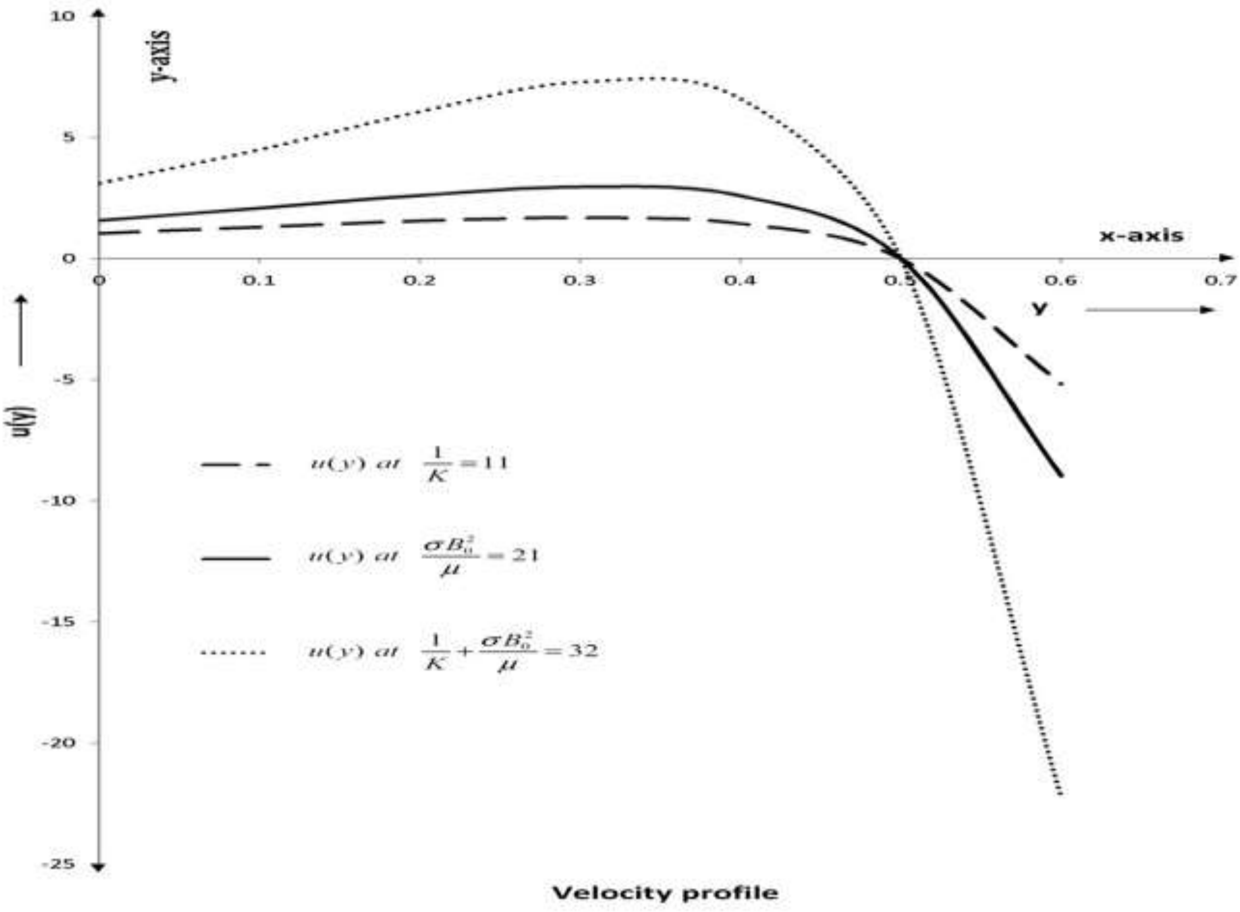


**Fig.-3**

**Table for Skin friction**

$$P=9, \quad \mu=.5, \quad h=.5, \quad \frac{v_0}{2\upsilon}=6, \quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{K}+\frac{\sigma B_0^2}{\mu}\right)}=2, \quad When \quad \frac{1}{K} \text{ p } \frac{\sigma B_0^2}{\mu}$$

$$\therefore \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\frac{\sigma B_0^2}{\mu}}=\sqrt{15}=A$$

