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Research Article

Improved approximation of PERT activity parameters and validation

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Abstract: In Project Evaluation and Review Technique (PERT) , the activity mean and variance are very useful to find the expected project duration and variance of the critical path. Project duration makes a large difference in the economic aspects of the project. Assuming activity durations in a project are beta distributed, new estimates of mean and variance of activity duration are derived. It is observed that the estimated mean is more moderate and the estimated variance is more conservative compared to other estimates of beta distribution. It is observed that, the new time estimates overcomes the problem of optimistic planning.
Keywords: PERT, Beta distribution, Activity times, Project duration.

INTRODUCTION

One of the most controversial issues in the antiquity of PERT is the distribution of activity durations and the approaches used to estimate the mean and the variance of activity times. As the activity mean and variance plays an important role in finding project duration and in turn project duration makes a large difference in the economic aspects of the project, many researchers estimated mean and variances using different distributions namely beta, triangular, uniform, normal, lognormal, etc. In 1959, Malcolm et al [9]., the creators of PERT considered beta distribution

$$f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}, \quad a < y < b, \quad \alpha, \beta > 0,$$

as a suitable distribution of the activity duration *y* where α and β are the parameters of the beta distribution and (a, b) is the domain of *f*. They have illustrated in a practical way that the mean and the variance of the activity duration *y* could be estimated as:

Mean of activity duration: $\mu_y = \dfrac{a+4m+b}{6}$

Variance of activity duration: $\sigma_y^2 = \dfrac{(b-a)^2}{36}$, where *a, m, b* are optimistic, most likely and pessimistic times of an activity determined by an expert.

During the period 1959-1987 researchers have tried to explain the relationship between the beta distribution and the estimates. Since 1987, several authors Farnum and Stanton [3],Ginzburg [6], Keefer and Verdini [8] have either modified the original PERT time estimates or proposed new ones to estimate the activity time more accurately. Cottrell [2] assumed the activity durations are normal distributed and estimated the mean and variance using most likely time and pessimistic time. Mohan et al. [10] presented a method that uses only two parameters, i.e., either most likely and optimistic or most likely and pessimistic time, assuming activity times are lognormally distributed. The experimental results showed that their method is better than the normal approximation. Garcia et al. [4] introduced a new distribution, namely Biparabolic (BP) and generalized this distribution in context of PERT methodology. Later Garcia et al. [5] introduced Standard Generalized Biparabolic distribution and studied suggested modified estimates of mean and variance assuming that the activity times belongs to subfamilies of BP distributions of constant variance and mesokurtic BP distributions. Herrerias et al. [7] approximated the estimate of variance by means of varying the condition of constant variance, while retaining the estimate of mean.

Ben-Yair [1] has theoretically justified the use of beta distribution by considering various comparative options, for man machine type activities. Moreover, the beta distribution can be estimated relatively easily from data on just the optimistic, pessimistic and most likely values. The managers and planners find it easier to estimate these three values than other statistical parameters and hence the beta distribution has been applied in many practical problems. Hence, our interest is to find better estimates of mean and variances using beta distribution.

EXISTING PERT APPROXIMATIONS

Traditional PERT Time Estimates

To determine the mean and variance of the activity duration distribution in PERT, Malcolm et al. [9] supposed that $\alpha - 1 = p,\ \beta - 1 = q$. Based on the statistical analysis and other intuitive arguments and the assumption $p + q \cong 4$ they showed that

$$\mu = \frac{a + 4m + b}{6}, \quad \sigma \approx \frac{b - a}{6}$$

Farnum and Stanton Time Estimates

Farnum and Stanton [3] stated that, on the basis of PERT assumption of constant variance $\sigma_x^2 = \frac{1}{36}$, σ_x^2 is not affected much by the shape parameters α and β, therefore the following assumption holds approximately

$$\sigma_x^2(\alpha, \beta) \cong \sigma_x^2(\alpha - 1, \beta - 1) \cong \frac{1}{36}$$

where $\sigma_x(\alpha, \beta)$ is the standard deviation of *x* with beta parameters α and β.

