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Research Article

All travelling wave solutions to the regularized short pulse equation

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Abstract: The complete discrimination system for polynomial method is applied to the RSPE, and we have obtained all of its possible exact traveling wave solutions including rational function type solutions, solitary wave solutions, triangle function type periodic solutions and Jacobian elliptic functions double periodic solutions, some of them are new solutions.
Keywords: traveling wave solution; complete discrimination system for polynomial method; the regularized short pulse equation

INTRODUCTION

In this paper, we consider the RSPE [1]

$$u_{xt} + \alpha u + (u^3)xx + \beta u_{xxxx} = 0, \tag{1}$$

where α and β are real constants. This system describes the nonlinear Maxwell equations with high-frequency dispersion. We apply the complete discrimination system for polynomial method to Eq.(1) and give the classification of its all single traveling wave solutions. The complete discrimination system for polynomial method was proposed by Liu [2-4] in the past several years. By Liu's method, many nonlinear partial differential equations [5-7] have been solved.

CLASSIFICATION

Taking the following transformation to the Eq.(1)

$$\mathrm{u} = \mathrm{u}(\xi),\ \xi = x - wt, \tag{2}$$

then we can get the following equations

$$-\mathrm{w}u'' + \alpha u + 3u^2u'' + \beta u'''' = 0. \tag{3}$$

By taking

$$(u')^2 = b_4u^4 + b_3u^3 + b_2u^2 + b_1u + b_0, \tag{4}$$

we have

$$(12b_4 + 24\beta {b_4}^2)u^5 + \left(\frac{21b_3}{2} + 30\beta b_3b_4\right)u^4 + \left(-20wb_4 + 9b_2 + 20b_2b_4\beta + \frac{15\beta b_3^2}{2}\right)u^3 + \left(-\frac{3wb_3}{2} + \frac{13b_1}{2} + 15\beta b_1b_4 + \frac{15b_2b_3\beta}{2}\right)u^2 + \left(-wb_2 + \alpha + 6b_0 + 12b_0\beta + \frac{9\beta b_2b_3\beta}{2} + b_2^2\beta\right)u - \frac{wb_1}{2} + 3b_0b_3 + \frac{b_1b_2\beta}{2} = 0. \tag{5}$$

Setting all coefficients of this polynomial to zero, we get a system of algebraic equations

$$\begin{cases} 12b_4 + 24\beta {b_4}^2 = 0, \\ \frac{21b_3}{2} + 30\beta b_3b_4 = 0, \\ -20wb_4 + 9b_2 + 20b_2b_4\beta + \frac{15\beta b_3^2}{2} = 0, \\ -\frac{3wb_3}{2} + \frac{13b_1}{2} + 15\beta b_1b_4 + \frac{15b_2b_3\beta}{2} = 0, \\ -wb_2 + \alpha + 6b_0 + 12b_0\beta + \frac{9\beta b_2b_3\beta}{2} + b_2^2\beta = 0, \\ -wb_2 + \alpha + 6b_0 + 12b_0\beta + \frac{9\beta b_2b_3\beta}{2} + b_2^2\beta = 0, \\ -\frac{wb_1}{2} + 3b_0b_3 + \frac{b_1b_2\beta}{2} = 0. \end{cases} \tag{6}$$

Solving this algebraic equation system, we obtain a family of value of parameters

$$b_4 = -\frac{1}{2\beta}, b_3 = 0, b_4 = \frac{w}{2\beta}, b_1 = 0, b_0 = -\frac{\alpha}{12\beta+6}, \qquad (7)$$

or

$$b_4 = b_3 = b_2 = b_1 = 0, b_0 = -\frac{\alpha}{12\beta+6}. \qquad (8)$$

Then we obtain

$$(u')^2 = -\frac{\alpha}{12\beta+6}, \qquad (9)$$

or

$$(u')^2 = -\frac{1}{2\beta}u^4 + \frac{w}{2\beta}u^2 - \frac{\alpha}{12\beta+6}. \qquad (10)$$

From Eq.(9), we can easily get

$$u = \sqrt{-\frac{\alpha}{12\beta+6}}(x - wt) + c, \qquad (11)$$

where c is a constant.
In order to solve Eq.(10), when $\beta < 0$, we take the transformation as follows

$$\omega = (-\frac{1}{2\beta})^{1/4}u, \xi_1 = (-\frac{1}{2\beta})^{1/4}\xi. \qquad (12)$$