**Table-4(for skin friction)**

| | y | 0 | .1 | .2 | .3 | .4 | .5 | .6 |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{K}=11$ | $\sigma_{xy}$ | **1.285** | **1.3** | **1.08** | **.107** | **-3.135** | **-13.24** | **-44** |
| $\frac{\sigma B_0^2}{\mu}=21$ | $\sigma_{xy}$ | **2.417** | **2.665** | **2.424** | **.675** | **-5.47** | **-23.63** | **-74.5** |
| $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ | $\sigma_{xy}$ | **6.215** | **7.62** | **7.695** | **3.3** | **-13.27** | **-60.3** | **-180.09** |

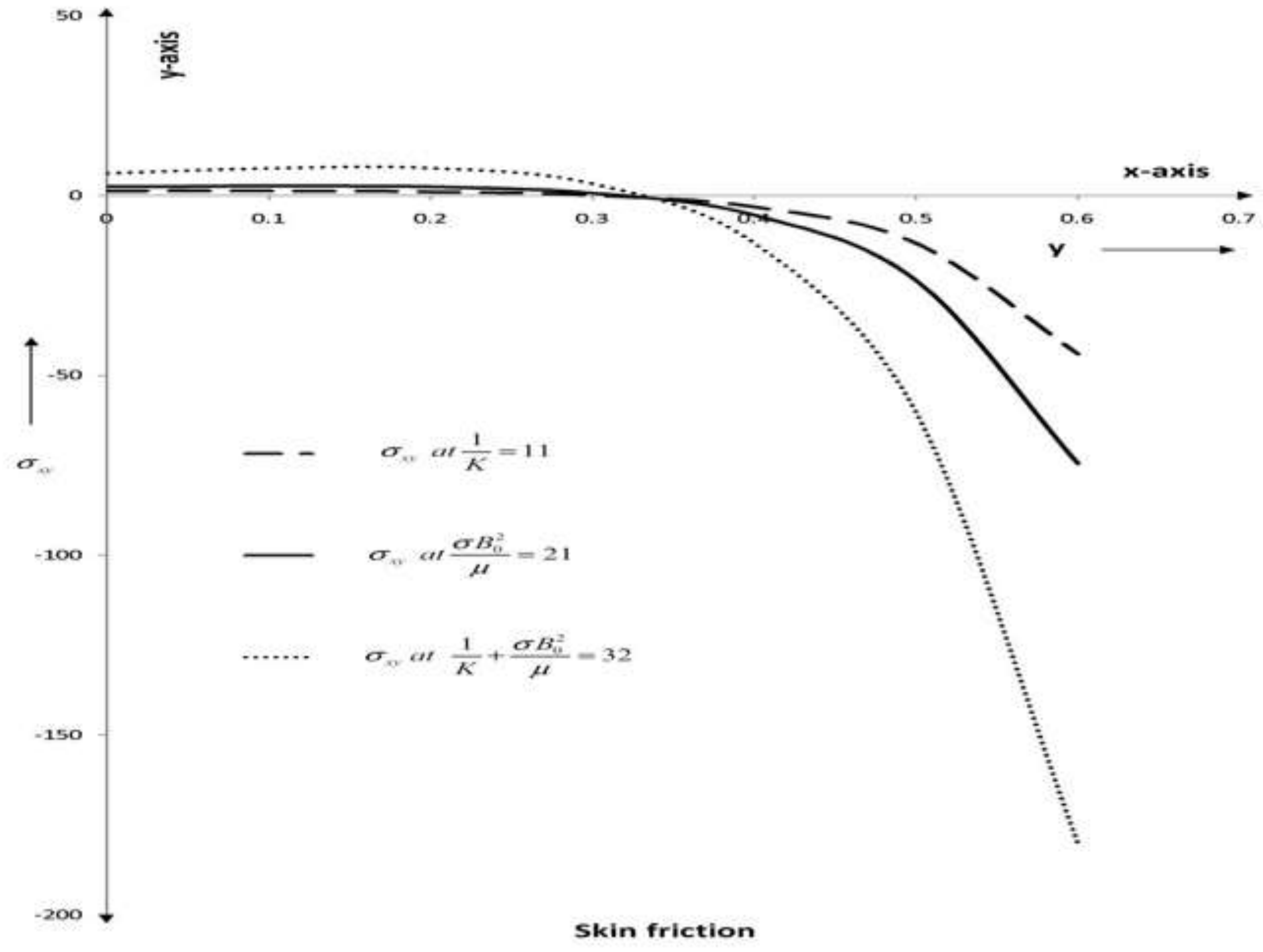


**Fig.-4**

**Table for velocity:**

$$P = 9, \quad \mu = .5, \quad h = .5, \quad \frac{v_0}{2\upsilon} = 6, \quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B_0^2}{\mu}\right)} = 2, \quad When \quad \frac{1}{K} \;\mathrm{f}\; \frac{\sigma B_0^2}{\mu}$$

$$\therefore \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2 - \frac{1}{K}} = \sqrt{15} = A$$

