

An Objective Study and Modelling of Combustion Operation of Bush-Type Fuel Material

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DOI: [10.36347/sjet.2020.v08i06.001](https://doi.org/10.36347/sjet.2020.v08i06.001)

| Received: 20.05.2020 | Accepted: 02.06.2020 | Published: 16.06.2020

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Abstract

Original Research Article

Bush fire modeling is a subset of the wild land fire operations. The need for its study remains imperative because of the positive and negative effects that encompass the act. This study is focused on the modeling and validation of combustion operation of a bush consisting majorly of cheat grass type. A finite bush model of length 70.8m having varying widths and heights according to the weights of the bush segments was set-up. The bush model was segmented at 60cm equal interval along the 70.8m length of the bush. The weight of each segment contained inside a vessel was determined with a spring balance after which the weight of the vessel was subtracted from the total measured weight. The set-up was set on fire. The time of fire-travel from one segment to another was noted with the aid of a stop watch. The values of the considered fire-travel time, distances, weights and fire propagation velocity were tabulated for 120 simulations/runs. Multiple regression analysis tool was employed in fitting a mathematical model for estimating any of the considered fire variables- velocity, distance, weight of fuel material and flow time. The developed model was validated using statistical tools such as: P-value, R-sq. value, mean bias error (MBE), root mean square error (RMSE), mean percentage error (MPE), the coefficient of correlation (r), Nash-Sutcliffe model (NSE), and t-statistic test. Sensitivity test was equally carried out using SIMLAB software version 2.2 to decipher the input variable with the most influence on the fire travel velocity using Extended Fourier Amplitude Sensitivity Test (FAST) approach. 10,000 sample elements of each independent variable- distance, flow time and weight of fuel material were randomly generated with their respective mean and standard deviation values normally distributed. The experimental results reveal a fluctuation in the rate of fire propagation along the segmented lengths of the bush and variations in the propagation time. These observations are attributed to some causative factors of wind speed variations, weight difference along the bush model segments, flame temperature variation, compatibility of the grass body and convection and radiation effects which were not under experimental control. The sum of the effects of these factors culminated to these observations. The developed mathematical model is suitable for estimating any of the considered fire variables as endorsed by the statistical tests conducted. From the performed sensitivity test, the weight of the finite grass model had the most influence on the fire propagation velocity. This was followed by the segmented distance of the grass model and lastly the flow duration. Weight is a function of density of the grass model and as such, the density is inversely related to the flame temperature. Therefore, the higher the fuel density, the lower the flame temperature and vice versa. This explains why weight of the grass model has the greatest effect on the fire travel velocity. This study will be of great help to fire fighters and educationists as it provides elucidated ideas on fire behaviors and operations.

Keywords: Combustion, modelling, statistical tests, sensitivity test.

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INTRODUCTION

Bush burning is the indiscriminate setting ablaze of vegetation cover and burning down of grassland and forest resources by fire. Bushfire is a class of the wildfire or wildland fire which occurs either

in the urban or rural areas where combustible vegetation is available. Depending on the type of vegetation where fire occurs, a wild fire can also be classified more specifically as a bush fire, desert fire, forest fire, grass fire, hill fire, peat fire and vegetable fire.



Fig-1: A burning bush

The bushfire and other classes of wildfires can be characterized in terms of the cause of ignition, their physical properties, the combustible materials present and the effect of weather on the fire. These factors solely determine the flame propagation temperature and the rate at which the fire spreads from one distance to another. Bush fire and other wildfire classes can be caused by: lightning, spark from rockfalls, spontaneous combustion, volcanic eruption and human factors.

