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Research Article

Design of Second Order Digital Integrator Using Simulation Annealing

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Abstract: In this paper, a new design of second order digital integrator is investigated using simulation annealing optimization algorithm. Transfer function of the second order digital integrator is derived from the use of simulation annealing algorithm. Then compare the results with the ideal second order integrator and digital IIR second order integrator using Richardson Extrapolation which is also obtained.
Keywords: Simulation Annealing (SA), Richardson Extrapolation, Infinite Impulse Response (IIR), Finite Impulse Response (FIR), Trapezoidal Integration.

INTRODUCTION

The digital integrator are found be used in many applications like in analogue computers to perform calculus operations, control systems, biomedical instrumentation, video processing, digital signal processing, audio processing.etc. The frequency response of ideal digital integrator can be given as

$$I(\omega) = \left(\frac{1}{j\omega}\right)^2 \qquad (1)$$

Where j = $\sqrt{-1}$ and ω is the angular frequency in radian/sec.

Because of expansion of digital signal processing discipline, there are variety of requirements which have to be met by digital integrators and differentiators. Digital integrators and differentiators can be classified as finite impulse response (FIR) and infinite impulse response (IIR) [5-7].However; digital IIR integrators are more preferred. Basically, digital IIR integrators and differentiators designs have been proposed by Newton Cotes integration rule. These can be designed directly or by transformation an analog to digital filters through methods like impulse invariance, bilinear transformation, forward difference, backward difference. Rectangular, Trapezoidal, Simpson 1/3, Simpson 3/8, and Boole are the basic integrators proposed

However, Optimization is a popular design method. This improves the performance of a system by reducing its runtime, bandwidth, memory requirement, or any other property. Optimization methods such as linear, simulated annealing, genetic algorithm, and pole zero optimization are used to design IIR digital integrators and differentiators.

In this paper, second order digital integrator is designed using SA and compare the result with second order digital integrator using Richardson extrapolation and second order ideal integrator. These comparisons of integrators are explained in result section.

EXPERIMENTAL SECTION

Use of Richardson Extrapolation to design second order Integrator

In the paper, first the trapezoidal integration rule [8] and differential equation are applied to derive the transfer function of the digital integrator. Then, the Richardson extrapolation [2-4] is applied to generate high-accuracy results while using low order formulas (LOF).So, the transfer function of the second order digital integrator can be approximated by

When replacing z by $e^{j\omega}$, the frequency response becomes

$$A_0(e^{j\omega},\alpha) = \left(\frac{\alpha}{2}\frac{1+\ e^{j\omega\alpha}}{1-\ e^{j\omega\alpha}}\right)^2 \quad (3)$$

$$A_0(e^{j\omega},\alpha) = -\frac{\alpha^2}{4}\left(cot\left(\frac{\alpha\omega}{2}\right)\right)^2 \quad (4)$$

Using the power-series expansion of the cotangent function

$$\cot(x) = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \cdots.. \quad (5)$$

Then the frequency response can be rewritten as

$$A_0(e^{j\omega},\alpha) = -\frac{\alpha^2}{4}\left[\frac{4}{\alpha^2\omega^2} - \frac{2}{3} + \frac{\alpha^2\omega^2}{60} + \frac{\alpha^4\omega^4}{1512} + \frac{\alpha^6\omega^6}{33600} + \cdots..\right]$$

$$A_0(e^{j\omega},\alpha) = \left(\frac{1}{j\omega}\right)^2 + \frac{\alpha^2}{6} - \frac{\alpha^4\omega^4}{240} - \frac{\alpha^6\omega^6}{6048} + \cdots.$$

$$A_0(e^{j\omega},\alpha) = I(\omega) + O(\alpha^2) \quad (6)$$

Where $O(\alpha^2)$ denotes the error term decays as fast as α^2. If parameter α approaches zero, we have the following result

$$\lim_{\alpha\to 0} A_0(e^{j\omega},\alpha) = I(\omega) \quad (7)$$

From the above expression, it is indicated that frequency response $A_0(e^{j\omega},\alpha)$ becomes the ideal integrator $I(\omega)$ when α tends to zero. Even so the convergence speed is slow. So in order to improve accuracy results, the Richardson extrapolation technique is applied to achieve high accuracy results from low order formulas (LOF). Now two expressions from (6) are given by

$$A_0(e^{j\omega},\alpha) = I(\omega) + c_1\alpha^2 + c_2\alpha^4 + c_3\alpha^6 + \cdots.. \quad (8)$$

$$A_0(e^{j\omega},2\alpha) = I(\omega) + 4c_1\alpha^2 + 16c_2\alpha^4 + 64c_3\alpha^6 + \cdots.. \quad (9)$$

