

## **Research Article**

# **An Analysis of Boundary Layer Flow over a Flat Plate Using Modified Adomian Decomposition Method**

**Adewoye Raphael Adebisi**

Rufus Giwa Polytechnic, Owoondo State, Nigeria

### **\*Corresponding author**

Adewoye Raphael Adebisi

Email: [sogbonwole@yahoo.com](mailto:sogbonwole@yahoo.com)

---

**Abstract:** In this paper, the boundary layer flow over a flat plate was presented and discussed. The equations governing the boundary layer and the energy equation were equally considered. Modified Adomian decomposition method discussed and used with the Runge-Kutta shooting method to solve the Blasius equation and the Energy equation arising from the equations governing the boundary layer flow. Numerical results were obtained and each of the methods used converges to the true solution as the velocity profile increased to a steady state. The obtained results for the Blasius equation revealed one of the features of boundary layer that the boundary layer increases from the edge of the boundary layer into the free steam. Different values of Prandtl number used for the obtained results for the energy equation revealed that as the Prandtl number used increases, the boundary layer thickness reduces which established the stability function of the Prandtl number on the temperature. Values for the skin friction (the measure of velocity profile at the wall) as well as the Nusselt number (the heat transfer rate at the wall) were calculated from the results obtained for the Blasius equation and the Energy equation. Using the Runge-Kutta shooting method as the standard value, it revealed that Modified Adomian Decomposition Method is a good approximation value to Nusselt number.

**Keywords:** Boundary layer flow, Modified Adomian Decomposition Method, Runge-Kutta shooting method

---

## **INTRODUCTION**

### **Background of the Study**

Boundary layer flows, according to Stern[1], are the external flows around streamlined bodies at high Reynolds numbers that have viscous (shear and no-slip) effects confined close to the body surfaces and its wake, but are nearly inviscid far from the body. Boundary layer flow, according to Burr *et al* [2], is the flow of that portion of a viscous fluid which is in the neighbourhood of a body in contact with the fluid and in motion relative to the fluid. That portion of a fluid flow, near a solid surface, where shear stresses are significant and the inviscid- flow assumption may not be used. Thus a fluid flow is retarded by a fixed solid surface, and is finite, slow-moving boundary flow is formed. A requirement for the boundary layer to be thin is that Reynolds number of the body be large, i.e  $10^3$  or more. Under these conditions the flow outside the boundary layer is essentially inviscid and plays the role of a driving mechanism for the layer. Boundary layer theory is applied in the fields of aerodynamics (airplanes, rockets, projectiles), hydrodynamics (ships, submarines, torpedoes), transportation (automobiles, trucks, cycles), wind engineering (building, bridges, water towers), and ocean engineering (buoys, breakwaters, cables)[1].

Over time, it has been discovered that most scientific problems and physical phenomena occur in the form of differential equations and many of these problems are either linear or nonlinear. Problems involving, especially, heat transfer, boundary layer (to mention a few), are basically nonlinear. Except for a few of these problems, finding the analytical solutions to majority of the problems is always difficult. However, these nonlinear equations can be solved by numerical techniques and other methods of approximation like the Adomian Decomposition Method (ADM) and the Variational Iteration method (VIM). Of recent, increased interest has been geared towards the variational and perturbation methods, where a small parameter is inserted into the equation. One major problem associated with this is how to find the small parameter and inserting it into the equation.

In this paper, Modified Adomian Decomposition Method (MADM) and the Runge-Kuta Shooting method were used in getting the approximate solutions to the Blasius equation and the Energy equation arising from equations governing the boundary layer flow over a flat plate.

## LITERATURE REVIEW

According to Veldman [3], it all started in 1904 at the International Congress in Heidelberg, when Ludwig Prandtl gave a lecture titled “*Über Flüssigkeitsbewegungen bei sehr kleiner Reibung*” (English: “On fluid flow with very little friction”). He explained that the viscosity of a fluid plays a role in a (very) thin layer adjacent to the surface, which he called “*Uebergangsschicht*” or “*Grenzschicht*”. Translated into English, the latter led to the term *boundary layer*. With this lecture, the understanding of fluid flow was significantly increased.

Many physical problems can be described by mathematical models that involve differential equations. Mathematical modelling involves physical observation, selection of the relevant physical variables, formulations of the equations, analysis of the equations, simulation, and finally, validation of the model. The means of mathematical models is an equation, or set of equations, whose solutions describe the physical behaviour of a related physical system. In other words, a mathematical model is a simplified description of physical reality expressed in mathematical terms. The behaviour of each model is governed by the input data for the particular problem: the boundary and initial conditions, the co-efficient functions of the differential equation, and the forcing function. This input data causes the solution of the model problem to possess highly localized properties in space, in time, or in both. Thus, the investigation of the exact or approximate solution helps us to understand the means of these mathematical models. In most cases, it is difficult, or infeasible, to find the analytical solution or good numerical solution of the problems. Numerical solution or approximate analytical solutions become necessary[4].

