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Applications of Kamal Transformation in Temperature Problems

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Abstract **Original Research Article**

Integral transformation plays important rule in science and engineering fields. Integral renovate method is expedient tool for particular calculations in mathematical and many other fields of sciences, as it transforms intricate problematical situations into a modest one. It is quiet easy to distinct the things of a stated function from the maternal function, after arrangement has been consigned. In this article, we are going to explained the effective behavior of Kamal's Transform in field of Mechanical, Chemical and many others type of engineering, which is applied to find solution of ordinary linear differential equations with coefficients of the variable value and its presentations in different era of engineering, Heat and temperature problems take place in chemical and many types of engineering are illustrated by a mathematical tool, called Kamal Transformation in this article. Descriptive properties and examples appear to show the efficiency of its suitability in solving differential equations.

Keywords: Kamal Transform, Time Derivative, Time Scaling, Differential Equations, Temperature Problem.

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1. INTRODUCTION

Integral transformation is important for calculations of mathematics, as it converts a difficult issue into a simple one. (Debnath & Bhatta, 2014, Necdet & Sinan, 2017) Transformations are straightforward to isolate the characteristics of particular parent functions. Following the integration has indeed been delegated. Finding a result to differential and integral solutions has become simple in recent research, thanks to the use of numerous integrated transformations. (Albukhuttar *et al.*, 2020) Integral transformations are transformations that are made up of two parts, with a lot of success used for over two centuries in there are several issues and branches in mathematics that may be found. These types of transformations are widely utilized in the field of knowledge and to solve a set of differential equations with an initial or initial condition and boundary conditions. (Belgacem *et al.*, 2017, Albukhuttar & Jaber, 2019) There are other integral transforms that may be used to solve differential equations, such as Laplace, Elzaki, Tameme, Sumudu, and others. (Kexue & Jigen, 2012, Mubarak *et al.*, 2021) When specific criteria are satisfied, such as when the initial values are zero, the Laplace Transform is used to discover simple solutions to partial differential equation of mixed partial derivatives more than two independent variables and differential equations. (Jost & Mulas, 2019) The differential equation is converted from the temporal framing into an algebraic expression inside the frequency. After the issue was resolved within the frequency, the results may be translated into the time - frequency domain, resulting in the ultimate solution of differential equation, where the Laplace equation can give a shorter approach for the resolution. It also gives you a way to manage the input system to getting out and as a result the foundation for control engineering. (Baeumer, 2003) The Laplace transform is a mathematical integral transform named after its originator, Pierre-Simon Laplace who developed the transform while working on probability theory. A function with a positive variable t is transformed into an imaginary frequency s using the Laplace transforms (Kufitting & Peter, 2003). The Laplace convolution theorem may be used to calculate the inverse transformation of a combination $H(s).J(s)$ where $H(s)$ and $J(s)$ are both identifiable as transformations of known functions.(Singh & Aggarwal, 2019) firstly developed the model of population growth rate equation and radioactive decay problem in form of ordinary linear differential equation and then solve it by Sawi and Laplace transform.

1.1 Temperature Problem in Differential Equation

It can be explained that the rate of change in temperature of a phenomenon or body is proportional to the change in temperature of the body and the medium where the body is placed. The differential equation to explain such phenomenon can be written as,

$$\frac{dT}{dt} = -\sigma(T - M),$$

With initial condition as

$$T(0) = T_0,$$

Where σ proportionality constant, T is the temperature of body and M is the temperature of body in any medium. T_0 Is the value of temperature at time t_0

2. METHODOLOGY

2.1. Def.

The Kamal transformation integral is defined as

$$K\{f(t)\} = \int_0^\infty e^{-\frac{t}{s}} f(t)dt = F(s). \quad (2.1)$$

The integral is said to be convergent if the limit of the above integral exits, (Abdelilah and Hassan, 2016).