If we multiply Equation (8) by 4 and subtract from Equation (9) then the terms involving c_1 cancel and the result is

$$4A_0(e^{j\omega},\alpha) - A_0(e^{j\omega},2\alpha) = 3I(\omega) - 12c_2\alpha^4 - 60c_3\alpha^6 + \cdots..$$

After some manipulation, the above equation can be rewritten as

$$A_1(e^{j\omega},\alpha) = \frac{4A_0(e^{j\omega},\alpha) - A_0(e^{j\omega},2\alpha)}{3}$$

$$A_1(e^{j\omega},\alpha) = I(\omega) - 4c_2\alpha^4 - 20c_3\alpha^6 + \cdots....$$

$$A_1(e^{j\omega},\alpha) = I(\omega) + O(\alpha^4) \quad (10)$$

In the above expression, the order of the error term is $O(\alpha^4)$ which gives faster convergence speed than $A_0(e^{j\omega},\alpha)$ when α approaches zero. For better convergence speed the term involving α^4 should be cancelled out. To see how it is removed; two expressions from Equation (10) are given by

$$A_1(e^{j\omega},\alpha) = I(\omega) + d_2\alpha^4 + d_3\alpha^6 + \cdots. \quad (11)$$

$$A_1(e^{j\omega},2\alpha) = I(\omega) + 16d_2\alpha^4 + 64d_3\alpha^6 + \cdots. \quad (12)$$

If we multiply Equation (11) by 16 and subtract from Equation (12) then the terms involving d_2 cancel and the result is

$$16A_1(e^{j\omega},\alpha) - A_1(e^{j\omega},2\alpha) = 15I(\omega) - 48d_3\alpha^6 + \cdots..$$

After some manipulation, the above equation can be rewritten as

$$A_2(e^{j\omega},\alpha) = \frac{16A_1(e^{j\omega},\alpha) - A_1(e^{j\omega},2\alpha)}{15}$$

$$A_2(e^{j\omega},\alpha) = I(\omega) - \frac{48}{15}d_3\alpha^6 - 16d_4\alpha^8 + \cdots.$$

$$A_2(e^{j\omega},\alpha) = I(\omega) + O(\alpha^6) \quad (13)$$

Here in this the order of error term is $O(\alpha^6)$. Which provides a faster convergence speed than $A_1(e^{j\omega},\alpha)$ when α approaches zero. Equivalently, the general solution can be obtain by recursive formula for Richardson extrapolation improvement process, which is given by

$$A_k(z,\alpha) = \frac{4^k A_{k-1}(z,\alpha) - A_{k-1}(z,2\alpha)}{4^k - 1}$$

Then, the frequency response can be defined in the unified form which is

$$A_k(e^{j\omega},\alpha) = I(\omega) + O(\alpha^{2k+2}) \quad (14)$$

The above expression is the general solution of the transfer function. The order of the error is $O(\alpha^{2k+2})$ which results in fastest convergence speed than above transfer function when α tends to zero. It is also indicated that the errors can be reduced by increasing k or decreasing α.