The ignition time, ignition temperature, moisture content, convection, radiation effects and wind affect the velocity of flame propagation during burning operation. The ignition time is defined as the time elapsed between fuel samples exposure to elevated temperatures and ignition. Ignition temperature is the fuel surface temperature when ignition occurs. Ignition is basically explained as the onset of a sustained, visible flame which occurs when molecules in the solid breakdown, enter the gas phase, mix with air and react. The processes involved during ignition are difficult to measure; hence, the ignition time and temperature are employed to determine the micro-structural operations of the fuel. The lower the ignition time and temperature, the faster the rate of flame propagation. The rate of flame propagation is reduced owing to the increase in the ignition time and temperature caused by moisture presence. According to Anderson [1], radiant heat flux contributes to flame propagation rate by about 40%. The energy is also transferred from one point of the grass to another through convection. Engstrom *et al.*, [2], showed experimentally that flaming ignition temperature can occur by convective heating without direct flame contact. Some other works has shown that convection contributes significantly to intermittent fuel pre-heating and downward fire spread [3]. Also, Vogel and Williams [4] explained that fire spread depends strongly on direct flame contact with un-burned fuel. This sums up to imply that there is yet an agreement on the fire spread modes [5].

A systemic control of the propagation/spreading of the flame is absolutely consequential in order to avoid damage to lives and properties. The control can be actualized through the use of models in predicting fire behaviors and effects. Sequel to this

concern, Ludwig *et al.*, [6], established an allometric solid fuel model which can be used to predict the general fuel properties, such as fuel loading, canopy height, relative amounts of live and dead fuels and biomass by size class. This model is disadvantaged by the limited nature of its applicability. A remote sensing device has been developed and improved on for use to enhance the modeling concept of individual plants and large areas. Also, a three-dimensional placement model that captures the natural structure of plants and the resulting local fuel-density fluctuations have been developed by researchers. Research has shown fuel bulk density to be an important variable in fire propagation [7]. Parson *et al.*, [8], illustrated the need for accurate three dimensional fuel characterization and proved that for the same mass and volume, fire spread behaves very differently between fuel beds with homogenous fuel density and those with variable fuel density. Studies have found that cutting dead fuel from the shrub canopy and placing it on the ground significantly reduced fire intensity, and thus concluded that canopy structure, not just fuel load, affects fire behavior. Weise and Wright [9] cited several other studies which indicate the importance of fuel arrangement. In addition, fuel element property model was equally established by researchers. This model describes the physical, chemical and shape properties of individual leaves or small branch segments. Chemical properties have received considerable attention, and include properties like heat capacity, thermal conductivity, and heat of combustion as well as chemical composition measurements like volatile contents, ash content, structural carbohydrates and ether extractives. Physical and shape properties have received less attention than chemical properties. The size, shape and orientation of fine fuel affect burning behaviors [10]. Fire behavior models have been developed by researchers in view of understanding the mechanism of fire spreading action. These includes: statistical, empirical, physical and simulation models. A statistical model is solely for test fires and contains no explicit physical information. The model is of two forms- those designed for specific fuels under specific conditions and those for all fuel species under various ranges of conditions. The first kind are often very accurate for the conditions and fuels specified, but provide little information outside those

conditions. The second kind provides ballpark information for a large number of fires, but aren't accurate enough to provide detailed information. The empirical model takes into account the fire variables such as wind, slope, fuel type and moisture content in predicting the rate of fire spread [10]. These models are essentially point-source models, where energy released by one fuel element is transferred to a neighbouring fuel element, thereby initiating the combustion sequence for that fuel element [11, 12 & 13]. Fon [14] was the first to model mathematically fire spread. The parameters he considered are the successive distances between fuel particles, particle ignition and ignition time. As a result of the shortcomings in the Fon's model, Rothermel [11] used the same premise as Fon's in defining how fire spread occurs but included much detail when he developed a model based on the data from Fraudsen [15]. Rothermel introduced a heat of ignition parameter that defines how much energy must be absorbed by a particle to raise the surface temperature to its measured ignition temperature, assuming water of vaporization occurs at 100°C. Rothermel's formulation forms the basis for most fire spread models developed in the last 40 years. Examples of these models used in the USA include: BEHAVE [11], FIRECAST and HFIRE [16]. One thing that differentiated Rothermel's model from others is the use of field measurement inputs regarding fuel type, fuel density, wind speed and others. However, Rothermel's model assumes homogeneous, continuous fuel that is contiguous to the ground, such as pine needle litter or grass, and ignores the effect of moisture content within the fuel except in delaying temperature rise while water evaporation occurs. Several models have been developed since Rothermel completed his model. Albini [12, 17] developed models that account for radiative pre-heating, pre-cooling and convective pre-cooling respectively and other models. The physical models are based principally on fundamental physics and chemistry principles [18]. This model provides the most useful and detailed information about fire spread behaviors. The simulation models work by taking a statistical or empirical model (usually one dimensional), generalize it to a two dimensional form, and provide an algorithm for fire spread on a landscape with inputs about the details of the landscape [19]. This model includes most basic geographical information of the experimental perimeter. The operational fire spread models that are said to be statistical or empirical (i.e., FARSITE) are actually a combination of a simulation model that propagates fire in a 2D space and 1D (usually) statistical or empirical fire spread model based on experimental data.