2.2. Linear Property

Kamal transform pairing $K\{f_i(t)\} = F_i(s)$, $i = 1, 2, 3, ...n$; the following holds for any variables $i = 1, 2, 3, ...n$

$$K\{\beta_1 f_1(t) \pm \beta_2 f_2(t) \pm ..... \pm \beta_n f_n(t)\}$$

$$= \beta_1 K\{f_1(t)\} \pm \beta_2 K\{f_2(t)\} \pm .... \pm \beta_n K\{f_n(t)\}$$

$$= \beta_1 F_1(s) \pm \beta_2 F_2(s) \pm ..... \pm \beta_n F_n(s) \quad (2.2)$$

2.3. Time Scaling

According to the property

$$K\{f(t)\} = F(s)$$

$$K\{f(bt)\} = \frac{1}{b} F(sb),\ b > 0. \quad (2.3)$$

2.4. Shifting of Frequency

This characteristic is provided by

$$K\{f(t)\} = F(s) \Rightarrow K\{f(t)e^{\lambda t}\} = F\left(\frac{s}{1-\lambda s}\right). \quad (2.4)$$

The frequency shift is represented by λ.

2.5. Time Derivatives

The differential feature of the Kamal transformation is among the most significant properties in linear systems thinking. It aids in the transformation of differential equations with constant coefficients into polynomial solutions with complex coefficients. We may find the output signal in frequency response by solving these algebraic problems. We can retrieve the matching time domain output signal by obtaining the reverse of Kamal transformation. The property of time derivatives says.

$$K\left\{\frac{df(t)}{dt}\right\} = \frac{1}{s}F(s) - f(0)$$

$$K\left\{\frac{d^2 f(t)}{dt^2}\right\} = \frac{1}{s^2}F(s) - \frac{1}{s}f(0) - f^{(1)}(0) \cdots\cdots\cdots\cdots\cdots\cdots$$

$$K\left\{\frac{d^n f(t)}{dt^n}\right\} = \frac{1}{s^n}F(s) - \frac{1}{s^{n-1}}f(0) - \frac{1}{s^{n-2}}f^1(0) - ... - f^{(n-1)}(0) \quad (2.5).$$

2.6. Transformation for some functions

We are going to find Kamal transformation for some functions, like fix function, Polynomial's, trigonometric function and other functions, (Sundhanshu & Sharma, 2019).

Table-1: Transformation for some functions

Serial no.	Function g (*t*)	Transformed function G(*s*) and their convergence
1.	1	$s, \quad s > 0$
2.	e^{bt}	$s/1-sb, \quad s > 0$
3	$t^n, \quad n = 1,2,3,......$	$n!s^{n+1}, \quad s \ge 0$
4.	$t^p, \quad p > -1$	$\Gamma(p+1)s^{p+1}, \quad s \ge 0$
5.	$\sqrt{t}$	$2\sqrt{\pi}s^{\frac{3}{2}}$
6.	$t^n e^{bt},\ n = 1,2,3,...$	$s^{n+1}n!/(1-sb)^{n+1}$
7.	$sin(bt), \quad t \ge 0$	$s^2b/1+(sb)^2, \quad s > 0$
8.	$cos(bt), \quad t \ge 0$	$s/1+(sb)^2, \quad s > 0$
9.	$sinh(bt), \quad t \ge 0$	$s^2b/1-(sb)^2, \quad s > \lvert b\rvert$
10.	$cosh(bt), \quad t \ge 0$	$s/1-(sb)^2, \quad s > 0$

3. RESULTS

The differential equation to explain such phenomenon can be written as,

$$\frac{dT}{dt} = -\sigma(T - M)\,, \qquad (3.1)$$

with initial condition as

$$T(0) = T_0\,, \qquad (3.2)$$

Where σ proportionality constant, T is the temperature of body and M is the temperature of body in any medium. T_0 Is the value of temperature at time t_0.