The authors defined the mean and variance as follows;

$$If \quad 0.13 \le m_x \le 0.87 \qquad \mu_x = \frac{4m_x + 1}{6} \quad and \quad \sigma^2{}_x = \frac{1}{36}$$

$$If \quad m_x < 0.13 \qquad \mu_x = \frac{2}{2 + \frac{1}{m_x}} \quad and \quad \sigma_x{}^2 = \left[\frac{m_x{}^2(1 - m_x)}{(1 + m_x)}\right]$$

$$If \quad m_x > 0.87 \qquad \mu_x = \frac{1}{3 - 2m_x} \quad and \quad \sigma^2{}_x = \left[\frac{m_x(1 - m_x)^2}{(2 - m_x)}\right]$$

Ginzburg Time Estimates

Ginzburg [6] pointed out that, the assumption $p + q = 4$ becomes poor because the actual variance is considerably smaller than $\frac{1}{36}$, especially in the tails of distribution. By assuming $p + q = k$ (a constant) and restricting the set of possible beta-distributions to those the alternative value is equal to $\frac{1}{36}$, he calculated the estimates μ_x and $\sigma_x{}^2$ as follows:

$$\mu_x = \frac{9m_x + 2}{13} \quad \text{and} \quad \sigma_x{}^2 = \frac{(22 + 81m_x - 81m_x{}^2)}{1268}$$

For the general beta distribution of the activity times, the mean, variance are given by

$$\mu_y = 0.2(3a + 2b) \quad \text{and} \quad \sigma_y{}^2 = 0.04(b - a)^2$$

Premchandra Time Estimates

Premchandra [11] developed a procedure to estimate mean and variance without violating the PERT assumptions.

He proposed the estimates of μ_x and σ_x as follows:

$$If \quad m_x \le 0.13, \quad \mu_x = \frac{2m_x}{1+2m_x} \quad and \quad \sigma_x^{\,2} = m_x^{\,2}(1-m_x)$$

$$If \quad m_x \ge 0.87, \quad \mu_x = \frac{1}{3-2m_x} \quad and \quad \sigma_x^{\,2} = m_x(1-m_x)^2$$

$$If \quad 0.13 < m_x < 0.87, \quad \mu_x = \frac{36m_x^{\,2}(1-m_x)+1}{36m_x(1-m_x)+2} \quad and \quad \sigma_x^{\,2} = \frac{1}{36}$$

Herrerias et al. Time Estimates

Herrerias et al. [7] developed an alternative for the PERT variance addressing one of the limitations of PERT assumption i.e. constant variance, while retaining the original PERT mean expression. He introduced PERT variance adjustment parameter $C(\delta)$, defining

$$C(\delta) = \frac{5}{7} + \frac{16}{7}\delta(1-\delta) \in \left[\frac{5}{7}, \frac{9}{7}\right]$$

where δ is the is the relative distance of the elicited most likely value *m* to the lower bound *a*, i.e.,

$$\delta = \frac{m-a}{b-a}, \quad \delta \in [0,1]$$

They showed that,

$$\sigma^2 = C(\delta) \times \frac{(b-a)^2}{36} \qquad \text{and} \qquad \mu = \frac{a+4m+b}{6}$$

PROPOSED PERT APPROXIMATIONS

It can be seen that the Traditional PERT, Farnum et al.,[3], Ginzberg[6] approximations are based on various assumptions on beta parameters. Therefore, new estimates of mean and the variance are derived without imposing any restrictions on the values of beta parameters α and β, and assuming that, the activity times are belonging to the subfamilies of constant variance.