The fire travel velocity during bush burning operation employing the determinant factors of weight of the grass, segmented fire travel distance and time of flame propagation need to be studied and modeled. Multiple regression analysis was employed in determining the functional relationship between the response variable- fire travel velocity and the

independent variables- flow time, weight and distance. The developed mathematical model was further validated statistically using tools like: mean bias error (MBE), root mean square error (RMSE), mean percentage error (MPE), the coefficient of correlation (r), Nash-Sutcliffe model (NSE), and t-statistic test. Sensitivity test was equally carried out to decipher the input variable with the most influence on the fire travel velocity. Hence, the basis of this research paper. It would help in understanding fire propagative behavior and control scheme using the developed model.

MATERIALS AND METHOD

The fire variables: fire travel velocity, flow time distance, successive distance of the grass and weight of grass sample were determined under 120 fire simulations. A dry bush which consists majorly of cheat grass type of length 70.8m was developed. The weights of the grass model were measured with a beam balance and various values were noted. The width and height of the model from 0m-0.60m were determined to be 0.4m and 0.25m respectively. Also, the width and height of the model from 0.60m-70.8m length were measured to be 0.55m and 0.35m. The experiment was performed during sun temperature of about 32°C as indicated by the thermometer in the dry season. The elemental bush was segmented into 60cm equal divisions uniformly across the 70.8m length of the entire developed bush model. These divisions were marked at each interval with the aid of a stick so as to determine the propagation rate of the fire along the elemental bush sections. The weight of each section was noted. The pressure at the start of the experiment remained at atmospheric degree (1atm). According to Guido C [20], the temperature at which cheat grass type burns and also propagates along certain marked distances is within the range of 200-300°C. The time it took the fire to move from one marked distance say 0m to 0.6m was measured with a stop watch. Other distances and fire travel-time were also noted and recorded. The model elemental bush section is shown in Figure-2.



Fig-2: Elemental bush section of varying weights (considered bush model)

Statistical Model Validation Tools

The most popular statistical tools are the mean bias error (MBE) and the root mean square error (RMSE). In this study, to evaluate the accuracy of the estimated data using the developed model for estimating fire travel velocity, the following statistical tests were used, MBE, RMSE, mean percentage error (MPE) and coefficient of correlation (r), to test the linear relationship between predicted and measured values. For better data modelling, these statistics should be closer to zero, but coefficient of correlation, r, should approach to 1 as closely as possible. The Nash-Sutcliffe equation (NSE) is also selected as an evaluation criterion. A model is efficient when NSE is closer to 1. However, these estimated errors provide reasonable criteria to compare models but do not objectively indicate whether a model’s estimate are statistically significant. The t-statistics allows models to be compared and at the same time it indicates whether or not a model’s estimate is statistically significant at a particular confidence level, so, t-statistics test of the model was carried out to determine statistical significance of the predicted values by the model. The validation tools are discussed thus:

The Mean Bias Error

This is determined using the relation:

$$MBE = \frac{1}{n} \sum_{i=1}^n (H_{ical} - H_{imeas}) \dots\dots\dots(1)$$

This test provides information on long-term performance. A low MBE value is desired. A negative value gives the average amount of underestimation in the calculated value. So, one drawback of these two mentioned tests is that overestimation of an individual observation will cancel underestimation in a separate observation.