### The Adomian Decomposition Method

In the 1980's, George Adomian (1923-1996) introduced a new powerful method for solving nonlinear functional equations. Since then, this method has been known as the Adomian decomposition method (ADM) [5-6]. The technique is based on a decomposition of a solution of a linear operator equation in a series of functions. Each term of the series is obtained from the polynomial generated from an expansion of an analytic function into a power series. The Adomian technique is simple in an abstract formulation but the difficulty arises in calculating the polynomials and in proving the convergence of the series of functions[7].

The Adomian Decomposition Method (ADM) allows exact solutions of nonlinear functional equations of various kinds (algebraic, differential, partial differential, integral, and others) without discretizing the equations or approximating the operators. The solution, when it exists, is found in a rapidly converging series form, and time and space are not discretized. The decomposition method yields rapidly convergent series solutions by using a few iterations for both linear and nonlinear deterministic and stochastic equations. The advantage of this method is that it provides a direct scheme for solving the problem, i.e., without the need for linearization, perturbation, massive computation and any transformation.

Convergence of the Adomian method when applied to some classes of ordinary and partial differential equations is discussed by many authors. For example, Abbaoui and Cherruault [8] proved the convergence of the Adomian method for differential and operator equations. Lesnic [9] investigated convergence of the ADM when applied to time-dependent heat, wave and beam equations for both forward and backward time evolution. He showed that the convergence was faster for forward problems than for backward problems. Al-Khaled and Allan [10] implemented the Adomian method for variable-depth shallow water equations with a source term and illustrated the convergence numerically.

## METHODOLOGY

In this session, we described the formulation of the model equations and the Modified Adomian Decomposition Method (MADM).

### Governing Equations

Boundary layer flow over a flat plate is governed by the continuity and the Navier-Stokes equations. Under the boundary layer assumptions and a constant property assumption, the continuity and Navier-Stokes equations become[11]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \quad (2)$$

Under a boundary layer assumption, the energy transport equation is also simplified.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

From Eqs. (2) and (3), the solutions of the energy and momentum equations are coupled[12]; however, the buoyancy force may be neglected if there is a pressure gradient perpendicular to the gravitational force. Thus, in the case

of the forced convection over a horizontal flat plate, the solution to the momentum equation is decoupled from the energy solution. However, the solution of the energy equation is still linked to the momentum solution. Introducing the following dimensionless variables in the transformation:

$$\eta = \frac{y}{\sqrt{x}} Re_x^{0.5}, \tag{4}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where  $\theta$  is non dimensional form of the temperature and the Reynolds number is defined as:

$$Re = \frac{u_\infty x}{\nu}. \tag{6}$$

Using Eqs. (1) through (5), the governing equations are reduced to two equations of third and second order differential equations where  $f$  is a function of the similarity variable ( $\eta$ ):

$$f''' + \frac{1}{2} f f'' = 0, \quad \epsilon \theta'' + f \theta' = 0 \tag{7}$$

Where  $\epsilon = \frac{1}{Pr}$  and  $f$  is related to the  $u$  velocity by

$$f' = \frac{u}{u_\infty}. \tag{8}$$

The reference velocity is the free stream velocity of forced convection. The boundary conditions are obtained from the similarity variables. For the forced convection case [14]:

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0. \tag{9}$$

**The Energy Equation**

Under a boundary layer assumption, the energy transport equation is also simplified as

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{10}$$

$$\begin{aligned} T &= T_w \quad \text{at} \quad y = 0 \\ T &= T_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned}$$

with the following dimensionless variables introduced in the transformation

$$\begin{aligned} T &= T_\infty + (T_w - T_\infty)\theta(\eta) \\ \theta(0) &= 1 \\ \theta(\infty) &= 0 \end{aligned} \tag{11}$$

where  $\theta$  is nondimensional form of the temperature and the Reynolds number is defined as:

$Re = \frac{u_\infty x}{\nu}$ , which is reduced to

$$\frac{1}{Pr} \theta'' + f \theta' = 0$$

**The Modified Adomian Decomposition Method (MADM) for Differential Equations**

To describe this method according to Ebaid and Al-Armani [13] we consider the second order differential equation:

$$u''(t) + p(t)u'(t) + q(t)f(t)u'(t) = 0, \tag{12}$$

Subject to the boundary conditions

$$u(a) = \alpha, \quad u(b) = \beta, \tag{13}$$

Where at least one of the functions  $p(t)$  and  $q(t)$  has a singular point and  $f(t)$  is an unspecified function. We first write (12) as

$$u''(t) = -p(t)u'(t) - q(t)f(t)u'(t). \tag{14}$$

Now, suppose that  $p(t)$  and  $q(t)$  have the same singular point ( $t = t_0$ , say), the following inverse operator was proposed to solve (14) with the boundary conditions in (13):

$$L^{-1}[\cdot] = \int_a^t d t' \int_c^t [\cdot] dt'' - \frac{t-a}{b-a} \int_a^b dt' \int_c^t [\cdot] dt'', \tag{15}$$

$a \neq b, \quad c$  (arbitrary)  $\neq t_0$ .