Use of Simulation Annealing to design second order Integrator

First of all let us consider a transfer function of integrator which can be given by equation (15). In this transfer function there are thirteen unknown coefficients. These unknown coefficients can be estimated by minimizing error function E_{int} .The expression of error function is given by equation (16). This error function is minimized using simulation annealing optimization technique [1]. After minimization Using SA, ten integrators are obtained which have less relative percentage error compare to the ideal one. All the coefficients of these ten designed integrators are shown in Tab.2.a and Tab.2.b and their relative error response is shown in Fig. 1. It is seen that all these integrators have less relative percentage error but design I and VI have minimum relative error in the entire Nyquist frequency range. Thus these are called as proposed integrators in this research paper.

$$H_{int}(z) = \frac{s(1) + s(2)z^{-1} + s(3)z^{-2} + s(4)z^{-3} + s(5)z^{-4} + s(6)z^{-5} + s(7)z^{-6}}{1 + s(8)z^{-1} + s(9)z^{-2} + s(10)z^{-3} + s(11)z^{-4} + s(12)z^{-5} + s(13)z^{-6}} \quad (15)$$

$$E_{int} = \int_0^{\pi} \left(\frac{1}{\omega^2} - \left|H_{int}(e^{j\omega})\right|\right)^2 d\omega \quad (16)$$

This error function E_{int} is basically the difference in the energy of the ideal second order integrator and the proposed second order integrator taken from the period [0 π].

RESULTS AND DISCUSSION

Design of digital integrator

The design of a digital integrator is a challenging task. Tab.1 shows all the parameters used to obtained best results in this paper. The designing is done using Simulation annealing optimization technique. The whole optimization process is carried out through MATLAB 7.

The various parameters of simulation annealing algorithm are given below in a table.

Table-1: Simulation Annealing parameters.

S.NO.	Parameter	Value
1.	Initial coefficients	[0 0 0 0 0 0 0 0 0 0 0 0 0]
2.	Annealing function	Fast annealing
3.	Temperature update function	Exponential update function
4.	Acceptance probability function	Simulated annealing acceptance
5.	Data type	Double
6.	Reannealing Interval	Default(100)
7.	Initial temperature	Default(100)
8.	Objective function	Error function $\boldsymbol{E_{int}}$

In the Tab.2.a and Tab.2.b, there are thirteen parameters which are obtained by minimizing error function given by equation (16).

Table-2a: Coefficients of designed integrators.

S.NO.	s(1)	s(2)	s(3)	s(4)	s(5)	s(6)	s(7)
I.	18.1052	36.7830	188.5033	383.9514	474.6985	112.5791	-85.1772
II.	-447.636	746.6806	1765.9817	-6234.357	-6720.986	-2478.244	-452.322
III.	436.2003	343.5358	-1309.501	-2232.793	-632.6399	-266.830	56.8518
IV.	-0.3197	52.2841	301.1164	483.9195	293.7661	-490.5094	68.80798
V.	6389.3846	5988.100	1139.0251	-2994.921	-1527.790	847.914	609.5788
VI.	-480.2038	-2241.71	-2790.941	-615.3302	-538.7708	588.6642	-175.081
VII.	13.7766	0.1117	-89.8687	-258.1554	-142.944	84.5409	-49.2791
VIII.	1512.2797	3464.110	2257.593	304.553	-220.913	-211.8039	-241.761
IX.	253.8706	330.3199	786.3090	-1686.625	-3615.8108	-2612.7034	-910.0218
X.	-60.4665	-233.045	-409.4368	-201.7827	81.6590	-62.9310	66.482

The error function minimization is carried out through a MATLAB optimization tool ‘optimtool’. The parameters which are required by optimtool, are mentioned in Tab.1. In this table, an objective is the function which has to be minimized.

Table-2b: Coefficients of designed integrators.

S.NO.	s(8)	s(9)	s(10)	s(11)	s(12)	s(13)
I.	161.573	244.703	-752.5987	71.972	236.0509	42.543
II.	-2356.344	-4325.732	12726.904	-4229.771	-1389.705	-443.198
III.	1524.574	-526.86	-4154.1892	3441.454	-200.485	-52.5099
IV.	54.8578	34.4854	-6.6938	-1443.844	2433.7747	-1076.227
V.	-5989.1721	6291.6573	4577.9393	1845.052	-11913.875	5155.466
VI.	2207.8758	-2585.8931	2033.7618	-5338.1621	4406.5891	-706.9709
VII.	-20.5433	37.2428	339.333	-645.7294	196.7216	92.7155
VIII.	303.3109	1107.3238	152.0971	-2771.8038	-1421.7991	2697.2036
IX.	2101.8028	369.3442	-4187.2056	514.2803	568.7703	717.5645
X.	-29.1725	-41.7105	-65.2885	-127.3344	808.0584	-548.8771

