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Research Article

Sobol' sensitivity indices of the structural parameters of symmetry arch under radial limit loads

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Abstract: Sensitivity analysis allows us to investigate the role, the contribution of the input factors to the objective output function. The results of the sensitivity analysis help the designer adjust the reliability of the structures In this research, we investigate the role and the contribution of the input structural parameters to the critical radial load of the symmetry span steel arch, section I. The Sobol' sensitivity analysis and Monte Carlo simulation method have been used to analysis the entire space of input factors. The characteristic of the structural parameters, cross section factors are considered random variables.
Keywords: Sensitivity, Steel, Stability, Imperfections, Uncertainty, Statistic, Simulation, Random, Variance, Sobol' Indices, Monte Carlo, Symmetry arch

INTRODUCTION

The problem of determining the uniform distributed radial limit loads on the symmetry arch is mentioned in [**Error! Reference source not found.**, **Error! Reference source not found.**]. The limit load depends on various factors, including structural, materials, geometry, etc. The investigation of the influence of these parameters to the limit loads is very important in case of fragment structures under heavy loads.

The sensitivity indices of the structural parameters is analyzed by determining the role and the contribution of the input factors Xi in the output model $Y = f\left(X_i\right)$ in order to investigate the influence of these parameters. There are different methods for analyzing the sensitivity indices, including screening method [**Error! Reference source not found.**], local sensitivity analysis and gradient method [**Error! Reference source not found.**, **Error! Reference source not found.**, **Error! Reference source not found.**]. A method for analyzing the sensitivity indices was proposed in 1993 by I.M. Sobol, a Russian mathematician [**Error! Reference source not found.**] which use variance-based sensitivity analysis. This method can be used to investigate the influence of various structural parameters and evaluate the correlation between the impact factors to the objective function $Y = f\left(X_i\right)$.

In this paper, the Sobol' sensitivity indices is used in combination with the Monte Carlo simulation to investigate the influence of different input factors to the uniform distributed critical load on the large-span symmetry steel arch. A calculation will be considered in case of the random standard-distributed input factors.

Stability of arches under uniform-distributed radial symmetry load

Considering an arch with the initial parameters shown Fig.1a. Under an uniform-distributed radial pressure *q*, there are only vertical force *q.r* [**Error! Reference source not found.**,**Error! Reference source not found.**]. In equilibrium state, the bending moments is calculated as followed:

$$M = A + B\sin\varphi + C\cos\varphi + qrw, \tag{2.1}$$

Where

M_0, T_0, N_0 is initial torque, shear force and axial force respectively

$$A = M_0 + N_0 r; \quad B = T_0 r - qru_0; \quad C = -\left(N_0 r + qrw_0\right)$$

Radial displacement *w* at the coordinates angle φ [**Error! Reference source not found.**, **Error! Reference source not**

found.].

$$w = D_1 \sin k\varphi + D_2 \cos k\varphi - \frac{Ar^2}{k^2 EI} + \frac{Br^2 \sin\varphi}{\left(1-k^2\right)EI} + \frac{Cr^2 \cos\varphi}{\left(1-k^2\right)EI} \quad (2.2)$$