Operating both sides of (14) with this inverse operator, we have

$$u(t) - u(a) - \frac{t-a}{b-a} [u(b) - u(a)] = -L^{-1}[p(t)u'(t) + q(t)f(t)u'(t)], \tag{16}$$

which can be rewritten as

$$u(t) = \alpha + \frac{t-a}{b-a} (\beta - \alpha) - L^{-1}[p(t)u'(t) + q(t)f(t)u'(t)]. \tag{17}$$

Based on Adomian's method, the solutions  $u(t)$  and  $f(t)$  of the system are assumed in the following form:

$$u(t) = \sum_{n=0}^{\infty} u_n(t), \quad f(t) = \sum_{n=0}^{\infty} f_n(t). \tag{18}$$

Inserting these series into (16), we obtain

$$\sum_{n=0}^{\infty} u_n(t) = \alpha + \frac{t-a}{b-a}(\beta - \alpha) - L^{-1}[p(t) \sum_{n=0}^{\infty} u'_n(t) + q(t) \sum_{n=0}^{\infty} \sum_{i=0}^n f_i(t) u'_{n-i}(t)] \tag{3.42}$$

Substituting  $p(t) = -1/(1-t)$  and  $q(t) = \gamma/(1-t)$  into (18) yields

$$\sum_{n=0}^{\infty} u_n(t) = \alpha + \frac{t-a}{b-a}(\beta - \alpha) + L^{-1} \left[ \left( \frac{1}{1-t} \right) \sum_{n=0}^{\infty} u'_n(t) - \gamma \left( \frac{1}{1-t} \right) \sum_{n=0}^{\infty} \sum_{i=0}^n f_i(t) u'_{n-i}(t) \right] \tag{19}$$

To overcome the difficulty of the singular point, we may replace the function  $1/(1-t)$  with the series form  $\sum_{n=0}^{\infty} t^n$ , where  $t \in [0, 1)$ . thus, we have

$$\sum_{n=0}^{\infty} u_n(t) = \alpha + \frac{t-a}{b-a}(\beta - \alpha) + L^{-1} \left[ \sum_{n=0}^{\infty} \sum_{i=0}^n t^{n-i} u'_i(t) - \gamma \sum_{n=0}^{\infty} \left( \sum_{j=0}^n \sum_{i=0}^j t^{n-j} f_i(t) u'_{j-i}(t) \right) \right] \tag{20}$$

According to the modified decomposition method, the solution  $u(t)$  can be evaluated by using the recurrence scheme:

$$u_0(t) = \alpha$$

$$u_1(t) = \frac{t-a}{b-a}(\beta - \alpha) + L^{-1}[u'_0(t) - \gamma f_0(t)u'_0(t)],$$

$$u_{n+1}(t) = L^{-1} \left[ \sum_{i=0}^n t^{n-i} u'_i(t) - \gamma \left( \sum_{j=0}^n \sum_{i=0}^j t^{n-j} f_i(t) u'_{j-i}(t) \right) \right], \quad n \geq 1. \tag{21}$$

The algorithms by (21) was applied in the next session to construct the approximate solutions.

## RESULTS

### The Modified Adomian Decomposition Method for the Blasius equation

Recall (7)

$$f''' + \frac{1}{2} f f'' = 0$$

Suppose we put  $t = 1 - e^{-\eta}$

Where  $\eta = 0, t = 0$  and  $\eta = \infty, t = 1$

$$\frac{d}{d\eta} = (1-t) \frac{d}{dt} \tag{22}$$

$$\frac{d^2}{d\eta^2} = (1-t)^2 \frac{d^2}{dt^2} - (1-t) \frac{d}{dt} \tag{23}$$

$$\frac{d^3}{d\eta^3} = \frac{d}{d\eta} \left[ (1-t)^2 \frac{d^2}{dt^2} - (1-t) \frac{d}{dt} \right] \tag{24}$$

$$\frac{d^3}{d\eta^3} = \frac{dt}{d\eta} \frac{d}{dt} \left[ (1-t)^2 \frac{d^2}{dt^2} - (1-t) \frac{d}{dt} \right] \tag{25}$$

$$\frac{d^3}{d\eta^3} = (1-t) \left[ -2(1-t) \frac{d^2}{dt^2} + (1-t)^2 \frac{d^3}{dt^3} - (1-t) \frac{d^2}{dt^2} \right] \tag{26}$$

$$\frac{d^3}{d\eta^3} = (1-t) \left[ -3(1-t) \frac{d^2}{dt^2} + (1-t)^2 \frac{d^3}{dt^3} + \frac{d}{dt} \right] \tag{27}$$

$$\frac{d^3}{d\eta^3} = (1-t)^3 \frac{d^3}{dt^3} - 3(1-t)^2 \frac{d^2}{dt^2} + (1-t) \frac{d}{dt} \tag{28}$$