**Table-5 (for velocity)**

| | y | 0 | .1 | .2 | .3 | .4 | .5 | .6 |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{K} = 21$ | **u(y)** | **1.58** | **2.09** | **2.616** | **2.965** | **2.597** | **0** | **-8.966** |
| $\frac{\sigma B_0^2}{\mu} = 11$ | **u(y)** | **1.05** | **1.31** | **1.56** | **1.69** | **1.45** | **0** | **-5.178** |
| $\frac{1}{K} + \frac{\sigma B_0^2}{\mu} = 32$ | **u(y)** | **3.11** | **4.5** | **6.07** | **7.29** | **6.6** | **0** | **-22.26** |

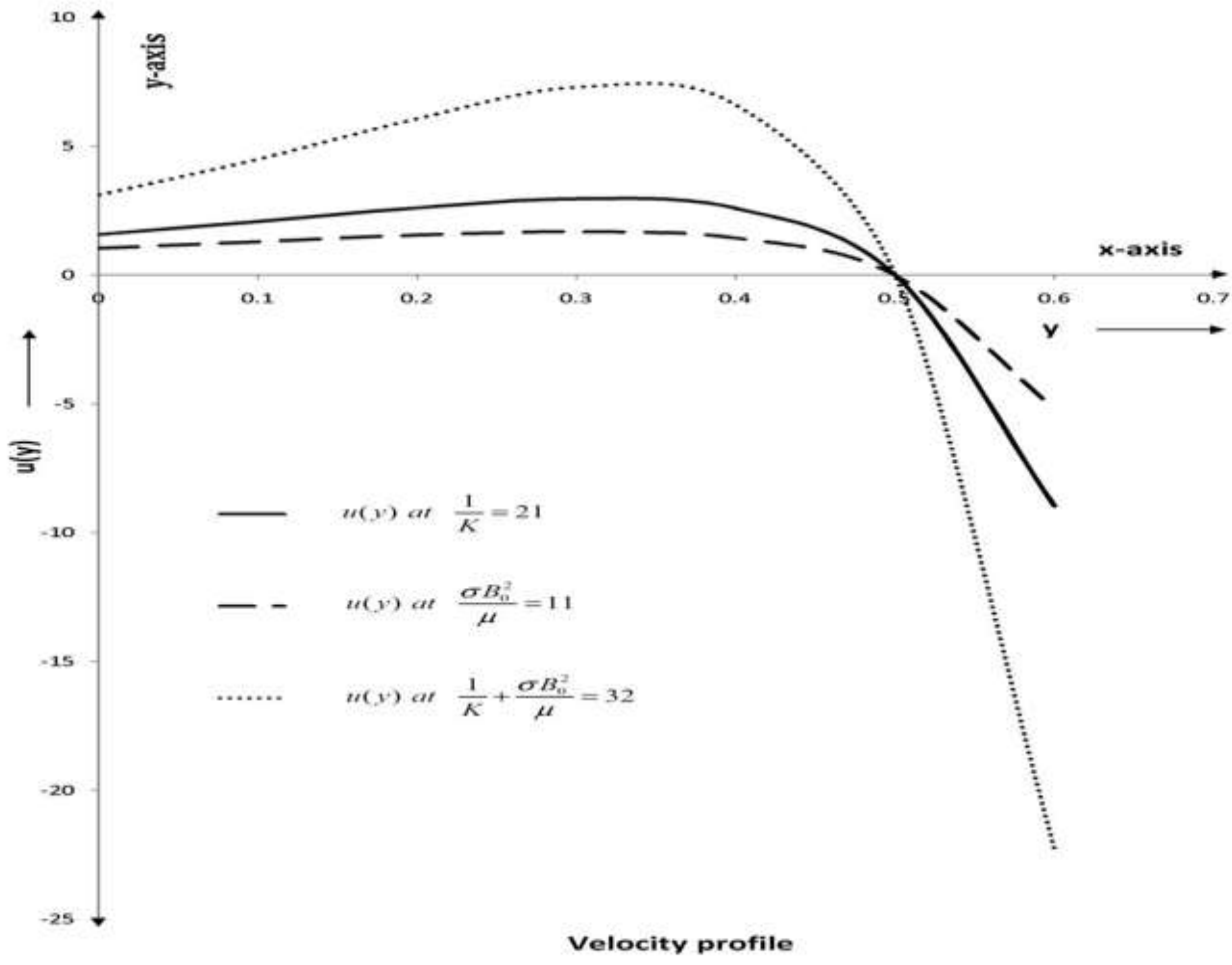


**Fig.-5**

**Table for Skin friction**

$$P=9,\quad \mu=.5,\quad h=.5,\quad \frac{v_0}{2\upsilon}=6,\quad \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\left(\frac{1}{K}+\frac{\sigma B_0^2}{\mu}\right)}=2,\quad When\ \ \frac{1}{K} \text{ f } \frac{\sigma B_0^2}{\mu}$$

$$\therefore \sqrt{\left(\frac{v_0}{2\upsilon}\right)^2-\frac{1}{K}}=\sqrt{15}=A$$

**Table-6(for skin friction)**

| | y | 0 | .1 | .2 | .3 | .4 | .5 | .6 |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{K}=21$ | $\sigma_{xy}$ | **2.417** | **2.665** | **2.424** | **.675** | **-5.47** | **-23.63** | **-74.5** |
| $\frac{\sigma B_0^2}{\mu}=11$ | $\sigma_{xy}$ | **1.285** | **1.3** | **1.08** | **.107** | **-3.135** | **-13.24** | **-44** |
| $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ | $\sigma_{xy}$ | **6.215** | **7.62** | **7.695** | **3.3** | **-13.27** | **-60.3** | **-180.09** |