The Mean Percentage Error

$$MPE(\%) = \frac{1}{n} \sum_{i=1}^n \left(\frac{H_{ical} - H_{imeas}}{H_{imeas}} \right) \times 100 \dots\dots\dots(2)$$

This is the relation used in the computation of MPE. A percentage error between -10% and +10% is considered acceptable.

The Root Mean Square Error

The value of RMSE is always positive, representing zero in the ideal case. The normalized root mean square error gives information on the short term performance of the correlation by allowing a term by term comparison of the actual deviation between the predicted and measured values. The smaller the value, the better is the model’s performance.

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^n (H_{ical} - H_{imeas})^2 \right]^{\frac{1}{2}} \dots\dots\dots(3)$$

Where H_{imeas} , H_{ical} and n are, respectively, the i^{th} measured values and i^{th} calculated values of fire travel velocity and the number of data points.

The Nash-Sutcliffe Equation

According to Chen *et al.*, [21], a model is more efficient only when NSE is closer to one.

$$NSE = 1 - \frac{\sum_{i=1}^n (H_{imeas} - H_{ical})^2}{\sum_{i=1}^n (H_{imeas} - \bar{H}_{mea})^2} \dots\dots\dots(4)$$

\bar{H}_{mea} = the mean measured fire travel velocity

The Coefficient of Correlation (COC)

The coefficient of correlation, r was used to determine the linear relationship between the measured and estimated values of global solar radiation. The correlation can be positive, negative, linear, perfect, positively perfect and negatively perfect. The approaches which can be applied in correlation determination of two variables are the: Karl Pearson’s coefficient of correlation, Spearman’s rank difference method, concurrent deviations method, two-way frequency table method and the scatter or dot diagrams. The Karl Pearson’s method was employed in this work and the relation is given as:

$$r = \frac{\sum XY}{\sqrt{(\sum X^2)(\sum Y^2)}} \dots\dots\dots(5)$$

Where:

X = the difference bewteen the measured fire travel velocity and the mean of the measured fire travel velocity

Y = the difference bewteen the estimated fire travel velocity and the mean of the estimated fire travel velocity.

t-Statistic Test

As defined by Bevington [22] in one of the tests for mean values, the random variable t with (n-1) degrees of freedom may be mathematically defined as:

$$t = \left[\frac{(n-1)(MBE)^2}{(RMSE)^2 - (MBE)^2} \right]^{\frac{1}{2}} \dots\dots\dots(6)$$

The smaller the value of t the better is the performance. To determine whether the developed model’s estimates are statistically significant, the t-statistics was done using a confidence interval (CI) of 95%.

Sensitivity Test of the Developed Model

SIMLAB software version 2.2 was used to conduct a sensitivity test of the model for estimating the fire travel velocity in quest of knowing the model variable with most significant effect on the velocity of fire propagation, thereby contributing highly in the uncertainty of the model’s estimation performance. This software allows the user to specify the distributions for each input factor of a model and generate a sample of elements of a given size N from a distribution set up. The sample generation can be made using a variety of methods, i.e., random sampling, quasi-random sampling, replicated Latin Hypercube, classic and extended FAST (Fourier Amplitude Sensitivity Test) and the Morris design. The sensitivity test of the model

employed a normal distribution of the input factors/regressors of the model using their respective mean and standard deviation values. Fourier Amplitude Sensitivity test approach was used and 10,000 simulations was done for sample elements generation using the extended FAST technique at element truncation range of 0.1-99.9% percentile. The main and total effects of the model's input variables on the response variable were studied.