in which D_1 and D_2 are the integral constants. And $k = \frac{qr^3}{EI} + 1$

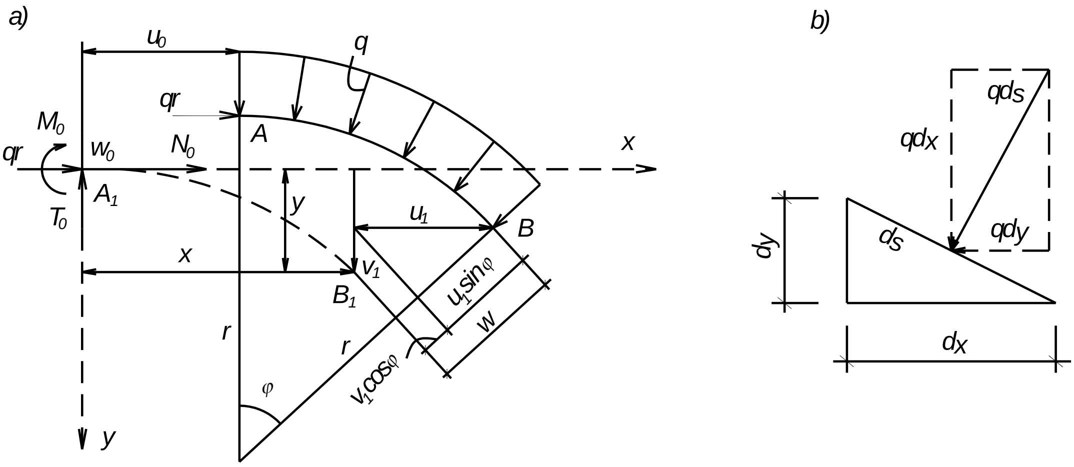

Fig.1 - Circular arc AB is instable under radial pressure

It has been shown that the minimum critical load corresponding with the asymmetric deformation [**Error! Reference source not found.**]. Therefore it is not necessary to consider the symmetry instability. When the system is unstable under asymmetric deformation, at the top of the arch we have the following conditions: $M_0 = 0;\ N_0 = 0;\ w_0 = 0.$

The bending moment (2.1) and the radial displacement (2.2) will be calculates as:

$$M = B\sin\varphi + qrw \quad (2.3)$$

and

$$w = D_1 \sin k\varphi + D_2 \cos k\varphi + \frac{Br^2 \sin\varphi}{\left(1-k^2\right)EI}, \quad (2.4)$$

Boundary conditions:

When $\varphi = 0$ (at the top of the arch), we have $w = 0$

From (2.4) we have $D_2 = 0$

When $\varphi = \alpha$ in which α is an input parameters, from (2.3) and (2.4) we have $M = 0$ and $w = 0$

From (2.3) we have $B\sin\alpha = 0$, so $B = 0$.

From (2.4) we have $D_1 \sin k\alpha = 0$. The arch is instability in case $D_1 \neq 0$ and $\sin k\alpha = 0.$

The minimum critical load in this case corresponds to the following condition: $k\alpha = \pi$

So replace $k\alpha = \pi$ in the calculation equation of*k*: $k = \frac{qr^3}{EI} + 1$, we have $k = \frac{qr^3}{EI} + 1 = \frac{\pi^2}{\alpha^2}.$

So the critical value of uniform-distributed radial load will be calculated as followed:

$$q_{cr} = \frac{EI}{r^3}\left(\frac{\pi^2}{\alpha^2} - 1\right) \quad (2.5)$$

SOBOL' SENSITIVITY INDICES

Definitions

Sensitivity analysis on the entire space of the input factors is to analyze the changes of the output model $Y = f(X)$ due to the influence of the input factors X_i. These analyses identify the components of output variances due to different input values. One of the most carefully formulated and coherent concept which can be applied to analysis the sensitivity indices. It was first proposed in 1993 with [**Error! Reference source not found.**].

Sobol' sensitivity indices

The indices S_i and S_{Ti} are called the first sensitivity and the total sensitivity indices of the input factor X_i. This concept was proposed by I.M Sobol and so called Sobol' sensitivity indices, determined by decomposing the output model $Y = f$ (X). Considering the input factor X_i is independent and defined on the interval [0,1] and the function $Y = f$ (X) is integrable on the interval $[0,1]^n$. f(X) can be performed as following:

$$f(x) = f_0 + \sum_i f_i(x_i) + \sum_{i<j} f_{ij}(x_i, x_j) + \ldots + f_{1\ldots n}(x_1,...,x_n), \tag{3.1}$$

where f_o is constant and f_i are defined by:

$$\int_0^1 f_{i_1\ldots i_s}(x_{i_1},...,x_{i_s})dx_k = 0 \text{ with } k = i_1,...,i_s \tag{3.2}$$

And

$$\int_0^1 f_{i_1\ldots i_s}(x_{i_1},...,x_{i_s}) f_{j_1\ldots j_t}(x_{j_1},...,x_{j_t})dx = 0 \tag{3.3}$$

