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Research Article

An Approximation Algorithms for Facility Location Problems with Multi-type Clients

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Abstract: A new model of facility location problem referred to as a facility location problem with diverse type of customers was proposed. The problem can be described as follows: There is a set of clients and a set of potential sites where facilities of uncapacitated can be set up. Each client demands to be satisfied by a set of facilities depending on which products it needs of the model, and one facility can be set up to supply only one product. Suppose that these facilities considered are relatively centralized ,under the assumption that the setting costs is zero and the shipping costs are in facilities centered metric space, it shows that the problem is NP-complete when k=2. Furthermore, an approximation algorithm is presented, worst case performance ratio was proved to be below 2-1/k for any integer k.
Keywords: Facility Location Problem; Approximation algorithms; Complexity.

INTRODUCTION

In the nineteenth century, Alfred Weber [1] proposed the Industry Location Problem , which was recognized as the beginning of Location Problem, so Location Problem was also called Weber problem. The essence of Location Problem is finding a reasonable allocation scheme and achieving some optimal solutions under the precondition of satisfying the given task. The algorithm of solving Location Problem is usually solved by the discrete integer programming model, such as the cutting plane method, the branch and bound method and so on. However, these methods only address some small instances problems. There are hundreds of constraints and variables in practical cases, which are limited by computer memory and computation time. These classic algorithms cannot solve these problems effectively. Most of the basic models of Location Problem are proved to be NP-complete.

In classic factory location problem, first of all, a subset used for setting up the factory was determined from a given set of addresses and then a factory providing products was assigned to each customer. In such problems, the general optimization goal is to minimize the total costs of services and the cost of setting up the factory. In recent years, supposed that the service cost satisfies the metric spaces such as non-negativity, symmetry, trigonometric inequality and so on, researchers have proposed some approximation algorithms with constant performance ratios. Shmoys [2] first proposed an approximation algorithm with an approximate ratio of 3.157 by using the method of rounding the score of linear programming problem. Subsequently, Guha [3] rose the approximate ratio given by Shmoys from 3.157 to 1.736. and proposed that the lower bound of any effective algorithm approximation ratio is 1.463. In consideration of using the original - dual algorithm to solve this problem, Jain [4] first increased the approximation ratio to 1.61 [5]. Mahdianand combined greedy algorithm and achieved MYZ algorithm with an approximate ratio of 1.52 [6]. Byrka combined MYZ algorithm and linear Planning relaxation and achieved 1.50 approximation algorithm [7] through the new random rounding techniques. Differed from the classical non-capacity-limited facility location problem that each customer only needs one product, some researchers put forward multi-product location problem [8]. Huang and Li [9] studied k kinds of products in the metric space. Each customer needs k kinds of products, but each factory can only provide customers with one product, that is, one customer's needs must be satisfied by k factories. Under the assumption that the setting costs is zero, the heuristic algorithm with the worst performance ratio of not more than $2k-1$ is proposed, an improvement work can be seen in the following article [10, 11] on the basis of the assumption.

In real life, each customer's needs are diverse. For example, customer A only needs products p_1 while customer B only needs products p_2, and customer C needs both. In this article, we discuss location problem of multi-products with different requirements of customers. In Section 2, a mathematical model is presented. An approximation algorithm is presented In Section 3, according to the problem that the setting cost of k kinds of product is zero, we give an approximate algorithm with the worst performance ratio of 2-1 / k. In Section 4, we make a conclusion and give prospects.

PROBLEM MODEL

In this section, we will consider the following problem:

Let D be the set of clients and F be the set of factories, and the set $P=\{p_1,p_2,\cdots,p_k\}$ represents k kinds of products; $\forall j\in D$,the set P_j represents the demand of customer j. Obviously, the set P_j is a subset of set P; each factory $i\in F$ may be set up to produce at least one product; the cost of setting up the factory to produce p_l is $f_i^l, i\in F, 1\le l\le k$; the cost of transportation between any two locations $i,j\in F\cup D$ is c_{ij}. Define the variable

$$x_{ij}^l=\begin{cases}1 & \textit{if factory i provides customer j with product } p_l,\\ 0 & \textit{otherwise,}\end{cases}$$

$$y_i^l=\begin{cases}1 & \textit{if facility i is set up to supply product } p_l,\\ 0 & \textit{otherwise,}\end{cases}$$

where, $\forall i\in F$, $j\in D$, $l\in\{1, 2, \cdots k\}$.