Estimating Mean and Variance of Activity Times

To obtain the activity mean and variance, it is assumed that

$p+q=z$ (a constant) ,where $p=\alpha-1, \; q=\beta-1$

On the basis of PERT assumption, that the standard deviation σ_x is not much affected by p,q. Hence it is assumed that,

$$\sigma_x^{\,2}(p+1,q+1) \cong \frac{1}{36} \qquad \text{Where} \quad \sigma_x^{\,2}(p+1,q+1) = \frac{(p+2)(q+2)}{(p+q+4)^2(p+q+5)}$$

Since the average value $\sigma_x^{\,2}(m_x)$ *for* $0<m_x<1$ has to be equal to $\frac{1}{36}$, i.e.

$$\int_0^1 \sigma_x^{\,2}(m_x)dm_x = \frac{1}{36}$$

and simplifying, we get

$$z^3 + 7z^2 - 16z - 64 = 0$$

Solving the cubic equation for *z*, we obtain $z \approx 3.4$ and the mean and variance of standardized beta distribution as,

$$\mu_x = \frac{17m_x+5}{27}, \quad \sigma_x^{\,2} = \frac{((3.4)m_x+2)(3.4-(3.4)m_x+2)}{(7.4)^2(8.4)}$$

The mean and variance of generalized beta distribution is,

$$\mu = \frac{5a + 17m + 5b}{27}$$

$$\sigma^2 = \frac{\left((3.4)m - (5.4)a + 2b\right)\left((5.4)b - 2a - (3.4)m\right)}{459.984}$$

$$\approx \frac{\left(17m - 27a + 10b\right)\left(27b - 10a - 17m\right)}{2300}$$

Validation of New Time Estimates

To validate the new times estimates theoretically; their performance is studied against mode $M, 0 < M < 1$ and compared with the estimates of beta distribution namely, Traditional, Ginzburg[6], Farnum and Stanton[3], Premchandra, and Herrerias et al[11].

Comparison of Means

Garcia et al. [5] pointed out that, an approximation of mean is more moderate when its estimated value is closer to 0.5. In order to validate the new estimate of mean and to determine, what of these time estimates is more moderate on average, the mean values, using different estimates corresponding to $M = \frac{m-a}{b-a}$, $0 < M < 1$, are computed and presented in Table 1. The graph of mean values corresponding to mode *M*, $0 < M < 1$ is shown in Fig.1

Table 1: The Mean values of different estimates for $0 < M < 1$

M	**Traditional**	**Ginzberg**	**Farnum et al.**	**Premchandra**	**Herrerias et al.**	**New time estimates**
0.05	0.2	0.19	0.09	0.09	0.2	0.22
0.1	0.23	0.22	0.17	0.17	0.23	0.25
0.15	0.26	0.26	0.27	0.26	0.27	0.28
0.2	0.3	0.29	0.3	0.28	0.3	0.31
0.25	0.33	0.33	0.33	0.31	0.33	0.34
0.3	0.36	0.36	0.37	0.34	0.37	0.37
0.35	0.4	0.39	0.4	0.38	0.4	0.40
0.4	0.43	0.43	0.43	0.42	0.43	0.44
0.45	0.47	0.46	0.47	0.46	0.47	0.47
0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.55	0.53	0.53	0.53	0.54	0.53	0.53
0.6	0.57	0.57	0.57	0.58	0.57	0.56
0.65	0.6	0.60	0.6	0.62	0.6	0.59
0.7	0.63	0.64	0.63	0.66	0.63	0.62
0.75	0.67	0.67	0.67	0.69	0.67	0.66
0.8	0.7	0.71	0.7	0.72	0.7	0.69
0.85	0.73	0.74	0.73	0.74	0.73	0.72
0.9	0.77	0.78	0.83	0.83	0.77	0.75
0.95	0.8	0.81	0.91	0.91	0.8	0.78

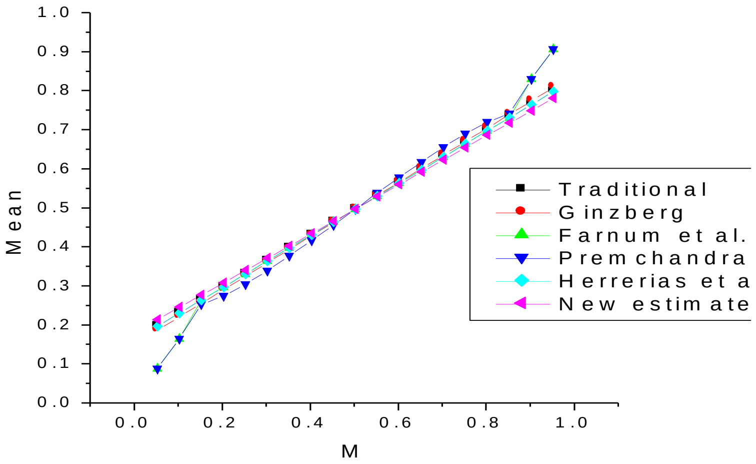

Fig 1: Comparison of means

From Fig. 1, it is observed that, the estimated mean is more moderate in mean throughout the interval $0 < M < 1$ compared to the other mean estimates.