Substituting (23) and (28) into (7)

that is

$$f''' + \frac{1}{2} f f'' = 0$$

We have

$$(1-t)^3 \frac{d^3 f}{dt^3} - 3(1-t)^2 \frac{d^2 f}{dt^2} + (1-t) \frac{df}{dt} + \frac{1}{2} f(t) \left[ (1-t)^2 \frac{d^2 f}{dt^2} - (1-t) \frac{df}{dt} \right] = 0 \tag{29}$$

$$(1-t)^3 \frac{d^3 f}{dt^3} - 3(1-t)^2 \frac{d^2 f}{dt^2} + (1-t) \frac{df}{dt} + \frac{1}{2} (1-t)^2 f \frac{d^2 f}{dt^2} - \frac{1}{2} (1-t) f \frac{df}{dt} = 0 \tag{30}$$

$$\frac{d^3 f}{dt^3} - \frac{3}{(1-t)} \frac{d^2 f}{dt^2} + \frac{1}{(1-t)^2} \frac{df}{dt} + \frac{1}{2(1-t)} f \frac{d^2 f}{dt^2} - \frac{1}{2(1-t)^2} f \frac{df}{dt} = 0 \tag{31}$$

$$\frac{d^3 f}{dt^3} = \frac{3}{(1-t)} \frac{d^2 f}{dt^2} - \frac{1}{(1-t)^2} \frac{df}{dt} - \frac{1}{2(1-t)} f \frac{d^2 f}{dt^2} + \frac{1}{2(1-t)^2} f \frac{df}{dt} \tag{32}$$

Given

$$\frac{1}{(1-t)} = \sum_{n=0}^{\infty} t^n, \quad \frac{1}{(1-t)^2} = \sum_{n=0}^{\infty} t^n \sum_{m=0}^{\infty} t^m = \sum_{n=0}^{\infty} \sum_{m=0}^n t^{n-m}$$

$$\text{and } \frac{d^2 f}{dt^2} = \sum_{n=0}^{\infty} f''_n$$

But  $(1+t)^n = 1 + nt + \frac{n(n-1)}{2!} t^2 + \dots$

$$= \sum_{n=0}^{\infty} (n+1)t^n$$

$$= 3 \sum_{n=0}^{\infty} \sum_{m=0}^n t^{n-m} f_m'' - n=0 \infty m=0 nj=0 mtn- mtm-jff' - 12n=0 \infty m=0 nj=0 mtn- mfm-jff'' + 12n=0 \infty m=0 nj=0 mk=0 jtn- mtm-jff-k f'k' = \sum_{n=0}^{\infty} \left\{ 3 \sum_{m=0}^n t^{n-m} f_m'' - \sum_{m=0}^n \sum_{j=0}^m t^{n-j} f_j' - \frac{1}{2} \sum_{m=0}^n \sum_{j=0}^m t^{n-m} f_{m-j} f_j'' + \frac{1}{2} \sum_{m=0}^n \sum_{j=0}^m \sum_{k=0}^j t^{n-j} f_{j-k} f_k' \right\} \tag{34}$$

$$f_{n+1} = \frac{f''(0)}{2} t^2 + f'(0)t + f(0) \tag{35}$$

$$= At^2 + \int \int \int \left\{ 3 \sum_{m=0}^n t^{n-m} f_m'' - \sum_{m=0}^n \sum_{j=0}^m t^{n-j} f_j' - \frac{1}{2} \sum_{m=0}^n \sum_{j=0}^m t^{n-m} f_{m-j} f_j'' + \frac{1}{2} \sum_{m=0}^n \sum_{j=0}^m \sum_{k=0}^j t^{n-j} f_{j-k} f_k' \right\} \tag{36}$$

$$f_0(t) = At^2 \tag{37}$$

$$f_{n+1} = \int \int \int \left\{ 3 \sum_{m=0}^n t^{n-m} f_m'' - \sum_{m=0}^n \sum_{j=0}^m t^{n-j} f_j' - \frac{1}{2} \sum_{m=0}^n \sum_{j=0}^m t^{n-m} f_{m-j} f_j'' + \frac{1}{2} \sum_{m=0}^n \sum_{j=0}^m \sum_{k=0}^j t^{n-j} f_{j-k} f_k' \right\} = 0 \tag{38}$$

(Using MAPLE 14 to solve)

$$6A - 2At - 1.000000000A^2t^2 + 1.000000000A^2t^3 = 0$$

$$= At^3 - \frac{1}{12}At^4 - \frac{1}{60}A^2t^5 + \frac{1}{120}A^2t^6$$

$$f_1(t) = At^3 - \frac{1}{12}At^4 - \frac{1}{60}A^2t^5 + \frac{1}{120}A^2t^6 \tag{39}$$

$$f_2(t) = \frac{1}{21600}A^3t^{10} - \frac{23}{60480}A^3t^9 + \frac{1}{8} \left( \frac{11}{2520}A^3 - \frac{1}{140}A^2 \right) t^8 + \frac{71}{2520}A^2t^7 + \frac{1}{6} \left( -\frac{3}{10}A^2 + \frac{1}{60}A \right) t^6 - \frac{1}{6}At^5 + At^4 \tag{40}$$