RESULTS AND DISCUSSIONS

The developed model for estimating the fire propagation velocity is shown in equation (7) and the

results of the application of the validation tools were equally discussed.

$$V = 0.0419 - 0.000241T + 0.00880W - 0.0143S \dots\dots (7)$$

Where:

- V = fire travel velocity (m/s)
- T = flow duration (secs)
- W = weight the grass volume (N)
- S = Segmented distance of the bush model (m)

The analysis of variance (ANOVA) of the model is shown in table 1 and the result of other validation tools were shown in Table-2.

Table-1: Analysis of variance (ANOVA) for the model

Source	DF	SS	MS	F	P
Regression	3	0.0062980	0.0020993	29.01	0.000
Residual error	116	0.0083945	0.0000724		
Total	119	0.0146925			

$$S = 0.00850684 \text{ R-Sq.} = 42.9\% \text{ R-Sq.(adj)} = 41.4\%$$

Table-2: Model Validation

	MBE	MPE (%)	RMSE	NSE	COC	T.stat. test
Model 1	3.33E-09	4.6988	0.008364	0.4287	0.6547	4.343E-06

From Table-1, the P-value of the model is 0.000 attesting that the model is statistically significant and can be employed for estimating fire travel velocity or any of the considered variable in the model. Sequel to that, the S-value is small implying also the suitability of the model and equally endorsing its estimation performance to be good. The R-Sq. value of 42.9% is fairly close to one but it is a good value. To further justify the estimation performance of the model, other statistical tools were used and their results are presented in table 2. It is evident from the table that the model passed all the applied statistical tools. The MBE value is very small, MPE value falls within -10% to 10%, RSME value is equally very small, NSE value is okay,

though it is not very much closer to one, the coefficient of performance (COC) attests that the measured and estimated values of fire travel velocity are positively correlated and the value is almost 0.7. The t-statistics test result is equally very small implying the estimation performance of the developed model to be good.

Graphical charts

The behavior of the fire propagation phenomena across the finite bush element under the considered variables of fire travel velocity, flow duration, distance and weight of the bush segment is vividly elucidated in Figures 3, 4 and 5.