This problem can be stated as the following integer program

$$(P_1)\quad \min \sum_{l=1}^{k}\sum_{i\in F} f_i^l y_i^l+\sum_{j\in D}\sum_{i\in F}\sum_{l\in P_j} c_{ij}x_{ij}^l \;; \quad (2-1)$$

$$\text{s.t.}\quad \sum_{i\in F} x_{ij}^l=1, \forall j\in D,\ l\in P_j; \quad (2-2)$$

$$x_{ij}^l\le y_i^l, \forall \mathrm{i}\in F, \mathrm{j}\in D, \mathrm{l}\in P_j \;; \quad (2-3)$$

$$\sum_{l=1}^{k} y_i^l=1, \ \forall \mathrm{i}\in F; \quad (2-4)$$

$$\mathrm{x}_{ij}^l, y_i^l\in\{0,1\}, \ \forall \mathrm{i}\in F, j\in D, \ l\in P_j \;; \quad (2-5)$$

The objective function (2-1) is the minimum value of the total transportation cost and the total cost of setting up the factories, the equality constraint (2-2) ensure that the demand of each customer is satisfied and each product of each customer can be supplied by one factory; if customer *j* gets the product from factory *i*, the inequality constraint (2-3) ensures the establishment of factory *i*; the constraint (2-4) ensures that each established factory can supply only one product.

Throughout the text, unless otherwise stated, it is assumed that the following condition holds:

1) $f_i^l\ge 0, \forall i\in F, l=1.2\cdots k$;
2) $c_{ij}\ge 0$,for each $i,j\in F\cup D$
3) $c_{ij}=c_{ji}$,for each $i,j\in F\cup D$
4) $c_{ik}\le c_{ij}+c_{kj}$,for each $i,j,k\in F\cup D$
5) $c_{i_1i_2}\le \min\{c_{i_1j},c_{i_2j}\}$。 $\forall i_1,i_2\in F, \forall j\in D$;

1) -4) mean that the ship costs are in metric space and condition. 5) represents that the factory set is relatively concentrated.

If do not consider the setting cost, that is $f_i^l=0$ $(i\in F, 1\le l\le k)$. Thus, a linear programming formula with zero setting cost can be seen as follows:

$$(P_2)\quad \min \sum_{j\in D}\sum_{i\in F}\sum_{l\in P_j} c_{ij}x_{ij}^l \;; \quad (2-6)$$

$$\text{s.t.}\quad \sum_{i\in F} x_{ij}^l=1, \quad \forall j\in D,\ l\in P_j; \quad (2-7)$$

$$x_{ij}^l\le y_i^l\ , \quad \forall i\in F,\ j\in D,\ l\in P_j; \quad (2-8)$$

$$\sum_{l=1}^{k} y_i^l=1, \quad \forall i\in F\ ; \quad (2-9)$$

$$x_{ij}^l, y_i^l\in\{0,1\}, \ \forall i\in F,\ j\in D,\ l\in P_j\ ; \quad (2-10)$$

Obviously, the orderly division of the factory set is one-to-one corresponding to the feasible solution of the (P2) factory, that is if and only if $y_i^l=1$ is $i\in S_l$. If $k=1$(P2) is simple, while when $k=2$, the structure of the problem becomes complex. Since the max-cut problem[12] is NP-complete, we can prove that (P2) is NP-complete by proving that $k=2$,and the (P2) problem is equivalent to the max-cut problem.

Theorem 1: Without considering the setting cost, ie: $f_i^l=0, \ \forall i\in F, l=1, 2, \cdots, k$, when $k=2$, the problem (P2) is NP-complete.

APPROXIMATION ALGORITHM

In order to further study the location problem of k kinds of products with different customer demands, firstly, the problem (P2) should be transformed. Suppose that all P kinds of products needed by each customer $j\in D$ are supplied by the factory set $|P_j|$. Considering $|P_j|=1, j\in D$ this special situation in model (P2), that is, each customer only needs one kind of product, but also different customer can demand for different products, so we divide the customer set into $D = D_1 \cup D_2 \cup \cdots \cup D_k$ so that all customers in D_l only need one kind of product p_l, $l = 1, 2, \cdots, k$, the special situation can be described by the following model (P3):

$$(P_3)\ \min \sum_{l=1}^{k} \sum_{i \in F} \sum_{j \in D_l} c_{ij} x_{ij}^l \ ; \qquad (3-1)$$

$$s.t. \quad \sum_{i \in F} x_{ij}^l = 1,\ \forall j \in D_l,\ l \in \{1, 2, \cdots, k\}; \qquad (3-2)$$

$$x_{ij}^l \le y_i^l,\ \forall i \in F,\ j \in D_l,\ l \in \{1, 2, \cdots, k\}; \quad (3-3)$$

$$\sum_{l=1}^{k} y_i^l = 1,\ i \in F; \qquad (3-4)$$

$$x_{ij}^l, y_i^l \in \{0,1\},\ \forall i \in F,\ j \in D_l,\ l \in \{1, 2, \cdots, k\}; \quad (3-5)$$