$$f(t) = f_0 + f_1 + f_2 + \dots$$

$$f(t) = At^2 + At^3 - \frac{1}{12}At^4 - \frac{1}{60}A^2t^5 + \frac{1}{120}A^2t^6 + \frac{1}{21600}A^3t^{10} - \frac{23}{60480}A^3t^9 + \frac{1}{8} \left( \frac{11}{2520}A^3 - \frac{1}{140}A^2 \right) t^8 + \frac{71}{2520}A^2t^7 + \frac{1}{6} \left( -\frac{3}{10}A^2 + \frac{1}{60}A \right) t^6 - \frac{1}{6}At^5 + At^4$$

$$f(t) = At^2 + At^3 + \frac{11}{12}At^4 - \frac{1}{60}A^2t^5 + \frac{1}{120}A^2t^6 + \frac{1}{21600}A^3t^{10} - \frac{23}{60480}A^3t^9 + \frac{1}{8} \left( \frac{11}{2520}A^3 - \frac{1}{140}A^2 \right) t^8 + \frac{71}{2520}A^2t^7 + \frac{1}{6} \left( -\frac{3}{10}A^2 + \frac{1}{60}A \right) t^6 - \frac{1}{6}At^5 \tag{41}$$

$$f'(t) = 2At + 3At^2 + \frac{11}{3}At^3 - \frac{1}{12}A^2t^4 + \frac{1}{20}A^2t^5 + \frac{1}{2160}A^3t^9 - \frac{23}{6720}A^3t^8 + \left( \frac{11}{2520}A^3 - \frac{1}{140}A^2 \right) t^7 + \frac{71}{360}A^2t^6 + \left( -\frac{3}{10}A^2 + \frac{1}{60}A \right) t^5 - \frac{5}{6}At^4 \tag{42}$$

Substituting  $t = 1 - e^{-\eta}$  into (41) and (42) respectively, we have

$$f(\eta) = A(1 - e^{-\eta})^2 + A(1 - e^{-\eta})^3 + \frac{11}{12}A(1 - e^{-\eta})^4 - \frac{1}{60}A^2(1 - e^{-\eta})^5 + \frac{1}{120}A^2(1 - e^{-\eta})^6 + \frac{1}{21600}A^3(1 - e^{-\eta})^{10} - \frac{23}{60480}A^3(1 - e^{-\eta})^9 + \frac{1}{8} \left( \frac{11}{2520}A^3 - \frac{1}{140}A^2 \right) (1 - e^{-\eta})^8 + \frac{71}{2520}A^2(1 - e^{-\eta})^7 + \frac{1}{6} \left( -\frac{3}{10}A^2 + \frac{1}{60}A \right) (1 - e^{-\eta})^6 - \frac{1}{6}A(1 - e^{-\eta})^5 \tag{43}$$

$$f'(\eta) = 2A(1 - e^{-\eta}) + 3A(1 - e^{-\eta})^2 + \frac{11}{3}A(1 - e^{-\eta})^3 - \frac{1}{12}A^2(1 - e^{-\eta})^4 + \frac{1}{20}A^2(1 - e^{-\eta})^5 + \frac{1}{2160}A^3(1 - e^{-\eta})^9 - \frac{23}{6720}A^3(1 - e^{-\eta})^8 + \left( \frac{11}{2520}A^3 - \frac{1}{140}A^2 \right) (1 - e^{-\eta})^7 + \frac{71}{360}A^2(1 - e^{-\eta})^6 - \frac{5}{6}A(1 - e^{-\eta})^4 \tag{44}$$

Substituting  $\eta = 5$  into (44), we have

$$7.744293347A - 0.1402308544A^2 + 0.001356525634A^3 = 1 \tag{45}$$

Solving (45), we have

$$A = 0.1294303030, 51.62279111 + 55.051268271i, 51.62279111 - 55.051268271i$$

∴  $A = 0.1294303030$  since other values are imaginary

Substituting  $A$  into (44), we have

$$f'(\eta) = 0.2588606060 - 0.2588606060e^{-\eta} + 0.3882909090(1 - e^{-\eta})^2 + 0.4745777777(1 - e^{-\eta})^3 - 0.1092546027(1 - e^{-\eta})^4 - 0.002030879110(1 - e^{-\eta})^5 + 0.000001003816090(1 - e^{-\eta})^9 - 0.000007421068950(1 - e^{-\eta})^8 - 0.0001101940435(1 - e^{-\eta})^7 + 0.003303906768(1 - e^{-\eta})^6 \tag{46}$$

$$\eta \rightarrow 0.2588606060 - 0.2588606060e^{-\eta} + 0.3882909090(1 - e^{-\eta})^2 + 0.4745777777(1 - e^{-\eta})^3 - 0.1092546027(1 - e^{-\eta})^4 - 0.002030879110(1 - e^{-\eta})^5 + 0.000001003816090(1 - e^{-\eta})^9 - 0.000007421068950(1 - e^{-\eta})^8 - 0.0001101940435(1 - e^{-\eta})^7 + 0.003303906768(1 - e^{-\eta})^6$$