Combining the expression (12) with Eq.(10) yield

$$\omega_{\xi_1} = F(\omega) = \omega^4 + p\omega^2 + q, \qquad (13)$$

where$p = \frac{w\sqrt{-2\beta}}{\beta}$, $q = -\frac{\alpha}{12\beta+6}$.
And if > 0, then we take the following transformation

$$\omega = (\frac{1}{2\beta})^{1/4}u, \xi_1 = (\frac{1}{2\beta})^{1/4}\xi, \qquad (14)$$

and we get

$$\omega_{\xi_1} = \pm(\xi_1 - \xi_0) = \int \frac{d\omega}{\sqrt{\varepsilon F(\omega)}} = \pm(\omega^4 + p\omega^2 + q), \qquad (15)$$

Then Eq.(9) and Eq.(10) become

$$\pm(\xi_1 - \xi_0) = \int \frac{d\omega}{\sqrt{\varepsilon F(\omega)}}, \qquad (16)$$

Write the discrimination of $F(w)$ as follows

$$\begin{cases} D_1 = 4 \\ D_2 = -p \\ D_3 = -2p^3 + 8pq \\ D_4 = 4p^4 - 32p^2q^2 \\ E_2 = -32pq. \end{cases} \qquad (17)$$

According to the discrimination , we give the corresponding traveling wave solutions to Eq.(15).
Case 1: $D_4 = D_3 = 0, D_2 < 0$, we have $F(\omega) = ((\omega - l)^2 + s^2)^2$, where *l* and *s* are real numbers. The corresponding traveling wave solutions of Eq.(15) is

$$\omega = \operatorname{stan} s(\xi_1 - \xi_0) = l. \qquad (18)$$

Case 2: $D_2 = D_3 = D_4 = 0$. $F(\omega) = \omega^4$, the corresponding traveling wave solutions of Eq.(15) is

$$\omega = -(\xi_1 - \xi_0)^{-1} \qquad (19)$$

Case 3: $D_4 = D_3 = 0, D_2 > 0, E_2 > 0$. We assume$F(\omega) = (\omega - \mu)^2(\omega - \nu)^2$, where$\mu$, $\nu$are real numbers,$\mu > \nu$.If$\omega > \mu$ or$\omega < \nu$, the corresponding traveling wave solutions of Eq.(15) is

$$\omega = \frac{\nu-\mu}{2}\left[coth\frac{(\mu-\nu)(\xi_1-\xi_0)}{2} - 1\right] + \nu \qquad (20)$$

when$\nu < \omega < \mu$, the corresponding traveling wave solutions of Eq.(12) is

$$\omega = \frac{\nu-\mu}{2}\left[tanh\frac{(\mu-\nu)(\xi_1-\xi_0)}{2} - 1\right] + \nu \qquad (21)$$

Case 4: $D_4 = 0, D_3 > 0, D_2 > 0$ and $E_2 = 0$. $F(\omega)$ has a real root of multiplicity three and a real root of multiplicity one. For the reason of when $E_2 = 0$ and $D_3 = 0$ occur at the same time, we have *p=0*, and it contradicts to $D_2 > 0$. So this condition does not exist in the present paper.
Case 5: $D_4 = 0$, $D_3D_2 < 0$.$F(\omega)$ has a real root of multiplicity two and a pair of conjugate complex roots. For the reason of $p^2D_4 = qD_3^2$ and $D_3 \neq 0$, we have *q*=0, but then we have $D_3D_2 = 2p^4 \geq 0$, so this condition does not exist either.
Case 6: $D_2 > 0$, $D_3 > 0, D_4 > 0$, we have

$$F(\omega) = (\omega - \alpha_1)(\omega - \alpha_2)(\omega - \alpha_3)(\omega - \alpha_4) \qquad (22)$$

where$\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$. When $\alpha_4 > 0$, if $\omega > \alpha_1$ or $\omega < \alpha_4$, the corresponding traveling wave solutions of Eq.(15) are

$$\omega = \frac{\alpha_2(\alpha_1-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-\alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-(\alpha_2-\alpha_4)}, \quad (23)$$

and if $\alpha_2 > \omega > \alpha_3$, then we get

$$\omega = \frac{\alpha_4(\alpha_2-\alpha_3)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-\alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-(\alpha_2-\alpha_4)}, \quad (24)$$

where, $m^2 = \frac{(\alpha_1-\alpha_4)(\alpha_2-\alpha_3)}{(\alpha_2-\alpha_4)(\alpha_1-\alpha_3)}$.
For $\alpha_4 < 0$, if $\alpha_1 > \omega > \alpha_2$, similarly we have the following solutions of Eq.(15)

$$\omega = \frac{\alpha_3(\alpha_1-\alpha_2)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-\alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-(\alpha_1-\alpha_3)}, \quad (25)$$

and when $\alpha_3 > \omega > \alpha_4$, we have

$$\omega = \frac{\alpha_1(\alpha_3-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-\alpha_4(\alpha_3-\alpha_1)}{(\alpha_3-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi_1-\xi_0),m\right)-(\alpha_3-\alpha_1)}, \quad (26)$$

where, $m^2 = \frac{(\alpha_1-\alpha_2)(\alpha_3-\alpha_4)}{(\alpha_1-\alpha_4)(\alpha_2-\alpha_4)}$. The expressions of (23)-(26) are elliptic functions double periodic solutions.
Case 7: $D_4 < 0,\ D_3D_2 \geq 0$. $F(\omega)$has two distinct real roots and a pair of conjugate complex roots, i.e.