In fact, if $j \in D$,for any situation of the model (P2), such as $|P_j| \ne 1$, $|P_j|$ replica of customer j can be substituted for demand product set of customer j ,so that each replica of the set P_j requires only a different kind of product, and the model (P2) can transforms equivalently into the model (P3). Therefore, Thus, the study of question (P3) leads to the corresponding conclusion of question (P2), and furthermore, if we remove the constraint that each factory is limited to producing only one kind of product, and each customer chooses the closest factory to provide its product, then the optimal solution of linear integer programming is the overlapping optimal solution of problem (P2).

$$(P_4)\ \min \sum_{l=1}^{k} \sum_{i \in F} \sum_{j \in D_l} c_{ij} x_{ij}^l \ ;$$

$$\text{s.t.} \quad \sum_{i \in F} x_{ij}^l = 1,\ \forall j \in D_l,\ l \in \{1, 2, \cdots, k\};$$

$$x_{ij}^l \in \{0,1\},\ \forall i \in F,\ j \in D_l,\ l \in \{1, 2, \cdots, k\};$$

Algorithm A:

Step 1: Each customer chooses the closest factory to provide product for them; we can get an overlapping optimal solution, let $S_1 = S_2 = \cdots = S_k = \Phi$

Step 2: As for $\forall i \in F$ if i doesn't provide any product in the overlapping optimal solution, then choose a S_l randomly and put i in it, otherwise let $B_i^l = \{ j \in D_j \mid j \text{ is supplied by in the overlap optimal solution}\}$,

$A_i^l = \sum_{j \in B_i^l} c_{ij}$, $A_i^s = \max\{A_i^l \mid l = 1, 2, \cdots, k\}$.

If $B_i^l = \Phi$, then $A_i^l = 0$. Set : $S_s := S_s \cup \{i\}, S_l := S_l, l \ne s, F := F \setminus \{i\}$, until $F = \Phi$.

Step 3: Factories in set S_l provide p_l, and for each customer $j \in D$, chooses the nearest factory in S_l as the supplier, $l = 1, 2, \cdots, k$.

Theorem 2: Algorithm A produces an integer feasible solution (x, y) of (P2), and the transportation cost of (x, y) is not more than $2 - 1/k$ times of the optimal solution .

Proof: As for $\forall j \in D_l, l = 1, 2, \cdots, k$, let $i_l(j)$ represents the nearest factory in S_l which provides product p_l to customer j; as for $\forall i \in F$,we denote that $B_i = \bigcup_{l=1}^{k} B_i^l$ represents the set of customers whose facroty i is selected as the supplier in the overlapping optimal solution. Now, we consider that the total cost A_i rising from the customer set B_i after the algorithm A ends. In the second step of the algorithm, it is not general, denoting $s = 1$, thus there are:

$$A_i = \sum_{l=1}^{k} \sum_{j \in B_i^l} c_{i_l(j)j} = A_i^1 + \sum_{l=2}^{k} \sum_{j \in B_i^l} c_{i_l(j)j}$$

$$\le A_i^1 + \sum_{l=2}^{k} \sum_{j \in B_i^l} (c_{i_l(j)i} + c_{ij}) \le A_i^1 + \sum_{l=2}^{k} \sum_{j \in B_i^l} (c_{ij} + c_{ij})$$

$$= A_i^1 + 2\sum_{l=2}^{k} A_i^l \le (2 - \frac{1}{k}) \sum_{l=1}^{k} A_i^l .$$

The second inequality is established above because the upper bound of the transportation costs between the two factories is no bigger than the lower bound of the transportation costs of between the two factories and any customer, and the last inequality is established because $A_i^s = \max\{A_i^l \mid l = 1, 2, \cdots, k\}$.

CONCLUSION AND PROSPECTS

Under the assumption that the service costs satisfy the metric space, this article analyzes non-capacity-limited location problem of multi-products with different requirements of customers, discusses the situation that the setting costs is zero, and establishes an approximate algorithm with the worst performance ratio of $2 - 1/k$. Further works will be made in the following two aspects: on the one hand, how to discuss the problem model under the situation that the factory supply of products with capacity constraints. On the other hand, under the assumption of the general metric space characteristic, we give the algorithm of k-independent constant performance ratio.

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