$\forall k \in \{1,...,n\}$ and $\{i_1,...,i_s\} \subseteq \{1,...,n\}$. The integral of a component of the decomposition over one of its variable x_{i_k} is zero and two components are orthogonal if at least one variable is not shared. According to equation (3.2) and (3.3) we have:

$$f_0 = \int f(x)dx \tag{3.4}$$

$$f_i(x_i) = \int f(x)\prod_{k\neq i} dx_k - f_0, \tag{3.5}$$

$$f_{ij}(x_i, x_j) = \int f(x)\prod_{k\neq i,j} dx_k - f_0 - f_i(x_i) - f_j(x_j), \tag{3.6}$$

$$f_{1,...,n}(x_1,...,x_n) = f(x) - f_0 - \sum_{i=1}^{n} f_i(x_i) - ... - \sum_{1\le i_1<...<i_{n-1}\le n} f_{i_1,...,i_{n-1}}(x_{i_1},...,x_{i_{n-1}}) \tag{3.7}$$

Where the last component $f_{1,...,n}(x_1,...,x_n)$ verifies the decompositionin Eq.(3.1). Therefore the variance of Y can be decompressed according the following theorem.

Theorem. The variance of the output function described in (3.1) can be performed as:

$$Var[Y] = \sum_{i=1}^{n} V_i + \sum_{1\le i<j\le n}^{n} V_{ij} + ... + V_{1\ldots n} \tag{3.8}$$

in which

$$V_i = \mathrm{Var}\left[E\left[Y|X_i\right]\right]$$

$$V_{ij} = Var\left[E|Y|X_i X_j\right] - V_i - V_j$$

$$V_{1\ldots n} = Var[Y] - \sum_{i=1}^{n} V_i - \sum_{1\le i<j\le n}^{n} V_{ij} - \sum_{1\le i_1<...<i_{n-1}\le n}^{n} V_{i_1\ldots i_{n-1}}$$

The components of the variance decomposition of Y are the variances of the components of the decomposition of $Y = f(X)$

$$V_{i_1\ldots i_s} = Var\left[f_{i_1\ldots i_s}(x_{i_1},...,x_{i_s})\right] \qquad \{i_1,...,i_s\} \in \{1,...,n\}$$

Sobol' sensitivity indices of the model $Y = f(X)$ are defined as following:

The first-order sensitivity indices

$$S_i = \frac{V_i}{\mathrm{Var}[Y]} \tag{3.9}$$

The second-order sensitivity indices

$$S_{ij} = \frac{V_{ij}}{Var\left[Y\right]} \tag{3.10}$$

The total sensitivity indices:

$$S_{Ti} = \sum_{k \# i} S_k;\ \text{or}\ S_{Ti} = S_i + S_{ij} + S_{ik} \tag{3.11}$$

Sobol' sensitivity indices determination using Monte Carlo simulation

Sobol' sensitivity indices can be determined by Monte Carlo simulation. Let us consider two samples N of X_i:

$$\tilde{X}_k^i = \left(x_{k1}^i, ..., x_{kp}^i\right)_{k=1,...N, i=1,2} \tag{3.12}$$

Each sample corresponds to a single point of the input factors with p dimensions. From these two N-samples-size realization of X, the order sensitivity indices S_i for the input $X_{(i)}$ is estimated in the following way [**Error! Reference source not found.**, **Error! Reference source not found.**]

$$\hat{S}_i = \frac{\hat{V}_i}{\hat{V}} = \frac{\hat{U}_i - \hat{f}_0^2}{\hat{V}} \tag{3.13}$$

Where the mean is estimated with

$$\hat{f}_0 = \frac{1}{N} \sum_{k=1}^{N} f\left(x_{k1}^1, ..., x_{kp}^1\right) \tag{3.14}$$

The variance is estimated by definition with

$$\hat{V} = \frac{1}{N} \sum_{k=1}^{N} f^2\left(x_{k1}^1, ..., x_{kp}^1\right) - \hat{f}_0^2 \tag{3.15}$$