### Modified Adomian Decomposition Method for the Energy Equation

Recall (7), that is,

$$\theta'' + Prf\theta' = 0$$

$$\theta(0) = 1$$

$$\theta(\infty) = 0$$

Recall

$$t = 1 - e^{-\eta}$$

But

$$\frac{d}{d\eta} = (1 - t) \frac{d}{dt} \tag{47}$$

$$\frac{d^2}{dt^2} = (1 - t)^2 \frac{d^2}{dt^2} - (1 - t) \frac{d}{dt} \tag{48}$$

$$\frac{1}{pr} \theta'' + f\theta' = 0 \text{ becomes}$$

$$\frac{1}{pr} \left[ (1 - t)^2 \frac{d^2\theta}{dt^2} - (1 - t) \frac{d\theta}{dt} \right] + f(t)(1 - t) \frac{d\theta}{dt} = 0 \tag{49}$$

$$(1 - t)^2 \frac{d^2\theta}{dt^2} - (1 - t) \frac{d\theta}{dt} + prf(t)(1 - t) \frac{d\theta}{dt} = 0 \tag{50}$$

$$\frac{d^2\theta}{dt^2} - \frac{1}{1-t} \frac{d\theta}{dt} + prf(t) \frac{1}{1-t} \frac{d\theta}{dt} = 0 \tag{51}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{1-t} \frac{d\theta}{dt} - prf(t) \frac{1}{1-t} \frac{d\theta}{dt} \tag{52}$$

$$\frac{d^2\theta}{dt^2} = (1 - prf(t)) \frac{1}{1-t} \frac{d\theta}{dt} \tag{53}$$

Recall

$$\frac{d^2\theta}{dt^2} = \sum_{n=0}^{\infty} \theta_n'', \quad \frac{d\theta}{dt} = \sum_{n=0}^{\infty} \theta_n' \text{ and } \frac{1}{1-t} = \sum_{n=0}^{\infty} t^n$$

Substituting into (53), we have

$$= (1 - prf(t)) \sum_{n=0}^{\infty} t^n \sum_{n=0}^{\infty} \theta_n' \tag{54}$$

$$= (1 - prf(t)) \sum_{n=0}^{\infty} \sum_{m=0}^n t^{n-m} \theta_m' \tag{55}$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^n t^{n-m} \theta_m' - pr \sum_{n=0}^{\infty} \sum_{m=0}^n \sum_{j=0}^m t^{n-m} \theta_m' f_{m-j} \tag{56}$$

$$\theta_{n+1} = \sum_{m=0}^n t^{n-m} \theta_m' - pr \sum_{m=0}^n \sum_{j=0}^m t^{n-m} \theta_m' f_{m-j} \tag{57}$$

Using MAPLE to solve (57) with the initial conditions, we have

$$\theta_0(t) = 1 + Bt \tag{58}$$

$$\theta_1(t) = \frac{1}{2} Bt^2 - \frac{71}{1200} ABt^4 \tag{59}$$

$$\theta_2(t) = \frac{19}{1492561} A^3 Bt^{11} - \frac{235}{7552073} A^3 Bt^{10} - \frac{469}{1695228} A^2 Bt^9 + \frac{1349}{420000} A^2 Bt^8 + \frac{1}{7} \left( \frac{71}{7200} AB + \frac{5041}{180000} A^2 B \right) t^7 - \frac{71}{3000} ABt^6 - \frac{497}{6000} ABt^5 + \frac{1}{3} Bt^3 \tag{60}$$

$$\theta_n(t) = 1 + Bt + \frac{1}{2} Bt^2 - \frac{71}{1200} ABt^4 + \frac{19}{1492561} A^3 Bt^{11} - \frac{235}{7552073} A^3 Bt^{10} - \frac{469}{1695228} A^2 Bt^9 + \frac{1349}{420000} A^2 Bt^8 + \frac{1}{7} \left( \frac{71}{7200} AB + \frac{5041}{180000} A^2 B \right) t^7 - \frac{71}{3000} ABt^6 - \frac{497}{6000} ABt^5 + \frac{1}{3} Bt^3 \tag{61}$$

But  $t = 1 - e^{-\eta}$ ,  $A = 0.1294303030$ , substituting into (61), we have

$$\theta_n(t) = 1 + B(1 - e^{-\eta}) + \frac{1}{2} B(1 - e^{-\eta})^2 - 0.007657959594B(1 - e^{-\eta})^4 + 2.76012922310^{-8} B(1 - e^{-\eta})^{11} - 6.74698254610^{-8} B(1 - e^{-\eta})^{10} - 0.000004634646998B(1 - e^{-\eta})^9 + 0.00005380648165B(1 - e^{-\eta})^8 + 0.0002493544800B(1 - e^{-\eta})^7 - 0.003063183838B(1 - e^{-\eta})^6 - 0.01072114343B(1 - e^{-\eta})^5 + \frac{1}{3} B(1 - e^{-\eta})^3 \tag{62}$$