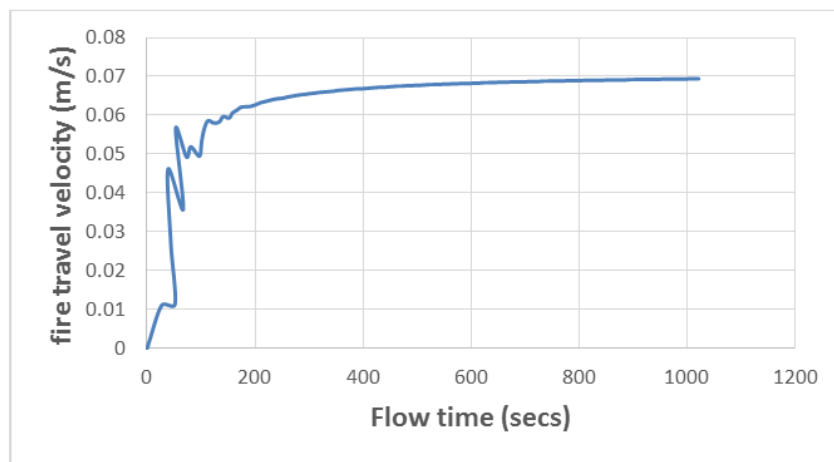


Fig-3: Fire travel velocity against flow time

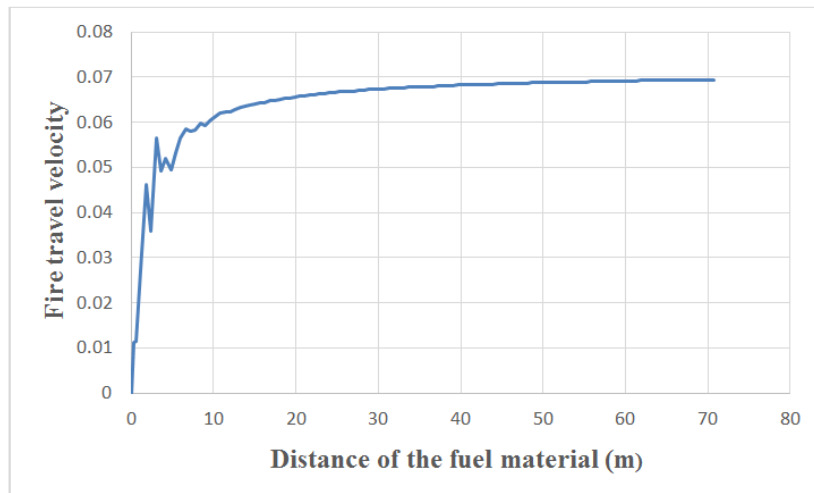


Fig-4: Fire travel velocity against distance of fuel material

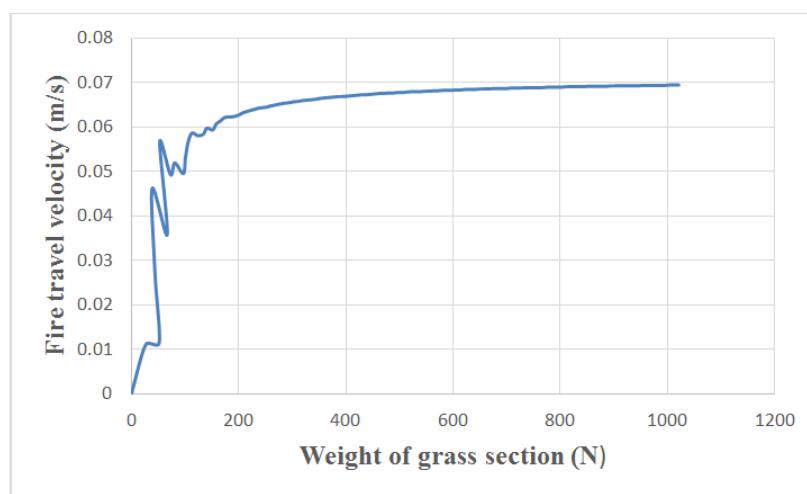


Fig-5: Fire travel velocity against weight of grass section

From Figures 3, 4 and 5, it is evident that each of considered fire variables- flow time, travel distance and weight has adverse effect on the fire propagation velocity. At the start of fire flow regime to a velocity point of about 0.065, the velocity of the flame fluctuates together with variables-time, distance and weight respectively. This fluctuation was noted to be because the experiment was conducted in an uncontrolled space, which thus allowed other external factors (not excluding the fuel factor) to influence the experimental process. These factors are: wind speed, varying weights of the finite bush segments, compatibility of the grass, convection effects, radiation effects and flame temperature. The high wind speed tends to increase the fire travelling rates and consequently reduce the spread time. The experiment was performed during the dry season when the wind velocity is high and unsteady and such contributed to the alternating values seen in the graphs. Also, as different weights of the grass were considered at each section of the finite model, the higher the grass weight, the higher the spread time and the propagation velocity. In addition, the compatibility of the grass model differs and such affects the energy

transfer rate from one body to another. The uniformity of the fire is also affected by low compatibility of the grass model. Furthermore, the energy transfer rate is either increased or decreased depending on the compatibility of the grass model. Sequel to that, the higher the flame temperature, the higher the velocity of the fire and the lower the spread time. The temperature at each section would differ owing to the varying weights of the grass sections. But at some point a bit above 0.065, the fire simulation across the finite grass section begins to flow uniformly (though a little bit wavy) in that the fire travel velocity increases proportionally with the flow time, distance and weight. This was because the momentum of the fire was large enough to cause a negligible effect of the mentioned influencing factors on its flow/travel process.

Sensitivity Test Result

To further determine which of the considered fire input variables has the most influence on the fire travel velocity and also contribute to the uncertainty of the model's outputs, Figures 6 and 7 illustrates this effect.