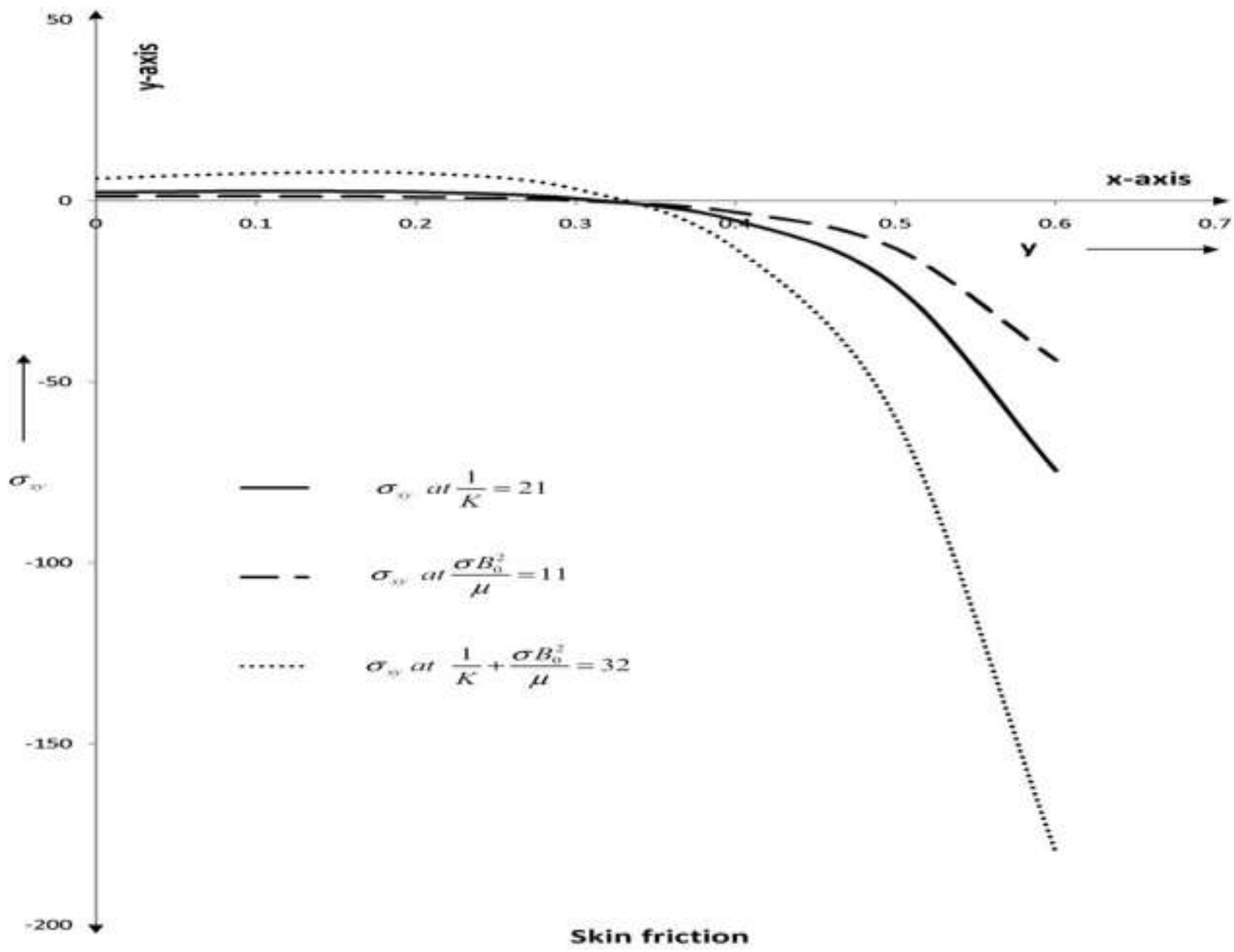


**Fig.-6**

**CONCLUSION AND DISCUSSION:**

In this paper, we have investigated the velocity by the graph of table-1 of equation (5). The velocity in porous medium and magnetic field at $\frac{1}{K}=\frac{\sigma B_0^2}{\mu}=16$ is less than the corresponding value of velocity in porous with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval. $0 \le y \le 0.3$ But the value of velocity is equal to zero in all mediums at $y=0.5$.

Again from the table-3 the value of velocity in porous medium at $\frac{1}{K}=11$ is less than the corresponding value of velocity in magnetic field at $\frac{\sigma B_0^2}{\mu}=21$ and is also less than the corresponding value of velocity in porous medium with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0 \le y \le 0.4$. The value of velocity in all mediums is equal to zero at $y=0.5$ and value of velocity (negatively) in porous medium is less than the value of velocity (negatively) in magnetic field and also is less than the velocity (negatively in porous medium with magnetic field at $y=0.6$.

Again from the table-5 the value of velocity in magnetic field at $\frac{\sigma B_0^2}{\mu}=11$ is less than the corresponding value of velocity in porous medium at $\frac{1}{K}=21$ and is also less than the corresponding value of velocity in porous medium with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0 \le y \le 0.4$.The value of velocity in all mediums is equal to zero at $y=0.5$ and value of velocity (negatively) in magnetic field is less than the value of velocity (negatively) in porous medium and also is less than the velocity (negatively in porous medium with magnetic field at $y=0.6$.