Using  $\eta = 5$ , we have

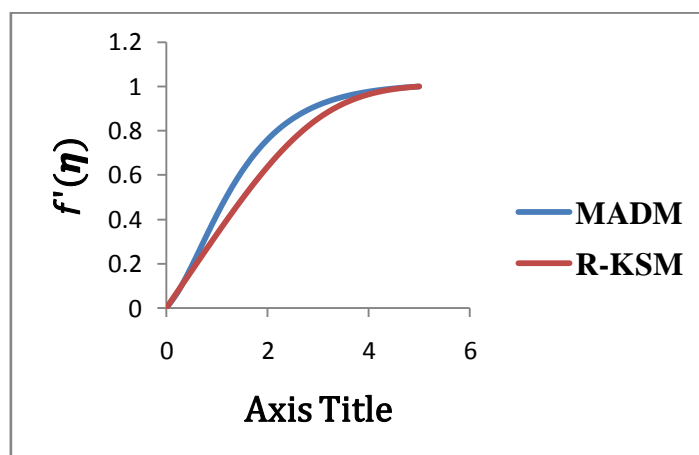
$$1 + 1.792712054B = 0, \quad B = -0.5578140660$$

Now substituting for B in (62), we have

$$\theta_n(t) = 0.4421859340 + 0.5578140660e^{-\eta} - 0.2789070330(1 - e^{-\eta})^2 + 0.004271717578(1 - e^{-\eta})^4 - 1.53963890510^{-8}(1 - e^{-\eta})^{11} + 3.76356176710^{-8}(1 - e^{-\eta})^{10} + 0.000002585271286(1 - e^{-\eta})^9 - 0.00003001401231(1 - e^{-\eta})^8 - 0.0001390934364(1 - e^{-\eta})^7 + 0.001708687032(1 - e^{-\eta})^6 + 0.005980404609(1 - e^{-\eta})^5 - 0.1859380220(1 - e^{-\eta})^3 \tag{63}$$

**Table-1: Table of results for the Blasius equation and Energy equation**

Pr=0.71				
□	BLASIVS EQUATION		ENERGY EQUATION	
	MADM	R-KSM	MADM	R-KSM
0	0	0	1	1
0.2	0.062390598	0.06722664	0.888619337	0.92450746
0.4	0.143255013	0.13440071	0.779199047	0.84909886
0.6	0.234890891	0.20138692	0.674767907	0.77398947
0.8	0.329480239	0.26796197	0.577944661	0.69953375
1	0.421209531	0.33381881	0.49037356	0.62621586
1.2	0.506439171	0.39857458	0.412748171	0.55463206
1.4	0.583271598	0.46178348	0.345032201	0.48546352
1.6	0.651031284	0.52295447	0.286704663	0.41943908
1.8	0.709824783	0.58157389	0.236965337	0.35728945
2	0.760214984	0.63713192	0.194885864	0.29969589
2.2	0.802997586	0.68915154	0.159511395	0.24723825
2.4	0.839056257	0.73721742	0.129923634	0.20034793
2.6	0.869273938	0.78100210	0.105276141	0.15927249
2.8	0.894482298	0.82028696	0.084810902	0.12405642
3	0.915436120	0.85497555	0.067862899	0.09454097
3.2	0.932803367	0.88509772	0.053857461	0.07038332
3.4	0.947164691	0.91080418	0.042303612	0.05109221
3.6	0.959018247	0.93235251	0.032785527	0.03607418
3.8	0.968787224	0.95008607	0.024953415	0.02468394
4	0.976828403	0.96440855	0.018514591	0.01627234
4.2	0.983440825	0.97575711	0.013225182	0.01022661
4.4	0.988873980	0.98457680	0.008882672	0.00599918
4.6	0.993335256	0.99129836	0.005319334	0.00312396
4.8	0.996996548	0.99632087	0.00239656	0.00122223
5	0.999999999	0.99999999	0.00000009	0



**Fig-1: Velocity Profile for the MADM and R-KSM methods (Pr=0.71).**

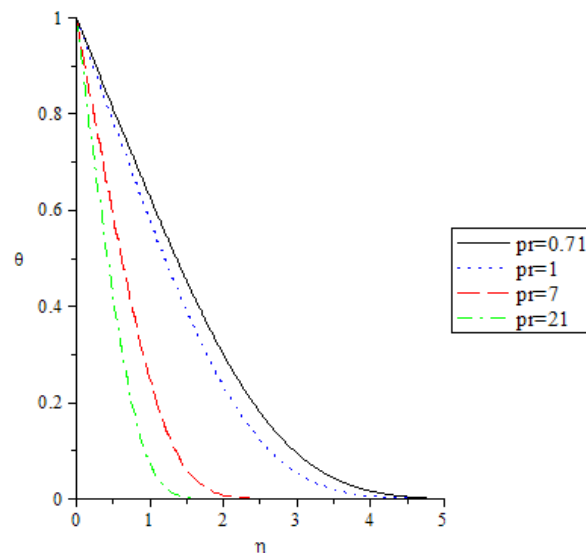


Fig-2: Temperature profile for various values of the Prandtl number

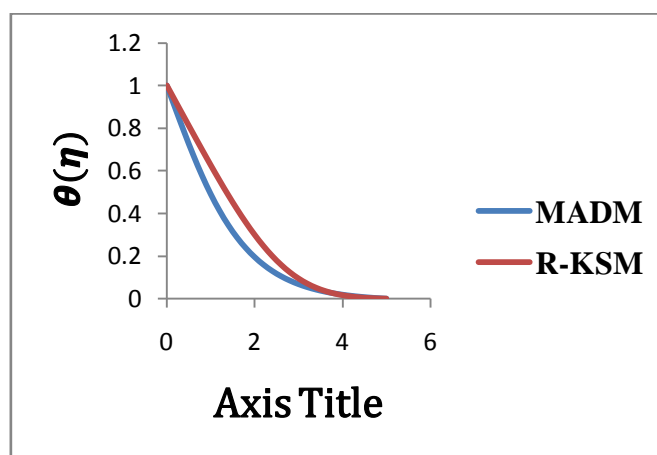


Fig-3: Temperature profile for MADM and R-KSM methods (Pr=0.71)

**DISCUSSION**

The discussion under this session was held under three subheadings in order to do justice to the discussion.