And finally, the term $\hat{U}_i$ is obtained by using the two N-sample-size realizations of X with [**Error! Reference source not found.**, **Error! Reference source not found.**]

$$\hat{U}_i = \frac{1}{N} \sum_{k=1}^{N} f\left(x_{k1}^1, ..., x_{k(i-1)}^1, x_{ki}^1, x_{k(i+1)}^1, ..., x_{kp}^1\right) \times f\left(x_{k1}^2, ..., x_{k(i-1)}^2, x_{ki}^2, x_{k(i+1)}^2, ..., x_{kp}^2\right). \tag{3.16}$$

The estimation of $\hat{U}_i$ allows us to evaluate the influence of the samples of the dimension i. The second-order sensitivity indices are estimated in the following manner [**Error! Reference source not found.**, **Error! Reference source not found.**]:

$$\hat{S}_{ij} = \frac{\hat{U}_{ij} - \hat{f}_0^2 - \hat{V}_i - \hat{V}_j}{\hat{V}} \tag{3.17}$$

In which

$$\begin{aligned} \hat{U}_{\text{ij}} = \frac{1}{N} \sum_{i=1}^{N} & f\left(x_{k1}^1, ..., x_{k(i-1)}^1, x_{k1}^1, x_{k(i+1)}^1, ..., x_{k(j-1)}^1, x_{kj}^1, x_{k(j+1)}^1, ..., x_{kp}^1\right) \\ & \times f\left(x_{k1}^2, ..., x_{k(i-1)}^2, x_{k1}^2, x_{k(i+1)}^2, ..., x_{k(j-1)}^2, x_{kj}^2, x_{k(j+1)}^2, ..., x_{kp}^2\right) \end{aligned} \tag{3.18}$$

Total sensitivity indices is estimated according to [**Error! Reference source not found.**]

$$\hat{S}_{Ti} = 1 - \frac{\hat{U}_i - \hat{f}_0^2}{\hat{V}} \tag{3.19}$$

With

$$\hat{U}_{\tilde{i}} = \frac{1}{N} \sum_{i=1}^{N} f\left(x_{k1}^1, ..., x_{k(i-1)}^1, x_{k1}^1, x_{k(i+1)}^1, ..., x_{kp}^1\right) \times f\left(x_{k1}^1, ..., x_{k(i-1)}^1, x_{k1}^2, x_{k(i+1)}^2, ..., x_{kp}^1\right). \tag{3.20}$$

SENSITIVITY ANALYSIS OF THE CRITICAL UNIFORM-DISTRIBUTED RADIAL LOAD

Problem inputs

In this section we applied the Sobol' sensitivity indices investigating method of the critical uniform-distributed radial load on large-span steel-arch, represented in Figure 2.

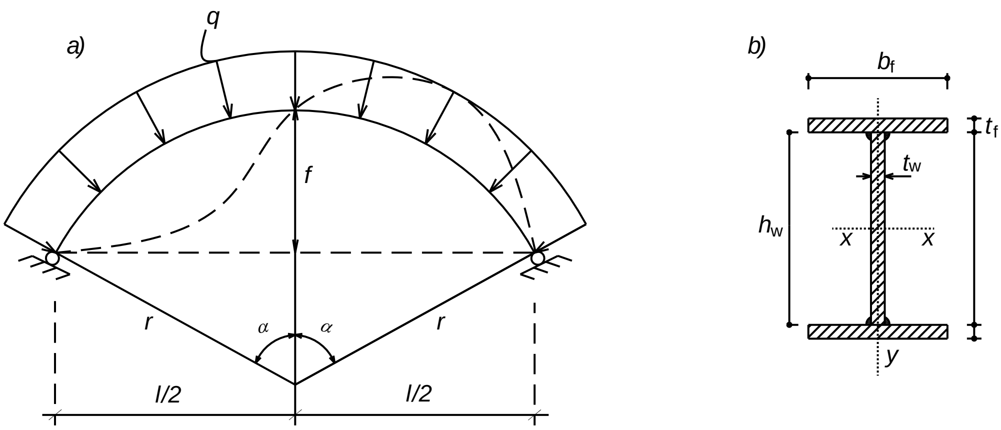

Fig.2 - Instability of two-joint arches and cross-section parameters

The critical load was determined by the equation (2.5)

$$q_{cr} = \frac{EI}{r^3}\left(\frac{\pi^2}{\alpha^2} - 1\right) \quad (4.1)$$

In which

E elasticity modulus of steel

r the radius of arch

I_x inertia moment. I_x can be determined by the following equation:

$$I_x = \frac{b_f\left(h_w + t_f\right)^3}{12} + 2\left[\frac{0,5\left(b_f - t_w\right)h_w^3}{12}\right] \quad (4.2)$$