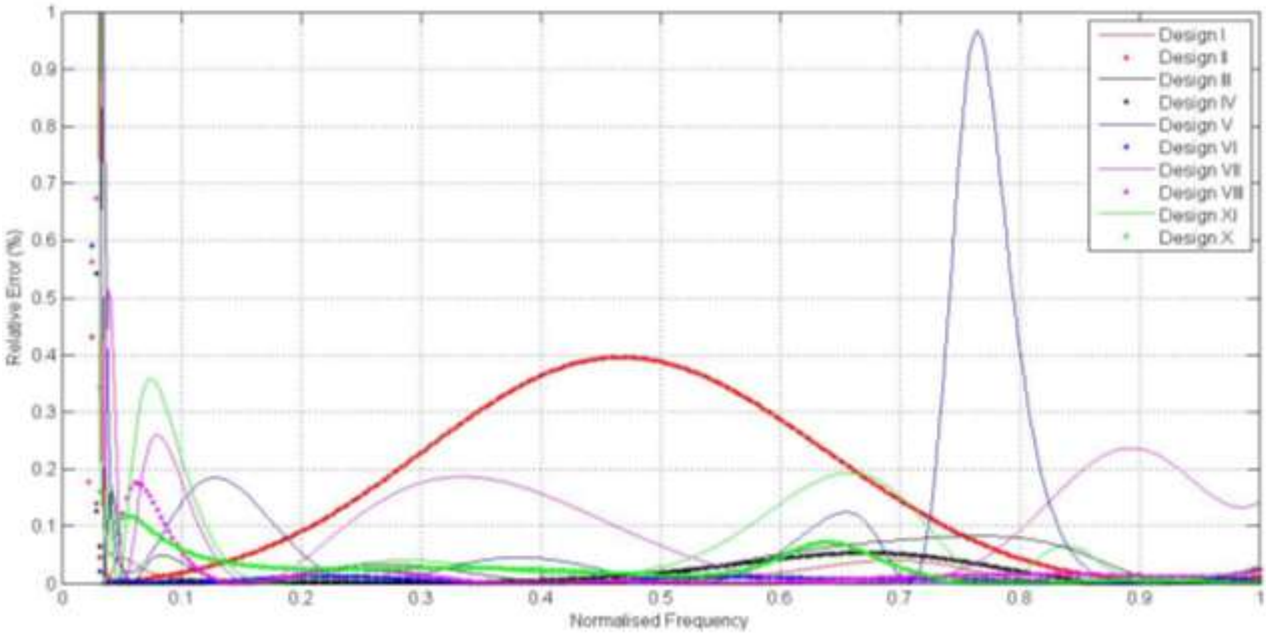

Fig-1: Relative percentage error response of all ten designed integrators

In the above figure, there are ten relative errors of integrators with reference to ideal second order integrator. It is found that only two designs I and VI out of ten integrators have $\text{Relative Error}(\%) \leq 0.1\%$ in entire range $0 \leq \omega \leq \pi$ among all other integrators. The transfer function of design I can be obtain by putting values of coefficients (s(1) to s(13))of serial number I of Tab.2.a and Tab.2.b jointly in equation (15) which is given below.

$$H_I(z) = \frac{18.1052 + 36.7830z^{-1} + 188.5033z^{-2} + 383.9514z^{-3} + 474.6985z^{-4} + 112.5791z^{-5} - 85.1772z^{-6}}{1 + 161.573z^{-1} + 244.703z^{-2} - 752.5987z^{-3} + 71.972z^{-4} + 236.0509z^{-5} + 42.543z^{-6}} \quad (17)$$

Similarly, we can write the transfer function for design VI which is given below

$$H_{VI}(z) = \frac{-480.2038 - 2241.71z^{-1} - 2790.941z^{-2} - 615.3302z^{-3} - 538.7708z^{-4} + 588.6642z^{-5} - 175.081z^{-6}}{1 + 2207.8758z^{-1} - 2585.8931z^{-2} + 2033.7618z^{-3} - 5338.1621z^{-4} + 4406.5891z^{-5} - 706.9709z^{-6}} \quad (18)$$