**Velocity Profile**

The Blasius equation was solved in order to obtain the velocity profile using three distinct approximate methods. The methods are the Modified Adomian Decomposition Method (MADM) and the Runge-Kutashooting method (R-KSM). Figure 1 depicts the velocity profile for the methods. It is observed that each of the methods converges to the true solution. It is to be noted that the velocity profile increases steadily until steady state is achieved for the methods. It is also noted that the boundary layer increases from the edge of the boundary layer into the free stream which is a feature of boundary layer flow. Numerical results obtained are also shown in Table 1

**Temperature profile**

The Energy equation was solved in order to obtain the Temperature profile using the two methods, Modified Adomian Decomposition Method (MADM) and the Runge-Kutta shooting method (R-KSM). Figure 3 represents the Temperature profile for the methods. It is observed that the methods also converge to the true solution. It is to be also noted that the Temperature profile decreases steadily until steady state is reached for the two methods. Numerical results obtained are also shown in Table 1.

**Temperature profile for the various values of the Prandtl number**

In Figure 2, we studied the impact (influence) of the dimensionless of the only thermo physical parameter arising from the flow, i.e. the Prandtl number, which depends on the properties of the substance or fluid. In this paper, the following values were used; Pr=0.71 for Air at the following degrees centigrade; 0, +50, 100 and 200, Pr=1 for certain gases, Pr=7



for water at 20<sup>0</sup>c and Pr=21 for certain class of lubricating oil. It is observed that as the Prandtl number increases, the boundary layer thickness reduces which shows that the Prandtl number plays important role in the boundary layer flow as it helps to stabilize the temperature.

## CONCLUSION

In this paper, the boundary layer flow over a flat plate was presented and the Blasius equation and the Energy equation arising from the governing equations were also solved using the two approximate methods. The methods are the Modified Adomian Decomposition Method (MADM) and the Runge-Kutta shooting Method (R-KSM). Modified Adomian Decomposition Method (MADM) is semi-analytic while the Runge-Kutta (R-KSM) shooting method which is a finite difference method. In the results obtained, it is revealed that in either case all the two methods converge to the true solutions.

## REFERENCES

1. Stern F; Boundary Layer Theory, 2010.
2. Burr KP, Akylas TR; Mei CC; Two dimensional Laminar boundary Layers, School-wide program on Fluid Mechanics Modules on High Reynolds Number Flows. I- Campus project, 2010.
3. Veldman AEP; Boundary Layers in Fluid Dynamics. Institute for Mathematics and Computer Science, University of Groningen, Netherlands, 2012.
4. Ghoreishi M, Ismail AIB, Alomari AK; Comparison between homotopy analysis method and optimal homotopy asymptotic method for nth-order integro-differential equation. *Mathematical Methods in the Applied Sciences*, 2011; 34(15):1833-1842.
5. Adomian G; *Nonlinear stochastic systems: Theory and Applications to Physics*. Kluwer Academic Press, 1998.
6. Adomian G; *Solving Frontier Problems of Physics: The Decomposition Method*. Kluwer Academic press, 1994.
7. Ahmed HMA; *Adomian Decomposition Method: Convergence Analysis and Numerical Approximations*. M.Sc Thesis, McMaster University, Ontario, 2008.
8. Abbaoui K, Cherruault Y; Convergence of Adomians method applied to differential equations. *Comp. Math Appl*, 1994a; 28: 103-109.
9. Lesnic D; The decomposition method for forward and backward time-dependent problems. *Journal of Computational and Applied Mathematics*, 2002; 147: 27-38.
10. Al-Khaled K, Allan F; Construction of solution for the shallow water equations by the decomposition method. *Mathematics and Computers in simulation*, 2005; 66(6): 479-486.
11. Sohr H, von Wahl W; On the regularity of the pressure of weak solutions of Navier-Stokes equations. *Archiv der Mathematik*, 1986; 46(5):428-439.
12. Esmailpour M, Ganji DD; Application of He's homotopy Perturbation Method to boundary layer flow and convection heat transfer over a flat plate. *Physics Letters A*, 2007; 372(1): 33-38.
13. Ebaid A, Al-Armani N; A new Approach for a Class of the Blasius Problem via a Transformation and Adomian's Method. *Abstract and Applied Analysis, the Scientific World Journal*, 2013.
14. Bejan A; *Convection Heat Transfer*, Wiley, New York, 1995