Again we have investigated the skin friction by the graph of table-2 of equation (8). The skin friction in porous medium and the magnetic field at $\frac{1}{K}=\frac{\sigma B_0^2}{\mu}=16$ is less than the corresponding value of skin friction in porous medium with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0 \le y \le 0.3$ and also skin friction (negatively) in porous medium and magnetic field at $\frac{1}{K}=\frac{\sigma B_0^2}{\mu}=16$ is less than the corresponding value of skin friction (negatively) in porous medium with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0.4 \le y \le 0.6$.

Again from the table-4 the value of skin friction in porous medium at $\frac{1}{K}=11$ is less than the corresponding value of skin friction in magnetic field at $\frac{\sigma B_0^2}{\mu}=21$ and is also less than the corresponding value of skin friction in porous with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval. $0 \le y \le 0.3$ and skin friction (negatively) in porous medium at $\frac{1}{K}=11$ is less than the corresponding value of skin friction (negatively) in magnetic field at $\frac{\sigma B_0^2}{\mu}=21$ and is also less than the corresponding value of skin friction (negatively) in porous medium with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0.4 \le y \le 0.6$.

Again from the table-6 the value of skin friction in magnetic field at $\frac{\sigma B_0^2}{\mu}=11$ is less than the corresponding value of skin friction in porous medium at $\frac{1}{K}=21$ and is also less than the corresponding value of skin friction in porous with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0 \le y \le 0.3$ and skin friction (negatively) in magnetic field at $\frac{\sigma B_0^2}{\mu}=11$ is less than the corresponding value of skin friction (negatively) in porous medium at $\frac{1}{K}=21$ and is also less than the corresponding value of skin friction (negatively) in porous medium with magnetic field at $\frac{1}{K}+\frac{\sigma B_0^2}{\mu}=32$ in the interval $0.4 \le y \le 0.6$. Also we have investigated shearing stress, the volumetric flow, drag coefficients and stream lines by the equations (7), (9), (11), (12),(13), (14) and (15) respectively. The fluid is rotational.

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