These equations represent optimum designed transfer function

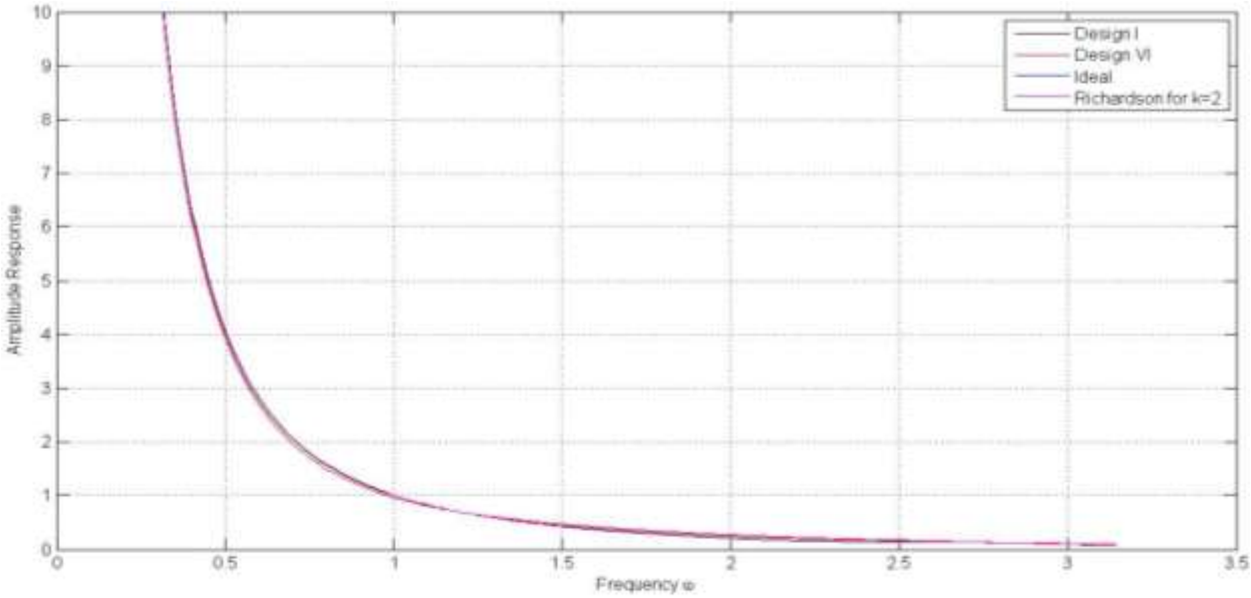

Fig-2: Amplitude Response of Design I, VI, Ideal and Richardson

In the above, Fig.2 plot shows amplitude responses of optimized designed, ideal and Richardson integrators. It is clear from above plot, design I and design VI and Richardson, ideal curves are overlapped. This shows that these curves are approximately same. However, Richardson extrapolation is more accurate than simulation annealing optimization technique because Richardson has better approximation than simulation annealing optimization technique. But the transfer function of integrator is designed by Richardson extrapolation has fractional order if $\boldsymbol{\alpha = 0.1}$. It is important to note that Richardson extrapolation method clearly explains, a fractional order filter can be constructed but it does not exist physically. If somehow we can realize a fractional order filter into an integer order filter by using appropriate method, then this integer order would be bigger, provided output is same. However, if we use simulation annealing algorithm for designing a transfer function for the same output filter, then the order of this transfer function will be lesser.

CONCLUSION:

In this paper, design of second order digital integrator is discussed with the help of simulation results shown by Fig.1 and Fig.2. These figures clearly indicate.

- It is observed that Richardson extrapolation method clearly explains, a fractional order filter can be constructed but it does not exist physically. If somehow we can realize a fractional order filter into an integer order filter by using appropriate method, then this integer order would be bigger, provided output is same. However, if we use simulation annealing algorithm for designing a transfer function for the same output filter, then the order of this transfer function will be lesser.

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