$$F(\omega) = (\omega-\mu)(\omega-\nu)((\omega-l)^2+s^2), \quad (27)$$

Where$\mu > \nu$ and *s>0*. Letting

$$\begin{cases} a = \frac{1}{2}(\mu+\nu)c - \frac{1}{2}(\mu-\nu)d \\ b = \frac{1}{2}(\mu+\nu)d - \frac{1}{2}(\mu-\nu)c \\ c = \mu - l - \frac{s}{m_1} \\ d = \mu - l - sm_1 \\ E = \frac{s^2+(\mu-l)(\nu-l)}{s(\mu-\nu)} \\ m_1 = E \pm \sqrt{E^2+1} \end{cases} \quad (28)$$

Then we can get the solution to Eq.(15)

$$\omega = \frac{a\ cn\left(\frac{\sqrt{\mp 2sm_1(\mu-\nu)}}{2mm_1}(\xi_1-\xi_0),m\right)+b}{c\ cn\left(\frac{\sqrt{\mp 2sm_1(\mu-\nu)}}{2mm_1}(\xi_1-\xi_0),m\right)+d}, \quad (29)$$

where $m^2 = \frac{1}{1+{m_1}^2}$.

Case 8 : $D_4 > 0,\ D_3D_2 \leq 0$. $F(\omega)$has two pair of conjugate complex root, i.e.

$$F(\omega) = ((\omega-l_1)^2+{s_1}^2)\ ((\omega-l_2)^2+{s_2}^2), \quad (30)$$

where $s_1 > s_2 > 0$. Letting

$$\begin{cases} a = l_1c + s_1d \\ b = l_1d - s_1c \\ c = -s_1 - \frac{s_2}{m_1} \\ d = l_1 - l_2 \\ E = \frac{(l_1-l_2)^2+{s_1}^2+{s_2}^2}{2s_1s_2} \\ m_1 = E + \sqrt{E^2+1} \end{cases} \quad (31)$$

Then we can get the solution of Eq.(15)

$$\omega = \frac{a\ sn(\eta(\xi_1-\xi_0),m)+b\ cn(\eta(\xi_1-\xi_0),m)}{c\ sn(\eta(\xi_1-\xi_0),m)+d\ cn(\eta(\xi_1-\xi_0),m)}, \quad (32)$$

where$m^2 = \frac{m_1^2-1}{m_1^2}$ and $\eta = \frac{s_2\sqrt{(c^2+d^2)(m_1^2c^2+d^2)}}{c^2+d^2}$.

Case 2.9:$D_2 > 0, D_3 > 0\ and\ D_4 = 0$. We assume$F(\omega) = (\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)^2$, where $\alpha_1 > \alpha_2$ and $\alpha_1 \neq \alpha_3, \alpha_2 \neq \alpha_3$. Letting

$$c = \frac{\alpha_1 - \alpha_2}{2}\left(\frac{\alpha_1 + \alpha_2}{2} - \alpha_3\right) \quad (33)$$

For $c^2 - 1 > 0$,the corresponding traveling wave solutions of Eq.(15) is

$$\xi_1 - \xi_0 = -\frac{1}{\sqrt{c^2-1}} \ln\left|\frac{y - c_1}{y + c_1}\right|, \quad (31)$$

when $c^2 - 1 < 0$, by using Eq.(15), we have

$$\xi_1 - \xi_0 = -\sqrt{1 - c^2}\, acrtan \frac{c+1}{1-c} y, \quad (32)$$

where$c_1 = \sqrt{\frac{c+1}{c-1}}$ and $y = \sqrt{1 - \frac{\alpha_1 - \alpha_2}{(\omega - \frac{\alpha_1 + \alpha_2}{2} + \frac{\alpha_1 - \alpha_2}{2})}}$.

CONCLUSION

In the present paper, we use the complete discrimination system for polynomial method to the RSPE, and we obtain the classification of traveling wave solutions. The results show that trial equation method is powerful for solving nonlinear problems.

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