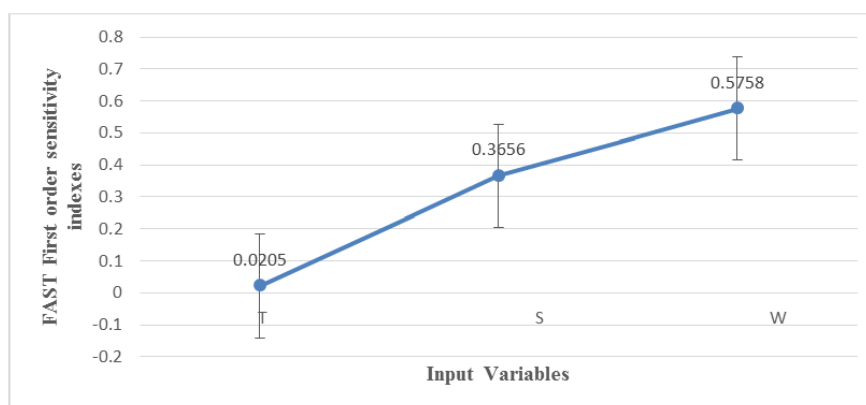


Fig-6: FAST first order sensitivity indexes of the model's input variables on the fire travel velocity

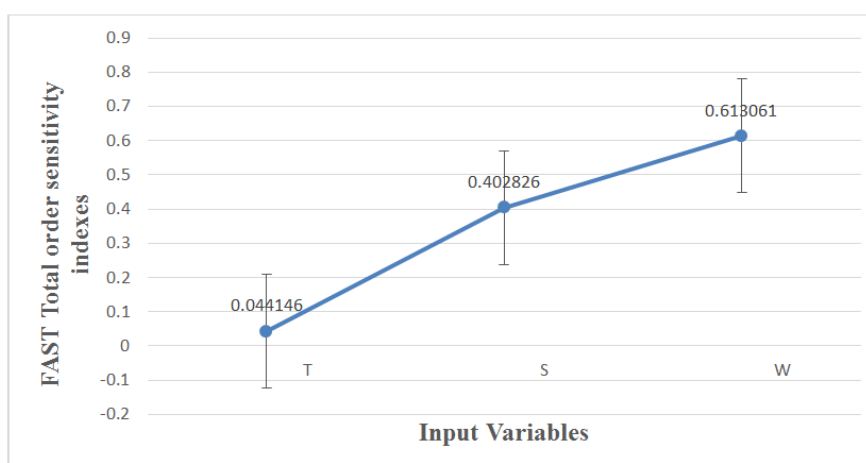


Fig-7: FAST Total order sensitivity indexes of the model's input variables on the fire travel velocity

Figures 6 and 7 both attest that the weight of the finite grass model has the most influence on the fire propagation velocity. This is followed by the segmented distance of the grass model and lastly the flow duration. Weight is a function of density of the grass model and as such, the density is inversely related to the flame temperature. Therefore, the higher the fuel density, the lower the flame temperature and vice versa. This explains why weight of the grass model has the greatest effect on the fire travel velocity.

CONCLUSION

Bush burning is indeed an activity being performed virtually everywhere especially in the African continent. The rate at which the set fire travels is solely affected by wind velocity, flame temperature, grass compatibility, convection and radiation effects, weight of the grass and moisture content. The energy transfer function is of great value if these factors are of high values. This study which led to the development of a mathematical model that governs fire behavior operation had thus proven that each of the fire variables- flow time, distance of fuel element from the other and the weight of the finite grass model has a significant level of effect on the fire propagation velocity, with the most effect from the weight of grasses, followed by the separating distances. The mathematical model developed is suitable for predicting

any of the fire variables considered in the model as endorsed by all the validation tools employed in testing its reliability. This model will be a helpful tool to fire fighters and all whose job are related to the study.