Design parameters, including $E, \alpha, h_w, t_w, b_f, t_f$ are random factors and described in the following table:

Table 1 - Random input factors

No	Symbol	Distribution	Mean value	Standard deviation
1	h_w	Histogram	0,25 m	0,00125 m
2	t_w	Histogram	0,008 m	0,00004 m
3	b_f	Histogram	0,15 m	0,00075 m
4	t_f	Histogram	0,012 m	0,00006 m
5	α	Gauss	60^0	$0,12^0$
6	E	Gauss	2,1E8 kN.m^2	0,126E8 kN.m^2

RESULT DISCUSSION

The analysis of the Sobol’ first-order sensitivity indices is represented in Fig.3. The results were calculated using Python application [**Error! Reference source not found.**]. The simulation calculations were performed with 100.000 runs.

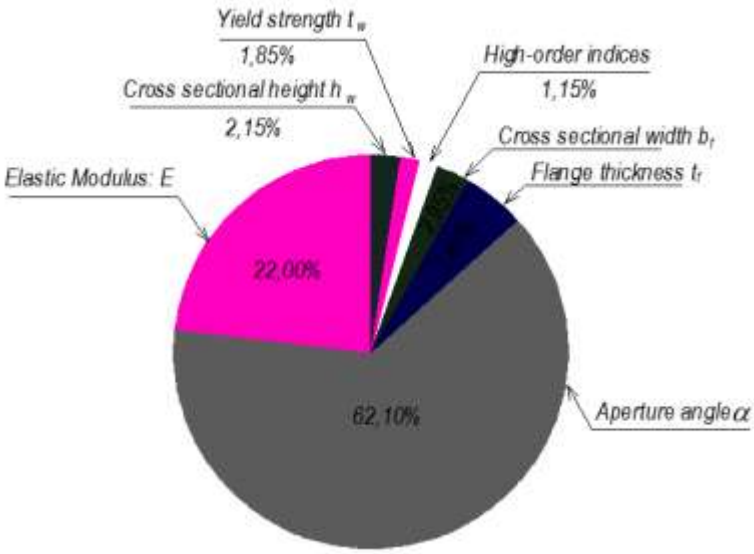

Fig. 3 - Sobol's first-order sensitivity indices for q_{rc}

The random uncorrelated input factors are prerequisites to using the Sobol' sensitivity analysis method.

The statistical characteristic of the variables h_w, t_w, b_f, t_f which has Histogram distribution, are experimental results.

The statistical characteristic of the elastic modulus *E*, α is determined by the standard deviation of Gaussian distribution. It is considered on the assumption of 95% of random factors.

From Figure 3, the sensitivity indices of first-order and high-order indices structural parameters are 100%. Meanwhile the total of the first-order sensitivity indices is $0.9815 < \sum S_i < 1.000$. It proved that the first-order sensitivity indices occupied a major component. The remaining high-order sensitivity indices was only about 1.85%.

The elastic modulus E and the aperture angle α has highest sensitivity indices, 22% and 62.1%, respectively. The influence of the cross-sectional parameters, including h_w, t_w, b_f, t_f, is insignificant.

CONCLUSION

In this paper the authors introduce a method to determine the sensitivity indices of the random independent input parameters by analysis output variance. Sobol' sensitivity indices and Monte Carlo simulations have been used. At present, the Sobol' sensitivity analysis is one of the most carefully formulated and most coherent concepts which can be applied to the analysis.

In the application we analyze the sensitivity of the critical radial loads acting on the steel arch, with I section, which is widely used in huge structures, including large-span building, stadium, etc. This type of structure is widely used for the large span structures, roof stadium ... Results in Fig.3 is reliable and can be used to adjust the structure, depending on specific circumstances, *i.e.* structure adjustment based on reliability or structure adjustment based on economic constraints.

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