Solution

As equation of temperature problem is given as,

$$\frac{dT}{dt} = -\sigma(T - M)$$

By applying the Kamal transform on (3.1)

$$K\left\{\frac{dT}{dt}\right\} = K\left\{-\sigma(T - M)\right\} \qquad (3.3)$$

By using (2.5),

$$\frac{1}{s}T(s) - t(0) = \left\{-\sigma T(s) - sM\right\} \qquad (3.4)$$

By using (3.2)

$$\frac{1}{s}T(s) - T_0 = \left\{-\sigma T(s) - sM\right\} \qquad (3.5)$$

$$\left(\frac{1}{s} + \sigma\right)T(s) = T_0 - sM \qquad (3.6)$$

$$T(s) = T_0\left\{\frac{s}{(1+\sigma s)}\right\} - \frac{s^2 M}{(1+\sigma s)} \qquad (3.7)$$

Taking inverse of transform

$$T(t) = T_0 e^{-\sigma t} + \frac{M}{\sigma}e^{-\sigma t} - \frac{M}{\sigma} \qquad (3.8)$$

$$T(t) = \left(T_0 + \frac{M}{\sigma}\right)e^{-\sigma t} - \frac{M}{\sigma} \qquad (3.9)$$

Eq. (3.9) provides the general solution of (3.2).

3.1. Example

A bar of metal placed at a temperature of $100^0 F$ in a room at constant temperature of $0^0 F$, if after 20 minutes the temperature of the bar is $50^0 F$ find

(a) The times required the bar to reach at $25^0 F$,

(b) The temperature of the bar after 10 minutes.

Solution:

As equation of temperature problem is given as,

$$\frac{dT}{dt} = -\sigma(T - M)$$

By using the result from (3.9)

$$T(t) = \left(T_0 + \frac{M}{\sigma}\right)e^{-\sigma t} - \frac{M}{\sigma}$$

As it is given that room temperature $M=0^0F$, eq. (3.9) becomes

$$T(t) = T_0 e^{-\sigma t}\,, \qquad (3.10)$$

By using the initial condition, we can get that

$$T_0 = 100$$

Eq. (3.10) becomes,

$$T(t) = 100e^{-\sigma t}\,, \qquad (3.11)$$

By using $T=50^0F$ at $t=20$,

K= 0.035

Eq. (3.11) becomes,

$$T(t) = 100e^{-(0.035)t} \qquad (3.12)$$

a) The times required the bar to reach at $25^0 F$.

From eq. (3.12),

$25 = 100e^{-0.035t}$, $t = 39.6$*min.*

b) The temperature of the bar after 10 minutes.

From eq. (3.12),

$$T = 100e^{-0.035(10)}\,,$$

$$T = 70.5^0 F\,.$$

Graphical explanation can be provided by below.

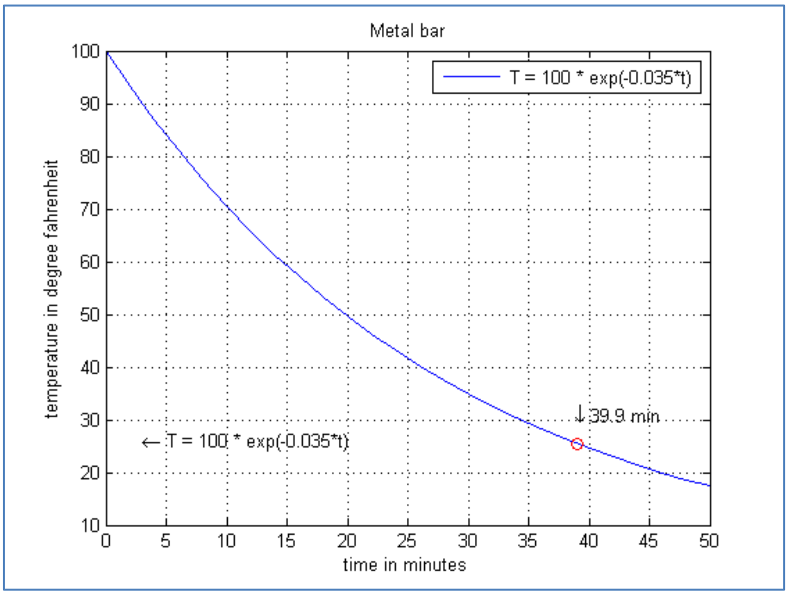

Fig-1: The above mention graph explains the situation.

4. CONCLUSION

In this article, we have seen that the first order linear differential equation plays vital role in temperature problems, and the Kamal transformation is important and a smart technique in solving temperature problem and able to describes the parameter involving in it. It can be seen that the temperature problems accrued in different fields of science and engineering, where Kamal transform can help as mathematical tool.

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