REFERENCES

1. Anderson HE. Heat transfer and fire Spread. Research Paper INT-69, Intermountain Forest and Range Experimental Station, Ogden, UT, USDA Forest Service, 1969.
2. Engstrom JD, Butler JK, Smith SG, Baxter LL, Fletcher TH, and Weise DR. Ignition behavior of live California Chaparral Leaves. Journal of Combustion Science and Technology, 2004; 176(9)1577-1591.
3. Finney MA. Fire growth using minimum travel time methods. Canadian Journal of Forest Research-Revue. 2002; 32(8), 1420-1424.
4. Vogel M, Williams FR. Flame propagation along matchstick arrays. Journal of Combustion Science and Technology. 1970; 1, 429-436.
5. Finney MA, Grenfell IC, McHugh CW, Seli RC, Trethewey D, Stratton RD, Brittain S. A method for ensemble wildland fire simulation. Environmental Modeling & Assessment. 2013; 16(2), 153-167.
6. Ludwig JA, Reynolds JF, Whitson PD. Size-biomass relationships of several chihuahuan desert

- shrubs. *American Midland Naturalist*. 1975; 94(2), 451-461.
7. Rothermel RC. A mathematical model for predicting fire spread in wildland fuels. Research Paper INT-115, Intermountain Forest and Range Experiment Station, Ogden, UT, USDA Forest Service, 1972.
 8. Parsons RA, Mell WE, McCauley P. Linking 3D spatial models of fuels and fire: effects of spatial heterogeneity on fire behavior. *Ecological Modelling*. 2011; 222(3), 679-691.
 9. Weise DR, Wright CS. Wild land fire emissions, carbon and climate: characterizing wildland fuels. *Forest Ecology and Management*. 2014; 317, 26-40.
 10. Lyons PRA, Weber RO. Geometrical effects on flame spread rate for wildland fine fuels. *Combustion Science and Technology*, 1993; 89(1-4), 153-165.
 11. Rothermel RC, Philpot CW. Predicting changes in chaparral flammability. *Journal of Forestry*. 1973; 71, 640-643.
 12. Albini, F. A. A model for fire spread in wildland fuels by radiation. *Journal of Combustion Science and Technology*. 1985; 42(5-6), 229-258.
 13. Catchpole WR, Catchpole EA, Butler BW, Rothermel RC, Morris GA, and Latham DJ. Rate of spread of free-burning fires in woody fuels in a wind tunnel. *Journal of Combustion Science and Technology*. 1998;131(1-6),1-37.
 14. Fon W. Analysis of fire spread in light forest fuels. *Journal of Agricultural Research*, 1946; 72, 93-121.
 15. Frandsen WH. Fire spread through porous fuels from the conservation of energy. *Combustion and Flame*. 1971; 16, 9-16.
 16. Peterson SH. Using HFIRE for spatial modeling of fire in shrublands. Research Paper PSW-RP 259, Pacific Southwest Research Station, Albany, CA, USDA Forest Service, 2009.
 17. Albini, F. A. Wildland fire spread by radiation - a model including fuel cooling by natural convection," *Combustion Science and Technology*. 1986;45(1-2), 101-113.
 18. Sullivan AL. Wildland surface fire spread modelling: Physical and Quasi- Physical Models. *International Journal of Wildland Fire*. 2009;18(4), 349-368.
 19. Sullivan AL. Wildland surface Fire spread modelling: Empirical and Quasi-Empirical Models. *International Journal of Wildland Fire*. 2009;18(4), 369-386.
 20. Ryan GB, Cliff WJ, Gabbiani G, Irle C, Montandon D, Statkov PR, Majno G. Myofibroblasts in human granulation tissue. *Human pathology*. 1974 Jan 1;5(1):55-67.
 21. Chen J, Wang W, Fang J, Varahramyan K. Variable-focusing microlens with microfluidic chip. *Journal of Micromechanics and Microengineering*. 2004 Mar 17;14(5):675.
 22. Bevington PR, Robinson DK. Data reduction and error analysis for the physical sciences McGraw-Hill. New York. 1969;1969:235-42.