Comparison of Variances

The variance of activity duration is an index of the probability of being able to carry out the activity in the predicted time. Garcia et al. [5] indicated that, the estimates with maximum variance are preferred in the view of PERT, i.e., it is better to approximately guess right rather than to making a mistake by reducing the variance. In this way, an estimate of variance is said to be more conservative when its estimated variance is greater. In order to validate the new estimate of variance and to determine, what of these time estimates are more conservative, the variances using different estimates corresponding to M, $0 < M < 1$, are computed and presented in Table 2. The graphical representation of variances corresponding to mode *M*, is shown in Fig. 2.

From Fig. 2, it is observed that, the new estimate of variance is more conservative for all *M*, $0 < M < 1$ compared to the other estimates. Hence it can be stated that, the alternative proposed is more moderate in mean and more conservative in variance throughout the interval ($0 < M < 1$).

Table 2: The variance values of different estimates for $0 < M < 1$

M	Traditional	Ginzberg	Farnum et al.	Premchandra	Herrerias et al.	New time estimates
0.05	0.023	0.020	0.002	0.002	0.019	0.123
0.1	0.025	0.023	0.008	0.009	0.023	0.129
0.15	0.028	0.025	0.028	0.028	0.028	0.133
0.2	0.03	0.027	0.03	0.03	0.032	0.137
0.25	0.032	0.029	0.032	0.032	0.036	0.141
0.3	0.033	0.031	0.033	0.033	0.039	0.144
0.35	0.034	0.032	0.034	0.034	0.042	0.146
0.4	0.035	0.033	0.035	0.035	0.044	0.147
0.45	0.035	0.033	0.035	0.035	0.045	0.148
0.5	0.036	0.033	0.036	0.036	0.046	0.149
0.55	0.035	0.033	0.035	0.035	0.045	0.148
0.6	0.035	0.033	0.035	0.035	0.044	0.147
0.65	0.034	0.032	0.034	0.034	0.042	0.146
0.7	0.033	0.031	0.033	0.033	0.039	0.144
0.75	0.032	0.029	0.031	0.031	0.036	0.141
0.8	0.03	0.027	0.03	0.03	0.032	0.137
0.85	0.028	0.025	0.028	0.028	0.028	0.133
0.9	0.025	0.023	0.008	0.009	0.023	0.129
0.95	0.023	0.020	0.002	0.002	0.019	0.123

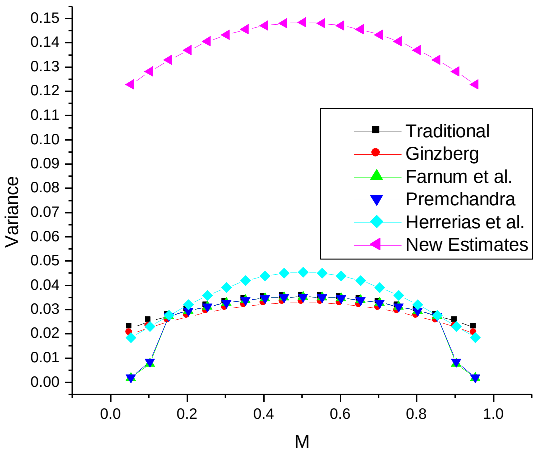

Fig. 2: Comparison of Variances

CONCLUSION

In this paper better estimates of mean and variance of activity duration are derived. It is shown that the mean is more moderate and variance is more conservative compared to other estimates of beta distribution. It is observed that, the new time estimates overcomes the problem of optimistic planning.

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